STRUCTURAL STEELWORK

FOR BUILDING AND ARCHITECTURAL STUDENTS

By the same authors

INTRODUCTION TO
STRUCTURAL MECHANICS
FOR BUILDING AND
ARCHITECTURAL STUDENTS

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By

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PREFACE TO SIXTH EDITION

FOR some time past various committees have been engaged on the preparation of 'recommendations' for the design of structures and the more economical use of constructional materials. For example, a committee of the Institution of Structural Engineers has been considering the question of the design of steel-framed buildings. The recommendations of such committees will be issued in reports with the probable title of 'Code of Practice.' It is likely that some new working stresses will be suggested and that in a few cases new calculation methods may be recommended. The 'Code of Practice' recommendations are not mandatory, but represent considered opinion on what should constitute good design. The recommendations have statutory effect if and when they become incorporated in local by-laws.

It is not probable that any major alterations will be made in the existing basic principles of steel-frame construction, but the reader is advised to watch carefully for any revision of local by-laws, such as those of the London County Council.

T. J. R.
L. E. K.

PREFACE TO THIRD EDITION

THE chapter on 'Welding' has been re-written in order to incorporate the regulations of the London County Council with respect to the use of metal arc welding.

By kind permission of the Institute of Welding some extracts from the 'Handbook for Welded Structural Steelwork' have been included. The authors wish to record their thanks to the Institute for this practical assistance.

A few of the working stresses in structural steelwork design have been raised for the period of the war emergency and these higher stresses may persist for a time after the emergency has ceased. The reader should consult B.S.S., etc., for any revision, temporary or permanent, which may take place in the permissible stress values.

T. J. R.
L. E. K.

PREFACE TO SECOND EDITION

THE L.C.C. By-laws affecting the design of steel-framed buildings came into force on January 1st, 1938. The By-laws supersede the regulations contained in the L.C.C. 'Code of Practice.'

The presentation of building regulations in the form of by-laws has not affected the general methods of calculation and design which have to be employed in structural steelwork construction. The authors have thoroughly revised the book, in order to use the phraseology of the L.C.C. By-laws, where appropriate.

It is desired to express thanks to the London County Council for permission to quote from the By-laws referred to, and also from the regulations dealing with the employment of metal arc welding. The thanks of the authors are also due to Messrs. Redpath, Brown & Co., Ltd., for their courtesy in permitting the use of certain diagrams from the new issue of their handbook.

T. J. R.
L. E. K.

PREFACE TO FIRST EDITION

DURING recent years a good deal of research work has been carried out with a view to improving the quality of the materials used in building construction and, in this connection, structural steel has received considerable attention. Improved quality in the materials of construction naturally leads to investigation into the possibility of their more economical use and, in the case of structural steelwork, much thought has been given to the question of better methods of design.

Building regulations have had to take into account the progress which has been made both in the quality of structural steel and in the knowledge of its most effective employment in steel-framed buildings.

As the regulations contained in the Building Acts of the London County Council are acknowledged throughout the country to represent a high standard of building construction, it will be useful to state the present position with regard to the regulations affecting steelwork construction in London.

In 1932 the Council issued a 'Code of Practice' for the use of structural steel and other materials in buildings, approved by the Council as a basis of consideration of applications for relief from the Third Schedule of the London Building Act, 1930. By the London Building Act (Amendment) Act, 1935 the Council has obtained powers to make byelaws with respect to a number of matters, including those affecting steelwork (and reinforced concrete) construction.

It is probable, therefore, that in course of time the 'Code' will lose its separate identity and its provisions become part of the matters dealt with by byelaws.

The L.C.C. 'Code of Practice' regulations and those contained in recent British Standard Specifications will be found at appropriate points in the text.*

* See Preface to Second Edition.

In writing this book the authors have endeavoured to show, in as simple a manner as possible, the relationship between the established principles of structural mechanics and modern methods of steelwork calculations—as exemplified in structures of not too difficult a character. It is written as a text-book for the student, whether he be a full-time student in a technical college or a young assistant in an office supplementing his practical experience by private, or part-time evening, study.

Throughout the book theoretical demonstration has been immediately followed by practical illustration in the form of a worked numerical example.

The mathematics employed has been of the simplest possible character consistent with effective demonstration and should present little difficulty to students in advanced building courses. On the few occasions in which theoretical investigation has involved the use of the methods of Calculus, the results have been clearly set out in simple language, so that their employment does not demand a knowledge of this branch of mathematics.

Students preparing for the examinations of the Institute of Builders, the Institution of Structural Engineers, and the Royal Institute of British Architects will, it is hoped, find the book of material assistance. Candidates for the Inter. R.I.B.A. examination should read Chapters I to VII, the part of Chapter IX dealing with maximum bending moments, and the more elementary portions of Chapter XI. For the final examination, architectural students will require to read Chapter XI more closely and also to take up those parts of the book which deal with practical design.

Building students preparing for the National Diploma or Certificate in Building should find the groundwork covered in the theory and design of structural steelwork.

The authors wish to acknowledge freely the many sources of the theoretical principles which, together with the results of practical experience, constitute the text of the book. The practical value of the book has been considerably enhanced by the assistance received from a number of well-

known constructional firms who have supplied diagrams, photographs, and other practical data. The authors' thanks are especially due to :

Messrs. Dorman, Long & Co., Ltd.; Messrs. Redpath, Brown & Co., Ltd.; Messrs. R. A. Skelton & Co. Steel & Engineering, Ltd.; Messrs. Dawnays, Ltd.; Messrs. The Kleine Company, Ltd.; Messrs. Caxton Floors, Ltd.; Messrs. The Quasi-Arc Company, Ltd.

The British Standards Institution kindly granted permission for extracts to be made from recent B.S.S. and this, together with the permission of the London County Council to quote from building regulations, notably the 'Code of Practice,' has made it possible for the authors to refer frequently to the practical considerations which influence purely theoretical results.*

Acknowledgment is also due to the British Steelwork Association for its courtesy in permitting the publication of certain property tables which will be found in the text, and to the Institution of Structural Engineers for permission to quote from the valuable report which it issued on the metallic arc welding of structural steelwork.

The authors wish to record their thanks to Mr. F. E. Drury, M.Sc., M.I.Struct.E., Principal of the L.C.C. School of Building, Brixton, for helpful advice given on this occasion, as on other occasions, whenever sought.

Finally it is desired to express appreciation of the interest taken by Dr. H. H. Burness in the preparation and production of the book.

1936.

T. J. R.
L. E. K.

* See Preface to Second Edition.

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CHAPTER I

STRESS, STRAIN AND ELASTICITY

Nature of Structural Steel

STEEL is not a simple element. It is mainly composed of iron, but the iron is alloyed, or associated with, various other materials. It is upon the nature and relative amounts of these special ingredients that the physical properties of the steel depend. For example, if the metal chromium be introduced into the composition, the resulting steel is able to exhibit, among other useful properties, a pronounced resistance to rusting and is given the name *stainless steel.* The element manganese, on the other hand, gives good wearing properties to steel, making it suitable for use in the manufacture of tram rails. There are, therefore, various types of steels, known respectively as *chromium steels, manganese steels,* and so on, according to the alloying elements which give the steels their characteristic properties.

A substance which plays an important part in the type of steel used in building construction is the element *carbon.* The percentage of carbon in steel directly influences its essential structural properties. An increase in carbon content results in an increase in strength, but this is accompanied by a marked decrease in ductility. Ductility, or absence of brittleness, is one of the important requisites of a structural steel. It promotes equalisation of load between the steel fibres of a member. Of such importance is this property of ductility that, in the commercial testing of structural steel, an upper limit of strength is prescribed for the steel in addition to a definite minimum value for the percentage elongation.

Steel which is to be used in general building construction is subject to a number of standardised requirements. The standards of quality required are laid down in specifications issued by the British Standards Institution. These specifica-

tions are known as **B.S.S. (British Standard Specification)** and the two relating to structural steel for general building work are **B.S.S. No. 15–1936 (Structural Steel)** and **B.S.S. No. 548–1934 (High Tensile Structural Steel)**. The former is concerned with the type of steel in common use at the present time and is referred to in such regulations as the L.C.C. By-laws (1938) (clause 15). The modern tendency is, however, to adopt steel of higher tensile strength, and for this steel the B.S.I. has prepared specification No. 548–1934.

Chromador Steel * is a specially manufactured steel which exhibits high tensile strength without lack of ductility. The comparative analyses given show that the metals chromium and copper are incorporated in the composition of the steel. The introduction of these elements leads also to an improved resistance to corrosion in the resultant steel.

'Mild' steel is steel conforming to the requirements of B.S.S. No. 15.

TYPICAL ANALYSES OF STRUCTURAL STEELS

PERCENTAGES

DESCRIPTION	CARBON	MANGANESE	CHROMIUM	COPPER	SILICON	SULPHUR	PHOSPHORUS	IRON
Mild Steel	0·2	0·5	—	—	0·04	0·04	0·04	99·18
Chromador Steel	0·3 Max.	0·7–1·0	0·7–1·1	0·25–0·5	0·2 Max.	0·05 Max.	0·05 Max.	% Difference

Stress

To understand the provisions of B.S.S. Nos. 15 and 548, and similar specifications, it is necessary to study the subject of *stress*.

Fig. 1 shows a tension member AB subjected to a pull of L tons at each end. Considering a typical section XX, we see that the load L tons, at the end A, is trying to detach the portion AX of the member from the portion XB. It is unable to do so, because of the numerous little pulls which the fibres of the material exert, and which are shown to the right

* A high tensile structural steel manufactured by Messrs. Dorman, Long & Co., Ltd.

of the section plane XX. Similarly the pull L tons at end B cannot effect separation at the section, because there are fibre pulls acting there, to the left. Such a system of actions and reactions, acting over the cross-sectional area of a member, constitute what is termed a *stress* at the section.

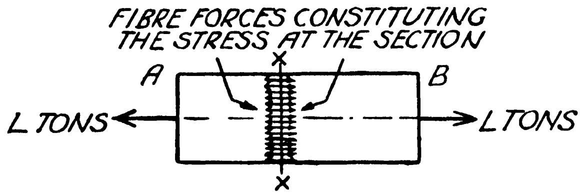


FIG. I.—DIAGRAM ILLUSTRATING THE NATURE OF STRESS.

If the cross-sectional area of the member at XX were A sq. ins., and if the load L tons were distributed uniformly over the section, the ratio L tons $\div$ A sq. ins. would give the *intensity of stress* at the section. Assuming the load to be 10 tons, and the sectional area to be 2 sq. ins., the *stress* at the section (the word *intensity* is always omitted in practice) would be $\frac{10 \text{ tons}}{2 \text{ sq. ins.}} = 5$ tons per sq. in.

Varying Stress.—The distribution of load over the section of a member may be of a non-uniform character. For example, in a beam section, not only is there variation of load value, but there is a complete reversal from *tension* (pulling) to *compression* (pushing). In such a case, the 'stress' cannot be obtained by the simple formula $\frac{\text{Load}}{\text{Area}}$: the stress will vary in value from point to point in the beam section. But, however its value may be obtained, a stress has always the nature of *load intensity per unit area.* When a phrase such as '*extreme fibre stress equals* 8 *tons per sq. in.*' is used, what is meant is simply that, were the fibres over a whole sq. in. to be subjected to this given stress value, the total load carried would be 8 tons. The structural designer does not think in terms of average stress values, but in terms of 'maximum stress intensity' in any single fibre.

Forms of Stress

There are three forms of simple stress: *tension, compression* and *shear.*

Tensile Stress.—This is the kind of stress induced in a member, when it is subjected to a pull.

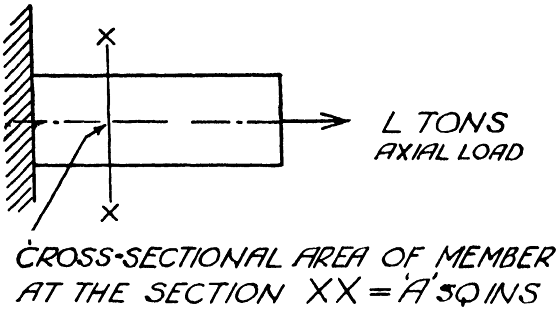


FIG. 2.—TENSILE STRESS.

$$\text{Tensile stress at section XX} = \frac{\text{Load}}{\text{Area}}$$

$$= \frac{\text{L tons}}{\text{A sq. ins.}} = \frac{\text{L}}{\text{A}} \text{ tons per sq. in.}$$

(Any convenient units of load and area may be used in stress calculations, but, as stress values in structural steelwork are nearly always expressed in *tons per sq. in.*, this unit has been adopted.)

ILLUSTRATIVE EXAMPLES

(1) *A solid circular steel tie-rod 2" diameter carries an axial load of 20 tons. Calculate the stress in the material of the rod.*

$$\text{Tensile stress} = \frac{\text{Load}}{\text{Area}}$$

$$\text{Sectional area of tie-rod} = \frac{\pi d^2}{4} = \frac{\pi \times 2^2}{4} \text{ sq. ins.}$$

$$= 3{\cdot}14 \text{ sq. ins.}$$

$$\text{Tensile stress} = \frac{20 \text{ tons}}{3{\cdot}14 \text{ sq. ins.}}$$

$$= 6{\cdot}37 \text{ tons per sq. in.}$$

(2) *Find the maximum safe axial load for a mild steel tie-bar, 2″ wide by ⅝″ thick, if the tensile stress is not to exceed 8 tons per sq. in.*

Sectional area of tie-bar = $2'' \times \frac{5}{8}'' = 1{\cdot}25$ sq. ins.

$$\text{Stress} = \frac{\text{Load}}{\text{Area}}$$

$$8 \text{ tons per sq. in.} = \frac{L \text{ tons}}{1{\cdot}25 \text{ sq. ins.}}$$

$$L = 8 \times 1{\cdot}25 \text{ tons} = 10 \text{ tons.}$$

(3) *Calculate the necessary thickness for a tie-bar 4″ wide, if it has to carry an axial load of 22·5 tons without the maximum stress exceeding 7·5 tons per sq. in.*

Sectional area of tie-bar = $4'' \times t'' = 4t$ sq. ins.

$$7{\cdot}5 \text{ tons per sq. in.} = \frac{22{\cdot}5 \text{ tons}}{4t \text{ sq. ins.}}$$

$$\therefore 4t \times 7{\cdot}5 = 22{\cdot}5.$$

$$t = \frac{22{\cdot}5}{30} = \frac{3}{4}''.$$

Compressive Stress.—Columns and *struts* (i.e. members which support thrusts) have their fibres in this condition of stress.

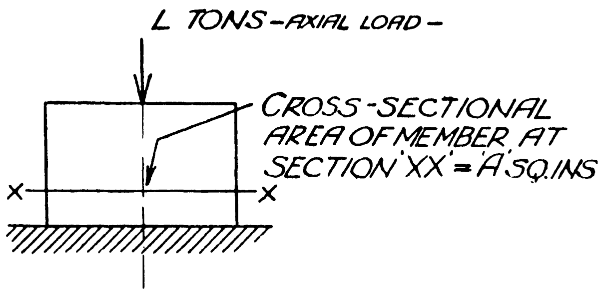


FIG. 3.—ILLUSTRATION OF COMPRESSIVE STRESS.

$$\text{Compressive stress at section XX} = \frac{\text{Load}}{\text{Area}}$$

$$= \frac{L \text{ tons}}{A \text{ sq. ins.}} = \frac{L}{A} \text{ tons per sq. in.}$$

Slender compression members are liable to failure by side bending or 'buckling,' in addition to direct crushing. This

type of member is fully dealt with later in this volume (Chapter XI).

EXAMPLE

A solid circular steel column supports 180 *sq. feet of floor area, for which the total inclusive load is* 3 *cwts. per sq. foot. Assuming it to be necessary to limit the maximum compressive stress in the column to* 2·5 *tons per sq. in., obtain the minimum permissible diameter for the column.*

Total load on column (assumed axial)

$= 3$ cwts. per sq. ft. $\times$ 180 sq. ft.

$= 540$ cwts. $= 27$ tons.

Let d'' = diameter of column.

$$\text{Sectional area of column} = \frac{\pi d^2}{4} \text{ sq. ins.}$$

$$\therefore 2{\cdot}5 \text{ tons per sq. in.} = \frac{27 \text{ tons}}{\frac{\pi d^2}{4} \text{ sq. ins.}}$$

$$\therefore 2{\cdot}5 \times \frac{\pi d^2}{4} = 27$$

$$d^2 = \frac{43{\cdot}2}{\pi}$$

$$d = \text{say, } 3\tfrac{3}{4} \text{ ins.}$$

Shear Stress.—When one portion of a member tends to slide over another portion at a given section, the fibres at the section are said to be in *shear stress.*

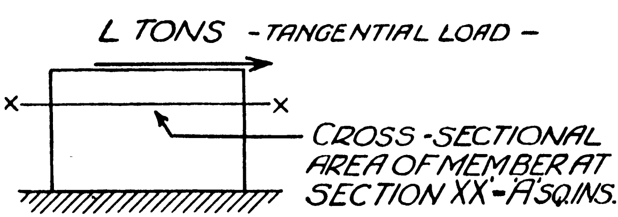


FIG. 4.—MEMBER SUBJECTED TO SHEAR STRESS.

$$\text{Shear stress at section XX} = \frac{\text{Shear Load}}{\text{Area under Shear}}$$

$$= \frac{\text{L tons}}{\text{A sq. ins.}} = \frac{\text{L}}{\text{A}} \text{ tons per sq. in.}$$

The rivets, in a simple riveted joint, are examples of structural units subjected to this form of stress.

EXAMPLE

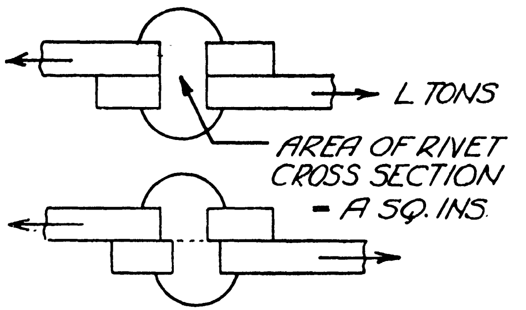


FIG. 5.—SHEAR STRESS IN A RIVET.

Assuming the load in Fig. 5 to be 2 tons, and the rivet diameter to be $\frac{3}{4}''$, find the shear stress in the rivet.

$$\text{Sectional area of rivet} = \frac{\pi d^2}{4} = \frac{\pi \times \cdot 75^2}{4} \text{ sq. ins.}$$

$$= \cdot 44 \text{ sq. ins.}$$

$$\text{Shear stress} = \frac{\text{Load}}{\text{Area}} = \frac{2 \text{ tons}}{\cdot 44 \text{ sq. ins.}}$$

$$= 4 \cdot 55 \text{ tons per sq. in.}$$

The subject of rivet strength is treated, in detail, in Chapter IV.

Strain

If we apply a load to a member, not only do we induce in the fibres a state of stress, but in some respect we alter the size, or shape, of the member.

The subject of *strain* is concerned with these geometrical alterations. Each of the stresses which has been referred to is accompanied by its corresponding strain.

Tensile Strain.—The tensile strain in the member in Fig. 6 is not measured by the extension x'' itself, but by the *ratio of this extension to the original length* of the member.

$$\text{Tensile strain} = \frac{\text{Extension}}{\text{Original length}}$$

$$= \frac{x''}{l''} = \frac{x}{l} \text{ (simply a number).}$$

The reader should carefully note that the value of the strain is not expressed in any dimensional unit. The two length measurements concerned in the computation may be in any

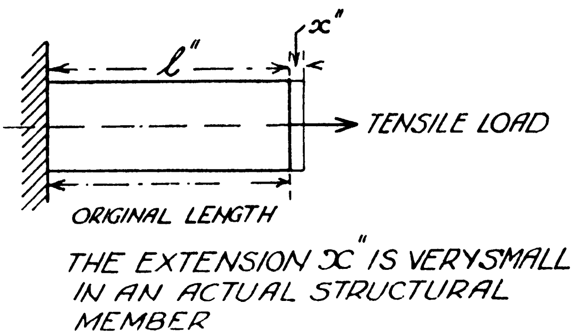


FIG. 6.—MEMBER SUBJECTED TO TENSILE STRAIN.

units, provided the same unit is employed for both. In the practical employment of a steel member, the extension x is extremely small, and is not discernible by the naked eye.

EXAMPLES

(1) *A tie member* 10 *ft. long is subjected to an axial load which stretches it* ·012 *ins. Calculate the strain in the material of the member.*

$$\text{Tensile strain} = \frac{\text{Extension}}{\text{Original length}}$$

$$= \frac{\cdot 012 \text{ ins.}}{(10 \times 12) \text{ ins.}} = \frac{\cdot 012}{120} = \cdot 0001 \text{ (a number).}$$

(2) *A tensile test specimen undergoes a strain of* ·0004. *Find the actual extension on a measured gauge length of* 8″.

$$\text{Tensile strain} = \frac{\text{Extension}}{\text{Original length}}$$

$$\cdot 0004 = \frac{\text{Extension in ins.}}{8''}$$

$$\therefore \text{Extension} = (8 \times \cdot 0004) \text{ ins.} = \cdot 0032 \text{ ins.}$$

Compressive Strain

$$\text{Compressive strain} = \frac{\text{Shortening in length}}{\text{Original length}} = \frac{x''}{l''} = \frac{x}{l}.$$

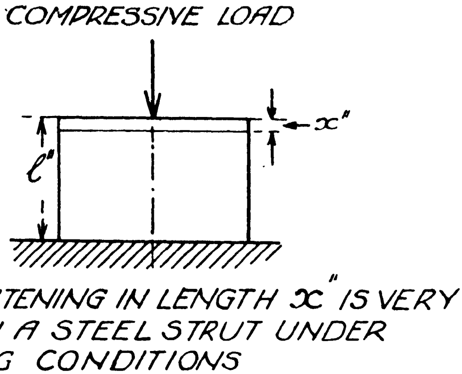


FIG. 7.—MEMBER SUBJECTED TO COMPRESSIVE STRAIN.

EXAMPLE. *A column, loaded axially, shortened by ·016 ins. If the resulting strain were ·0002, find the original length of the column.*

$$\text{Compressive strain} = \frac{\text{Shortening}}{\text{Original length}}.$$

$$\cdot 0002 = \frac{\cdot 016 \text{ ins.}}{\text{length in ins.}}.$$

$$\therefore \text{Length} = \frac{\cdot 016 \text{ ins.}}{\cdot 0002} = 80 \text{ ins.} = 6'\ 8''.$$

Shear Strain.—The two strains already discussed involve the change in length of a member. Shear strain is concerned with

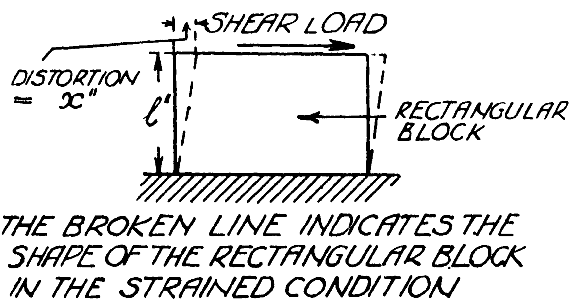


FIG. 8.—NATURE OF SHEAR STRAIN.

the change of shape, or the distortion, which results from shear stress.

The value of the shear strain is given by the ratio $\frac{x''}{l''} = \frac{x}{l}$.

It is more important at the present stage for the reader to have a correct appreciation of the nature of shear strain, than to possess a knowledge of its exact determination. It is the type of strain induced in a workshop shaft which is transmitting a twisting moment, or in a key when we are attempting to turn it in a stiff lock.

Relationship between Stress and Strain

It will now be clear that the terms 'stress' and 'strain' refer, respectively, to two quite different physical conditions of a loaded member. Within certain limits, however, there is a definite, and simple, relationship between the corresponding values of these quantities. The relationship is more clearly defined in some building materials than in others. Steel possesses the property of *elasticity* in a high degree, and obeys the *elastic law* very closely.

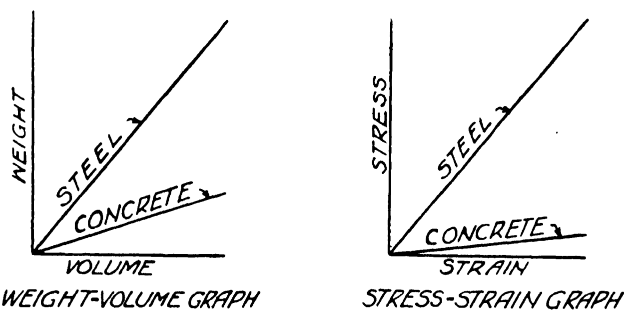
Elasticity.—A piece of material is said to be *elastic* if, having been deformed by an applied force, it regains its original size and shape when the deforming force is removed. An experimenter, named Hooke, discovered (about the year 1676) that an elastic body would always stretch by an amount which was directly proportional to the applied tensile load, provided the experiment were not conducted beyond a certain maximum limit of stretching. This law of relationship between 'load' and 'extension' is exemplified in the tension spring balance scales used in shops. The graduation marks on such scales are all equidistant, indicating, for example, that the spring stretches 10 times as much for a 10-lb. weight as it does for a 1-lb. weight. A steel member may be regarded as being, within limits, a very strong and accurate spring, whose extensions (or compressions) are so small that special instruments are required to measure them.

We have seen that the stress in a member is directly proportional to the applied load $\left(\text{stress} = \frac{\text{load}}{\text{area}}\right)$ and that the strain is directly proportional to the extension $\left(\text{strain} = \frac{\text{extension}}{\text{original length}}\right)$.

Hooke's load-extension law may therefore be expressed in terms of ' stress ' and ' strain.' This is the form in which it is usually remembered and quoted, and the law is expressed by the statement that **' stress varies as strain.'** The law will equally apply in tension and compression for all steel members, up to a stress value known as the *elastic limit* for the particular steel.

Young's Modulus of Elasticity

The reader will, perhaps, more readily understand this very important physical property, if a comparison be made between it and another physical property with which he is already familiar.



FIGS. 9 AND 10.—ANALOGY BETWEEN THE TWO PHYSICAL CONSTANTS, ' DENSITY ' AND ' YOUNG'S MODULUS.'

If we took a number of different pieces of any given material and weighed them, a graph could be drawn showing the variation of ' weight ' with ' volume.' The graph would be a straight line (Fig. 9), as weight would increase uniformly with volume. The value of the ratio $\frac{\text{weight}}{\text{volume}}$ would be the same for any pair of corresponding values of weight and volume, taken at any point in the graph. The ratio would give us the *physical constant* for the material known as its *density.* But stresses and strains follow the same type of law as weights and volumes do.

The stress-strain graph will be a straight line, and the ratio of any particular stress to its corresponding strain will be a

constant value for all points in the graph. The actual value of this constant does not depend on the size of the member undergoing stress and strain, but simply on the nature of the material of the member, just as *density* is independent of the dimensions of the substance concerned.

The physical constant, obtained from the stress-strain ratio, is given the name *Young's Modulus*, and is denoted, in calculations, by the letter E.

$$E = \frac{\text{Stress}}{\text{Strain}}.$$

Hooke's elastic law holds also in the case of shear stress and shear strain, but the value of the constant $\frac{\text{stress}}{\text{strain}}$ differs in this case from that obtained in tension and compression, and is termed the *shear modulus* of the given material.

Units of Young's Modulus.—As the value of strain is simply expressed as a number, the units of E will be those of stress. If strain = 1, E = stress, so that Young's modulus may, theoretically, be defined as the stress value required to produce unit strain in a tensile specimen of the particular material. Unit strain, however, involves an extension equal to the original length of the specimen. Young's modulus has no significance beyond the elastic limit of the material, which, in the case of steel, represents a strain of the order of ·001. Although unit strain is impracticable in attainment, the terms of the definition serve to emphasise the nature of the constant E.

The value of Young's modulus for structural steel, in tension or compression, may be taken as 13,000 tons per sq. in. A lower value of about 12,000 tons per sq. in. is sometimes taken in calculations involving the deflection of steel beams.

The following important facts of elasticity will now be appreciated by the reader:

(1) **Stress cannot exist without strain, nor strain without stress.**

(2) **A given elastic stress value is always accompanied, in any particular type of material, by the same value of strain.**

(3) **Young's modulus is the physical constant which enables**

us to calculate exactly how much strain accompanies a given stress value, and vice versa.

EXAMPLES

(1) *Calculate the value of E from the following results of a steel tensile test.*

Sectional area of specimen = ·44 *sq. ins.*
Measured gauge length on specimen = 8″.
Applied tensile load = 1·43 *tons.*
Corresponding elastic extension = ·002 *ins.*

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{1{\cdot}43 \text{ tons}}{{\cdot}44 \text{ sq. ins.}} = 3{\cdot}25 \text{ tons per sq. in.}$$

$$\text{Strain} = \frac{\text{Extension}}{\text{Original length}} = \frac{{\cdot}002 \text{ ins.}}{8 \text{ ins.}} = {\cdot}00025.$$

$$\text{E} = \frac{\text{Stress}}{\text{Strain}} = \frac{3{\cdot}25 \text{ tons per sq. in.}}{{\cdot}00025} = 13{,}000 \text{ tons per sq. in.}$$

(2) *Find the elongation produced in a circular tie-rod,* 10 *ft. long and* $\frac{7}{8}''$ *diameter, when subjected to an axial load of* 4 *tons.* E = 13,000 *tons per sq. in.*

Let x'' = the extension.

Sectional area of a $\frac{7}{8}''$ diameter rod = ·6013 sq. ins.

$$\text{Stress in rod} = \frac{\text{Load}}{\text{Area}} = \frac{4 \text{ tons}}{{\cdot}6013 \text{ sq. ins.}}$$
$$= 6{\cdot}65 \text{ tons per sq. in.}$$

$$\text{Strain in rod} = \frac{\text{Extension}}{\text{Original length}}$$
$$= \frac{x''}{(10 \times 12)''} = \frac{x}{120}$$

$$\text{E} = \frac{\text{Stress}}{\text{Strain}}$$

$$13{,}000 = \frac{6{\cdot}65 \text{ tons per sq. in.}}{\frac{x}{120}}$$

$$13{,}000 \times \frac{x}{120} = 6{\cdot}65$$

$$x = {\cdot}061''.$$

(3) *In a test, to determine the stress induced in a member of a steel frame by the load carried, an instrument was fixed to the*

member in order to measure the shortening produced in it. Assuming a contraction in length of ·005″ to have been measured on a 10″ gauge length, deduce the stress in the member.

$$\text{Strain in member} = \frac{\text{Shortening}}{\text{Original length}}$$

$$= \frac{\cdot 005''}{10''} = \cdot 0005.$$

$$E = \frac{\text{Stress}}{\text{Strain}} \qquad \therefore \text{Stress} = E \times \text{Strain}.$$

Taking E to be 13,000 tons per sq. in.,
Stress = 13,000 × ·0005 = 6·5 tons per sq. in.

The above example illustrates the method employed in research work to ascertain the stress at any part of a loaded specimen. The extremely small alterations in length caused by the application of load are measured by instruments termed ' extensometers.'

The illustration given in Fig. 13 shows a model steel roof truss being tested in a 100-ton Riehlé testing machine. A Lamb's roller extensometer is fixed to one of the struts in order to find the extension on a known gauge length, i.e. the ' strain ' (and hence the ' stress ' and ' load ') in this member.

EXERCISES I

(1) A tie-bar in a steel truss carries a load of 9 tons. The section of the bar is rectangular, $3'' \times \frac{3}{8}''$. Calculate the tensile stress in the material of the bar.

(2) How many steel suspension bars, $4'' \times \frac{3}{4}''$, would be required to support a load of 72 tons, assuming the load equally divided between the bars ? Maximum stress not to exceed 8 tons per sq. in.

(3) Find the necessary diameter for a steel column of solid circular section which has to carry an axial load of 75 tons, the maximum allowable stress being 6 tons per sq. in.

(4) The base of a column is carried on a square concrete slab. The load transmitted to the ground beneath the slab is 64 tons. Assuming a safe bearing pressure on the ground of 4 tons per sq. ft., find the minimum dimensions, in plan, for the slab.

(5) Find the maximum safe value for P, in the gusseted con-

nection given in Fig. 11, from the point of view of (*a*) the tension in the tie-bar ($2\frac{1}{4}''$ gross width), (*b*) the tension at section XX in the gusset plate.

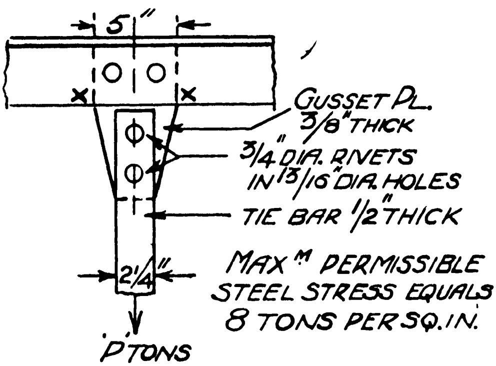


Fig. 11.

(If x'' = width of tie-bar and d'' = diameter of rivet hole, a section taken through a rivet hole will have an 'effective area' of $(x - d)'' \times$ 'thickness of tie-bar.' The safe load for the tie-bar must be computed from this 'net' area.)

(6) Find the shear stress in the tie-bar rivets of the given

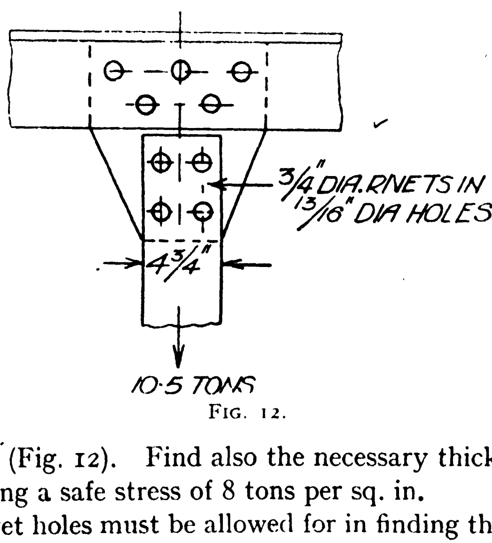


FIG. 12.

connection (Fig. 12). Find also the necessary thickness of the tie-bar, using a safe stress of 8 tons per sq. in.

(Two rivet holes must be allowed for in finding the net width of the tie-bar.)

(7) Calculate the strain in a column which is shortened by ·0312″ under the applied axial load, the original length of the column being 13′. Also determine the contraction in length corresponding to a strain of ·0004.

(8) Obtain Young's modulus from the following results of a practical test:

Diameter of circular specimen = $\frac{3}{4}$″.
Gauge length on specimen = 8″.
Applied load = 2·86 tons.
Corresponding extension (measured on gauge length) = ·004.″

(9) Distinguish between the terms 'stress' and 'strain.' Give the three forms of simple stress with examples of typical structural members in which they respectively occur. Name and write down the law which, within certain limits, governs the relative values of these two physical properties, and explain the meaning and nature of 'Young's modulus.'

(10) Calculate the extension in a steel tie-rod, 1″ diameter and 8′ long, for an axially applied load of 3 tons. E = 13,000 tons per sq. in.

(11) In a test to determine the 'live load' carried by a member of a steel lattice girder—due to the passage over the girder of a travelling crane—an extensometer, of the direct-reading dial type, was employed. Taking the following test results, determine the live load referred to.

Gauge length on member = 10″.
Value of one dial division on extensometer = $\frac{1}{10000}$″.
Difference in readings due to passage of load = 20 divisions.
Sectional area of member = 4 sq. ins.
Take the usual value for E.

(12) An uncased steel member 20′ long was subjected during a fire to a temperature rise of 30° C. Assuming the ends of the member to have been fixed in such a way as not to allow of any expansion, calculate the stress induced in the steel.

Coefficient of linear expansion of steel = ·00001.
Assume E = 13,000 tons per sq. in.

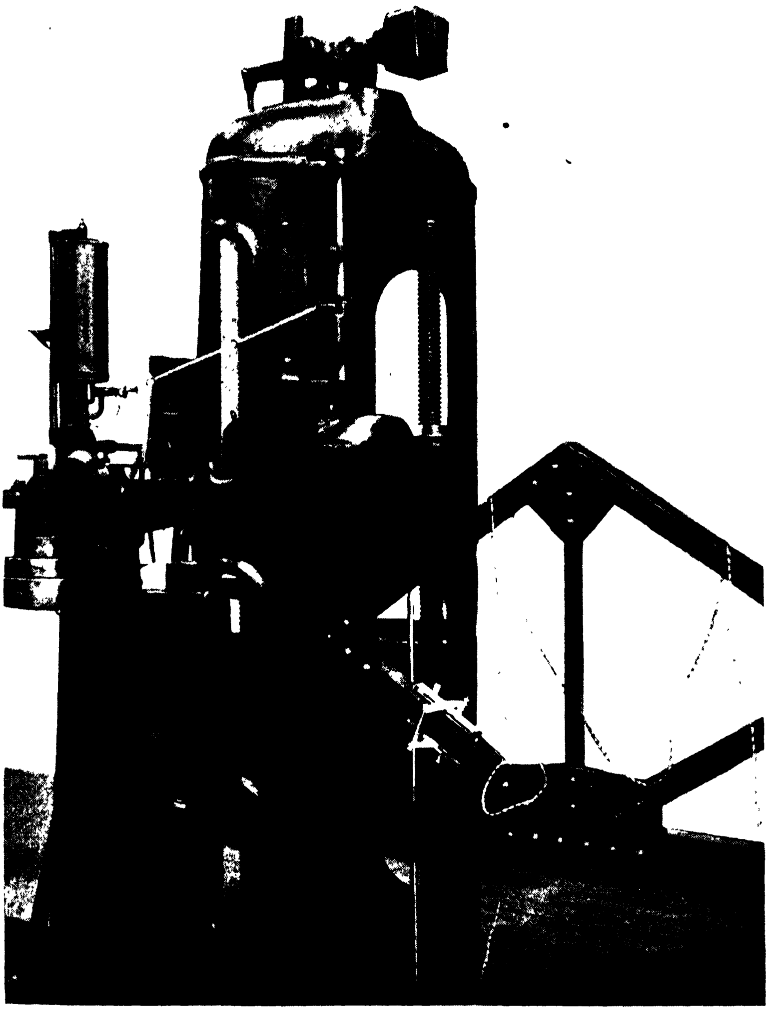


FIG. 13.—100-TON RIEHLÉ UNIVERSAL TESTING MACHINE.

CHAPTER II

ULTIMATE STRESS, FACTOR OF SAFETY AND WORKING STRESS

Stress-Strain Graph for Tensile Test

IF a mild steel specimen were placed in a testing machine and a tensile load applied steadily until the specimen fractured, the stress-strain graph would have the general character given in Fig. 14.

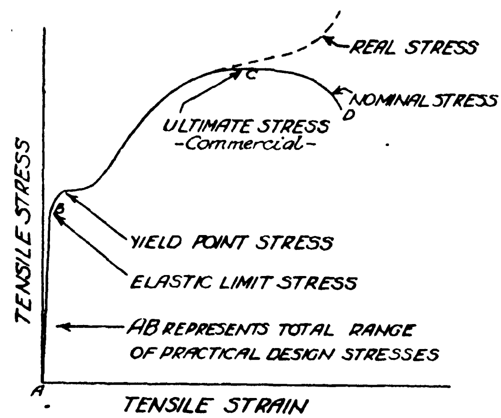


FIG. 14.—STRESS-STRAIN GRAPH FOR A MILD STEEL SPECIMEN TESTED TO FRACTURE.

The graph may be divided into three parts, indicated respectively in the figure by AB, BC and CD.

(i) *A to B.* Between A and B the graph is a straight line. The point B fixes the upper limit of proportionality between stress and strain. The stress value corresponding to this limit is known as the *Elastic Limit* stress for the steel. For structural steel its value will be about 15 tons per sq. in. The part AB of the graph is very important from the point of view of design, as the stresses involved are those corresponding to the elastic strain of the steel—the strain condition of all

steel in practical structures. It will be clear that no formula in design, based on the assumption of Hooke's law, will hold for a stress exceeding the elastic-limit stress.

(ii) *B to C*. When the stress has reached a value slightly higher than the elastic limit, a definite yield takes place in the specimen. The strain value increases without corresponding increase in the stress. The stress at this point is known as the *Yield Point* stress. The yield point is made apparent in a practical test by the sudden drop of the lever arm of the testing machine, and the temporary refusal of the specimen to take up load. The term *commercial elastic limit* is sometimes used for this stress. After passing the yield point the stress increases, and the graph takes the form indicated in the figure.

(iii) *C to D*. Throughout the test lateral strain accompanies the longitudinal strain, but extensometers would be required to measure both these strains—inside the elastic limit. Between C and D, however, there is a marked contraction of cross-sectional area, easily visible to the naked eye. Just before fracture the specimen will have the appearance shown in Fig. 15.

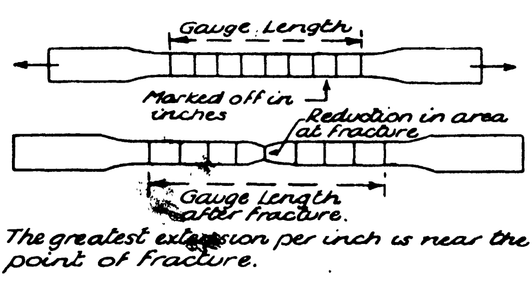


FIG. 15.—LABORATORY TENSILE TEST SPECIMEN.

It will be found that to maintain the lever arm *floating* between its stops, during this part of the test, load will have to be taken off the specimen. If we calculate stress values on the original sectional area of the specimen, these values will decrease with the decreasing load. This explains the apparent drop in stress before fracture, as represented in the graph. Actually the stress increases up to the point of fracture, and the

broken-line graph would be obtained if the reduced cross-sectional area of the specimen were taken into account. Stress calculations, in commercial testing, are based on original dimensions.

Ultimate Stress.—In a practical test to determine the strength of steel, the maximum load carried by the specimen is the important quantity, not the actual load at fracture. This applies equally to 'compression' and 'shear' tests. The ultimate stress is obtained by dividing the maximum load during the test by the original sectional area of the specimen.

$$\text{Ultimate stress} = \frac{\text{Maximum load}}{\text{Original sectional area}}.$$

Commercial Testing of 'Structural Steel'

B.S.S. No. 15–1936 gives precise details of the nature of the tests to be carried out, and of the general procedure which has to be followed. The reader is referred to this specification for fuller particulars than can be given here.

Some of the steel sections, etc., being rolled to an order, are made longer than necessary. The extra lengths are then cut off, as required, for test specimens.

There are two tests imposed by the B.S.S., (i) a tensile test and (ii) a cold bend test.

Extract from Clauses relating to the Tensile Test.

(*a*) **Plates, Sections (e.g. Angles, Tees, Beams, Channels, etc.) and Flat Bars.**—*The tensile breaking strength of all plates, sections (such as angles, tees, beams, channels, etc.) and flat bars, shall be between the limits of* 28 *and* 33 *tons* (62,700 *and* 73,920 *lb.*) *per square inch of section. The elongation measured on the Standard Test Piece A shall be not less than* 20 *per cent. for steel of* 0·375 *inch in thickness and upwards, and not less than* 16 *per cent. for steel below* 0·375 *inch in thickness.*

For plates, sections and flat bars under 0·25 *inch in thickness bend tests only shall be required.*

(*b*) **Round and Square Bars.**—*The tensile breaking strength of round and square bars (other than rivet bars) shall be between the limits of* 28 *and* 33 *tons per square inch of section, with an elongation of not less than* 20 *per cent. measured on the Standard Test Piece B,*

or not less than 24 *per cent. measured on the Standard Test Piece F. For bars under* 0·375 *inch diameter or thickness for concrete reinforcement, the elongation measured on test piece B shall be not less than* 16 *per cent. For bars under* 0·375 *inch diameter or thickness for other purposes, bend tests only are required. The bars may be tested the full size as rolled.*

(*c*) **Rivet Bars.**—*The tensile breaking strength of rivet bars shall be between the limits of* 25 *and* 30 *tons* (56,000 *and* 67,200 *lb.*) *per square inch of section, with an elongation of not less than* 26 *per cent. measured on the Standard Test Piece B, or not less than* 30 *per cent. measured on Standard Test Piece F. The bars may be tested the full size as rolled.*

Standard Test Pieces.

A.—A test piece of prescribed pattern with a gauge length of 8″ for elongation measurements.

B.—A standard test piece in which the minimum gauge length is given as 8 times the diameter.

F.—For test pieces over 1″ diameter. The gauge length is not to be less than 4 times the diameter.

Extract from Clauses relating to the Cold Bend Test.—*For cold bend tests, except in the case of round bars* 1 *inch in diameter and under, the test piece shall withstand, without fracture, being doubled over either by pressure or by blows from a hammer until the internal radius is not greater than* 1½ *times the thickness of the test piece, and the sides are parallel. In the case of round bars,* 1 *inch in diameter and under, the internal radius of the bend shall be not greater than the diameter of the bar. For sections having flanges less than* 2 *inches wide these bend tests may be made from the flattened section.*

Laboratory Steel Testing

It will be noted that in the tensile test—in commercial testing to B.S.S. (No. 15 specification)—only two quantities are determined, viz. *ultimate tensile stress* and *percentage elongation at fracture.* In laboratory testing two other values are usually found, viz. the *Elastic Limit* (*or Yield Point*) *Stress* and the *percentage contraction in area at fracture.* Interesting tests on a range of carbon steels (steels in which the variation of

carbon content provides the characteristic properties) may with advantage be carried out to illustrate how the Y.P. and ultimate stresses are increased in value, and the ductility constants (percentage elongation and percentage contraction in area) decreased, as the percentage carbon content increases.

EXAMPLES ON STEEL TESTS

(1) *Calculate the usual test constants from the following results of a steel tensile test, and state whether the steel represented by this specimen would satisfy the requirements of the B.S.S. for structural steel.*

Diameter of specimen = ·75″.
Distance between gauge points = 8″.
Load at yield point = 7·48 *tons.*
Maximum load during test = 12·76 *tons.*
Gauge length (after fracture) = 10·08″.
Diameter at fracture = ·52″.

$$(a)\ \text{Yield point stress} = \frac{\text{Load at Y.P.}}{\text{Original area of specimen}}.$$

$$\text{Original area of specimen} = \frac{\pi d^2}{4} = \frac{\pi \times \cdot 75^2}{4}\ \text{sq. ins.}$$
$$= \cdot 44\ \text{sq. ins.}$$

$$\text{Y.P. stress} = \frac{7\cdot 48\ \text{tons}}{\cdot 44\ \text{sq. ins.}} = 17\ \text{tons per sq. in.}$$

$$(b)\ \text{Ultimate stress} = \frac{\text{Maximum load during test}}{\text{Original area of specimen}}$$
$$= \frac{12\cdot 76\ \text{tons}}{\cdot 44\ \text{sq. ins.}} = 29\ \text{tons per sq. in.}$$

$$(c)\ \text{Percentage elongation at fracture} = \frac{\text{Elongation}}{\text{Gauge length}} \times 100$$
$$= \frac{(10\cdot 08 - 8)''}{8''} \times 100 = \frac{2\cdot 08''}{8''} \times 100 = 26.$$

(*d*) Percentage contraction in area at fracture

$$= \frac{\text{Contraction in area}}{\text{Original area}} \times 100.$$

$$\text{Area at fracture} = \frac{\pi \times \cdot 52^2}{4}\ \text{sq. ins.} = \cdot 21\ \text{sq. ins.}$$

$$\therefore\ \text{Contraction in area} = (\cdot 44 - \cdot 21)\ \text{sq. ins.} = \cdot 23\ \text{sq. ins.}$$

$$\text{Percentage contraction} = \frac{\cdot 23\ \text{sq. ins.}}{\cdot 44\ \text{sq. ins.}} \times 100 = 52\cdot 2.$$

Results for (*b*) and (*c*) are within the limits of the B.S.S. requirements, hence the steel would pass the tensile test.

(2) *For the specimen given in the last example, evaluate the maximum and minimum test loads, respectively, which would have been permissible for the specimen to pass the B.S.S. test. Also determine the minimum elongation on 8″ gauge length.*

Sectional area of specimen = ·44 sq. ins.

∴ Maximum test load = 33 tons per sq. in. × ·44 sq. ins.
= 14·52 tons.

Minimum test load = 28 tons per sq. in. × ·44 sq. ins.
= 12·32 tons.

Minimum elongation = 20% of 8″ = 1·6 ins.

High Tensile Structural Steel

B.S.S. No. 548–1934 defines the procedure to be adopted in testing this type of steel, and should be consulted for detailed information. It follows, generally, the lines of B.S.S. No. 15–1936, but the test results required are, of course, different. The limits '37' to '43' tons per sq. in. appear in place of 28–33 tons per sq. in. A minimum 'yield point' stress is given in the tensile test, the value depending upon the thickness of material in the test specimen. For example, in the case of plates and sections from ¼″ to 1¼″ in thickness, the minimum yield point stress allowed is 23 tons per sq. in. This is also the figure given for round and square bars up to 1″ diameter, or side, respectively.

A cold bend test is specified, as in B.S.S. No. 15. In the case of B.S.S. No. 548, such test applies also to rivet bars, which are exempt from this form of test in the former specification.

Factor of Safety and Working Stress

It will be clear that the stress to be used in the actual dimensioning of structural members will have to be somewhat less than the corresponding ultimate stress for the material. Some of the principles governing the suitable margin of safety to be allowed, in any particular case, are indicated later. The stress used in practical design is termed the *safe working stress* or simply *working stress*. This stress value is obtained from the

appropriate ultimate stress by dividing it by a selected number, known as the *factor of safety.*

$$\text{Working stress} = \frac{\text{Ultimate stress}}{\text{Factor of safety}}.$$

Thus if we take 32 tons per sq. in. as the ultimate stress for structural steel and adopt a factor of safety of 4, the working stress will be

$$\frac{\text{Ultimate stress}}{\text{Factor of safety}} = \frac{32 \text{ tons per sq. in.}}{4} = 8 \text{ tons per sq. in.}$$

Choice of a Factor of Safety.—The value of the factor of safety will be influenced by such considerations as the following:

(i) The ultimate stress of a material is not really the best stress upon which to base a working stress value. The limit of useful stress is given by the *elastic limit stress.* Formulæ and methods of design are definitely based on the assumption of Hooke's elastic law. In the case of structural steel, this fact alone requires a factor of safety of about ' 2.'

(ii) Materials such as timber and cast iron are not so reliable as steel, and are less likely to exhibit, throughout any considerable quantity, the standard of quality represented by the tested specimens. From this point of view, therefore, some materials will require a higher factor of safety than others.

(iii) Structural members may be temporarily overloaded during abnormal conditions. The margin of safe stress is useful then.

(iv) It is not always possible to calculate the actual load a member will have to carry. The load application may be of a doubtful character, and, possibly, the theoretical principles involved be based on assumptions not wholly justifiable.

(v) Design calculations are made on the basis of a high standard of workmanship in the fabrication of the members. Also (as in the case of reinforced concrete) it is assumed that the details given on the working drawings will be strictly adhered to, in the assembly of the members. The factor of safety makes some allowance for the human element in this respect.

Working Stresses.—For *dead loading* (i.e. loading applied steadily and not intermittently) the factor of safety for structural steel is 4. A higher factor is required for *live load* and in special circumstances, as in the design of long columns.

However, the designer has little to do with the factor of safety itself, but is supplied with a list of working stresses directly. Various lists of working stresses are issued, the most important being those of the British Standards Institution, the London County Council and the Institution of Structural Engineers.

B.S.S. No. 449–1937 is concerned with the use of structural steel in building and the reader is strongly advised to make himself acquainted with its contents. With the permission of the British Standards Institution, extracts will be given, where appropriate, from this specification.

The London Building Act, 1930, Third Schedule, contains clauses dealing with the working stresses to be used in constructional work carried out in the area under L.C.C. jurisdiction. The present working stresses for steelwork are given in the London County Council By-laws (1938).

The Institution of Structural Engineers issues reports from time to time, as the results of research work, carried out under its promotion, become established.

B.S.S. No. 449 (Clause 10) gives the following working stresses for Mild Steel B.S.S. No. 15. This steel must be made by the Open Hearth Process (Acid or Basic),* and must not show on analysis more than ·06 per cent. of Sulphur, or of Phosphorus. The stress values in the specification are accompanied by certain provisos, which will be referred to in the practical use of the stresses later. The L.C.C. By-laws (clause 81) give the same values as the B.S.S. as far as this list of working stresses is concerned. For the corresponding values of working stresses for High Tensile Steel B.S.S. No. 548 the reader should refer to B.S.S. No. 449–1937.

(a) **For Parts in Tension**	TONS PER SQ. INCH
On the nett section for axial stress or extreme fibre stress of all beams	8
On the nett section of rivets for axial stress, in the case of rivets driven at the Works where the steelwork is fabricated .	5
On the nett section of rivets for axial stress in the case of rivets driven at the site	4
On the nett section of bolts for axial stress . . .	5

* B.S.S. No. 15–1936 permits Acid Bessemer Process.

	TONS PER SQ. INCH
(*b*) **For Compression Flanges of Beams**	
On the gross section for extreme fibre stress of beams embedded in a concrete floor or otherwise laterally secured .	8
On the gross section for extreme fibre stress of uncased beams where the laterally unsupported length L is less than twenty times the width b of the compression flange .	8
On the gross section for extreme fibre stress of uncased beams where L is greater than twenty times b	$11{\cdot}0 - 0{\cdot}15 \frac{L}{b}$

(*In no case may the ratio* $\frac{L}{b}$ *exceed* 50.)

(*c*) **For Parts in Shear**	
On the gross section of webs	5
On shop rivets and tight-fitting turned bolts . . .	6
On field rivets	5
On black bolts	4

(*The strength of rivets and bolts in double shear may be taken as twice that for single shear.*)

(*d*) **For Parts in Bearing**	
On shop rivets and tight-fitting turned bolts . . .	12
On field rivets	10
On black bolts	8

The stresses to be used in the design of grillage beams, filler floor beams, and in columns, will be described in the following chapters.

EXAMPLES

(1) *Taking the ultimate stress for mild steel as* 30 *tons per sq. in. and adopting a factor of safety of* 4, *calculate the safe axial load for a mild steel tie-bar,* 4″ × ⅝″.

$$\text{Working stress} = \frac{\text{Ultimate stress}}{\text{Factor of safety}}$$

$$= \frac{30 \text{ tons per sq. in.}}{4} = 7\tfrac{1}{2} \text{ tons per sq. in.}$$

Sectional area of tie-bar = 4″ × ⅝″ = 2·5 sq. ins.

∴ Safe axial load = (2·5 × 7·5) tons = 18·75 tons.

(2) *A tie-bar, 2″ × ⅜″ section, is used in a structure to carry 5·625 tons. In a test on the same quality steel, the maximum load carried was 13·64 tons, the test specimen having a sectional area of ·44 sq. ins. Find the factor of safety used in the design.*

$$\text{Ultimate stress for the steel} = \frac{\text{Maximum load}}{\text{Sectional area}} = \frac{13{\cdot}64 \text{ tons}}{{\cdot}44 \text{ sq. ins.}}$$
$$= 31 \text{ tons per sq. in.}$$

$$\text{Actual working stress} = \frac{5{\cdot}625 \text{ tons}}{(2 \times \frac{3}{8}) \text{ sq. ins.}} = 7{\cdot}5 \text{ tons per sq. in.}$$

$$\text{Factor of safety used} = \frac{\text{Ultimate stress}}{\text{Working stress}}$$

$$= \frac{31 \text{ tons per sq. in.}}{7{\cdot}5 \text{ tons per sq. in.}} = 4{\cdot}13.$$

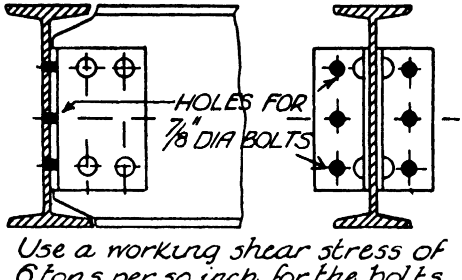
(3) *Find the safe shear load for one ¾″ diameter rivet, assuming the shearing tendency to be across one section of the rivet, and the rivet to have been put in position in the 'shop.'*

From the table of working stresses given, we find that the stress in such a case can be taken as 6 tons per sq. in.

$$\text{Sectional area of rivet} = \frac{\pi d^2}{4} = \frac{\pi \times {\cdot}75^2}{4} \text{ sq. ins.}$$
$$= {\cdot}442 \text{ sq. ins.}$$
$$\therefore \text{Safe shear load} = ({\cdot}442 \times 6) \text{ tons}$$
$$= 2{\cdot}65 \text{ tons.}$$

(Tables of rivet and bolt strengths, for various working stresses, are given in Chapter IV.)

(4) *Calculate the safe load, from the point of view of shear in the bolts, for the cleat connection given in Fig. 16.*



ANGLE CLEAT CONNECTION FOR BEAM TO BEAM

FIG. 16.

Sectional area of one $\frac{7}{8}''$ diameter bolt $= \cdot 6013$ sq. ins.
Working stress for bolts $= 6$ tons per sq. in.
$\therefore$ Safe load per bolt $= (6 \times \cdot 6013) = 3 \cdot 6078$ tons.
$\therefore$ Safe load for 6 bolts $= 21 \cdot 65$ tons.

The following B.S.S. are referred to in this chapter: No. 15–1936, No. 548–1934, No. 449–1937. The extracts have been made by permission of the British Standards Institution, 28 Victoria Street, London, S.W.1 (see Appendix I), from whom official copies of the specifications may be obtained, price 2*s*. 2*d*. each, post free.

Exercises 2

(1) Express the relationship between the following three quantities: ultimate stress, working stress and factor of safety. Assuming a working stress of 8 tons per sq. in. to represent a factor of safety of 4, obtain the ultimate stress. What would be the working stress in this case for a factor of safety of 5 ?

(2) Work out a complete set of test results for the following steel test, and show that the steel would satisfy the requirements of B.S.S. No. 15.

Diameter of specimen (round)	$= \cdot 74''$
Gauge length	$= 8''$
Load at yield point	$= 7 \cdot 25$ tons
Maximum load	$= 12 \cdot 54$ tons
Gauge length (after fracture)	$= 10 \cdot 3''$
Diameter at fracture	$= \cdot 50''$

(3) A specimen of mild steel gave the following calculated results in a tensile test: Y.P. stress $= 18$ tons per sq. in.; ultimate stress $= 30$ tons per sq. in.; percentage elongation at fracture (on $8''$ gauge length) $= 28$; percentage reduction in area at fracture $= 58$. The original sectional area of specimen being $\cdot 56$ sq. ins., evaluate the experimental results obtained in the test.

(4) A piece of mild steel of rectangular section, $2'' \times \frac{5}{8}''$, fractured at a maximum tensile load of 40 tons. Using a factor of safety of 4, determine the safe working stress and hence find

the necessary thickness for a tie-bar of the same quality steel, 3″ wide, to carry safely an axial pull of 18 tons.

(5) A flat bar, 1·5 sq. in. in sectional area, has two bolt holes drilled in it 12 ft. apart. Assuming that the load in the bar, when in position in a structure, is 19,530 lb., and that E for the material of the bar is 30,000,000 lb. per sq. in., show that the bolt holes will be $\frac{1}{16}$″ out of true with bolts spaced at 12′ centres.

(6) A specimen of rivet steel, ·4 sq. in. in section, sheared in a test at a load of 10·8 tons. Adopting a factor of safety of 5, obtain the safe shear load for four $\frac{5}{8}$″ diameter rivets which are resisting shear, as indicated in Fig. 5, Chapter I.

(7) In a tensile test, a specimen of steel of rectangular section, 2″ × $\frac{3}{4}$″, broke at a maximum load of 48 tons. A tie-bar of the same quality steel, and having a rectangular section 4″ × $\frac{5}{8}$″, is used to carry an axial load of 20 tons. What factor of safety does this represent ?

(8) By an error in printing, the following was given as a problem in ' elasticity ' :

Sectional area of mild steel tensile member	= ·5 sq. ins.
Length before application of load	= 10″
Load applied	= 12·5 tons
E for material	= 13,000 tons per sq. in.

Calculate the extension in length produced.

Explain why this problem is not capable of solution. Substitute a possible correct set of values.

FIG. 17.—STEEL PLATE ROLLING MILL.

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CHAPTER III

FABRICATION OF STEELWORK

British Standard Sections

CONSTRUCTIONAL steelwork is built up, or fabricated, from units of standardised shape and dimensions. The British Standards Institution issue B.S.S. for the various *sections* employed. B.S.S. No. 4–1932 is the revised standard specification for the dimensions and properties of *British Standard Channels and Beams for Structural Purposes.* B.S.S. No. 4A–1934 gives similar details for *British Standard Equal Angles, Unequal Angles and T-bars for Structural Purposes.*

The various sections are produced from white-hot steel ingots by passing them through rolls in a *rolling mill.* The photograph facing this page shows the soft-steel slab ready to be drawn through the rolls, just like a garment in the ordinary domestic mangle. The mill shown in the foreground is the *finishing mill,* and it has reduced the steel to the form of a *plate.* The rolls have grooves cut in them when the sections

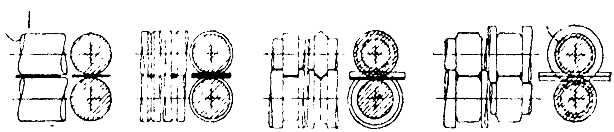


FIG. 18.—FORMS OF ROLLS USED IN THE ROLLING OF STEEL SECTIONS.
Reproduced by permission and courtesy of Messrs. R. A. Skelton & Co., Ltd.

rolled are of the flanged type. Fig. 18 gives the types of rolls used for the various steel sections used in structural work. The diagrams show the rolls used for (i) plates and sheets ; (ii) squares and rounds ; (iii) flats, angles, etc. ; (iv) flanged sections.

The term *rolled steel section* is applied to constructional units manufactured in the manner indicated, and the section in the form of the letter I is commonly called a *rolled steel joist* (R.S.J.). Some steel firms roll special sections which are not

'British Standard,' in addition to the range of British Standard sections. It is customary to denote the latter by prefixing the letters **B.S.** Thus **B.S.E.A.** will mean *British Standard Equal Angle* and **B.S.B.** will indicate *British Standard Beam.* It is possible with most sections to slightly increase the thickness of certain parts by spacing the rolls farther apart. B.S.S. Nos. 4 and 4A give the section properties corresponding to various thicknesses thus obtained.

Choice of Sections.—In the practical choice of a section for a particular job in a structure, several factors have to be considered, irrespective of the question of strength suitability. The section chosen should be one which does not require the steelmaker to change the rolls in his rolling mill, as this is an expensive operation. The section should therefore be a *standard* one, and, not only so, it should be one fairly frequently rolled. Steelmakers indicate in their lists those sections which are most readily obtainable. They also issue lists of *extras* which have to be paid, for sizes and weights which are outside certain limits in the case of any particular section.

It is not possible, owing to the high temperature of the sections when dealt with—and the usual mode of cutting—to obtain the exact dimensions and weights which might be specified in an order. The reader is referred to B.S.S. No. 15 for full details of the maximum allowable variations. For bars and sections to be cut to specified lengths, the margin is one inch, under or over, with a two-inch margin over, when the length specified is a minimum. When *exact* lengths are specified, the sections are to be cold-sawn or machined to one-eighth of an inch, over or under. An *extra* has to be paid for cutting to exact lengths. The tolerance for weights of flat bars or sections (not stated to be either a maximum or a minimum) is $2\frac{1}{2}\%$, over or under the specified weight. The same rolling margin applies to plates over $\frac{1}{4}''$ thick, and round and square bars over $\frac{3}{8}''$ in diameter or thickness.

Commercial Data

The authors are indebted to the British Standards Institution for permission to reproduce details respecting the profiles of

the B.S. sections. Messrs. R. A. Skelton & Co., Steel and Engineering, Ltd., have very kindly assisted in the compilation of the commercial data given, and readers interested in this important side of practical design are recommended to obtain Handbook No. 19, issued by this firm. It should be remembered that commercial data is subject to possible variation from time to time.

British Standard Beam (B.S.B.).—It will be observed, from an inspection of the profile given in Fig. 19, that the flange thickness is measured at a point half-way between the extreme edge of the flange and the nearer side of the web.

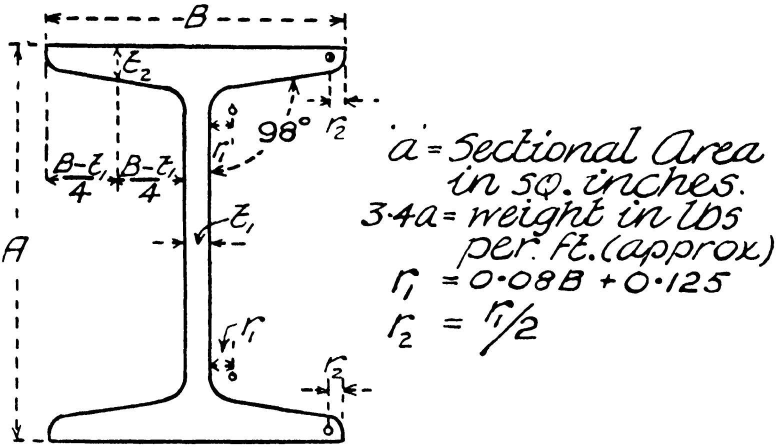


FIG. 19.—BRITISH STANDARD BEAM SECTION.

In naming a beam section the *depth* is given first. Thus a $5'' \times 4\frac{1}{2}'' \times 20$ lb. B.S.B. is a British Standard Beam, having $5''$ overall depth, $4\frac{1}{2}''$ width of flange and a weight of 20 lb. per foot of length. The smallest B.S.B. is a $3'' \times 1\frac{1}{2}'' \times 4$ lb. section, and the largest, $24'' \times 7\frac{1}{2}'' \times 95$ lb. It is possible, in a few cases, to obtain two B.S.B.s, having the same overall dimensions, but different weights per foot (and different *section properties*) owing to differences in respective flange and web thicknesses. When ordering it is usual to specify the depth, flange width and weight per foot. Thicknesses cannot be ordered together with the weight per foot. Some firms roll

beam sections with specially wide flanges. The Broad Flange Beams, Grey Process, manufactured by Messrs. R. A. Skelton, are referred to later.

Stock Sizes and 'Extra' Sizes of Beams.—Sections 5″ × 3″ × 11 lb. and up are usually stocked by steel firms in even lengths up to 40′. Smaller sizes up to about 36′. 'Extras' are charged for beams less than 5″ × 3″ or greater than 16″ × 6″ and for a few intermediate sizes. Extras are also listed for lengths exceeding 50′ or under 10′.

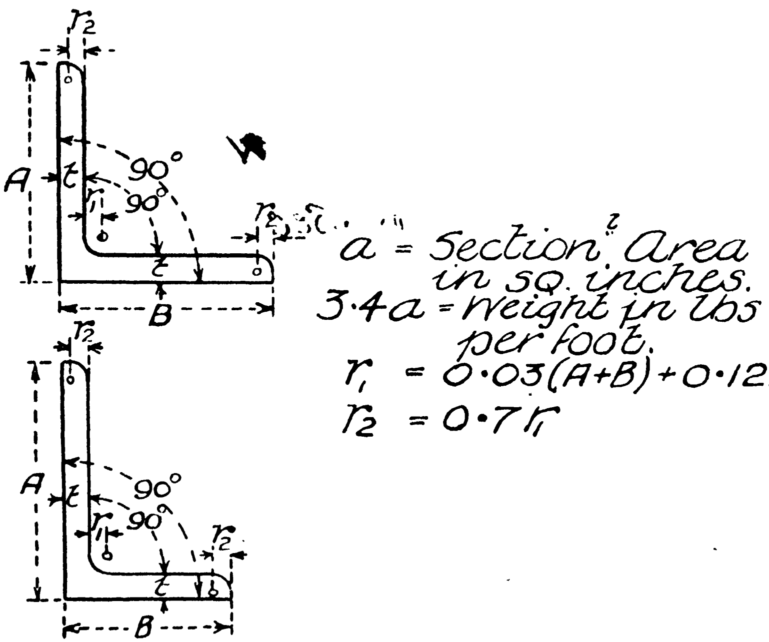


FIG. 20.—BRITISH STANDARD EQUAL ANGLE AND UNEQUAL ANGLE SECTIONS.

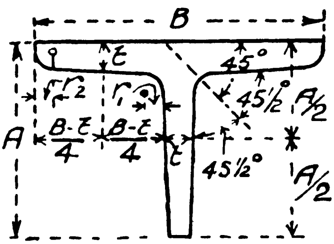
British Standard Equal Angles (B.S.E.A.) and Unequal Angles (B.S.U.A.).—As indicated in diagrams (Fig. 20), an equal angle means one with equal legs, and an unequal angle one in which the legs are of unequal length. An angle section is named by giving the two leg lengths and the thickness thus: 4″ × 4″ × ½″ or 5″ × 3″ × ⅜″. The angle between the legs is 90°, though a variation from 89° to 91° is accepted. It is possible to obtain rolled angle sections with

acute angles of about 45° upwards, and obtuse angles up to 135°, but these are not often rolled.

The smallest B.S.E.A. is 1″ × 1″ × ⅛″ and the largest 8″ × 8″ × ⅞″. B.S.U.A.s are rolled as small as 2″ × 1½″ × $\frac{3}{16}$″ and as large as 8″ × 6″ × ¾″ or 9″ × 4″ × ¾″.

Stock Sizes and 'Extra' Sizes of Angles.—Angle sections 2½″ × 2½″ and larger are usually stocked in even lengths up to about 36′ or 40′. Smaller sections are in stock up to 18′ long.

Angles between 6″ and 12″ combined leg lengths, by ⅜″ thickness and upwards, are charged at *basis* price. Extras are charged when the united leg lengths are outside the stated limits. Angles less than ⅜″ thick are subject to an extra. The maximum and minimum lengths, without extras, are 60′ and 10′ respectively.



a - Section Area in sq. inches
3·4a = weight in lbs per foot
$r_1 = 0{\cdot}03(A+B) + 0{\cdot}12$
$r_2 = 0{\cdot}7 r_1$

FIG. 21.—BRITISH STANDARD T-SECTION.

British Standard T-Bars (B.S.T.).—In naming ~~a T-section the~~ flange dimension is given first. Thus a 6″ × 3″ × ½″ T-bar would indicate a flange width (B) of 6″, a web depth (including flange thickness) of 3″ (A) and a thickness of ½″. The thickness of the flange must be measured at a point half-way between the extreme edge of the flange and the nearer side of the web, and the web thickness is measured half-way between the extreme edge of web and the farther side of the flange. As indicated in the diagram (Fig. 21) the flange and web have a taper of ½°. B.S.T.s range from 1½″ × 1½″ × ¼″ to 6″ × 6″ × ⅝″. Special Tees can be obtained with the flange thickness different from that of the web, but only in considerable quantities of a size.

Stock Sizes and 'Extra' Sizes of Tees.—Tees 3″ × 3″ and

larger are stocked in lengths up to 36′, smaller sizes up to about 20′.

Tees having the sum of their overall dimensions from 6″ to 12″, and thickness ⅜″ and upwards, are charged an extra over the 'angle' basis price. A further extra is charged if the 'united inches' is less than 6″, or over 12″. Thicknesses less than ⅜″, and lengths over and under 60′ and 10′ respectively, lead to extra charges.

British Standard Channels (B.S.C.).—The profile given in Fig. 22 indicates that the flanges have a taper of 5°. Standard

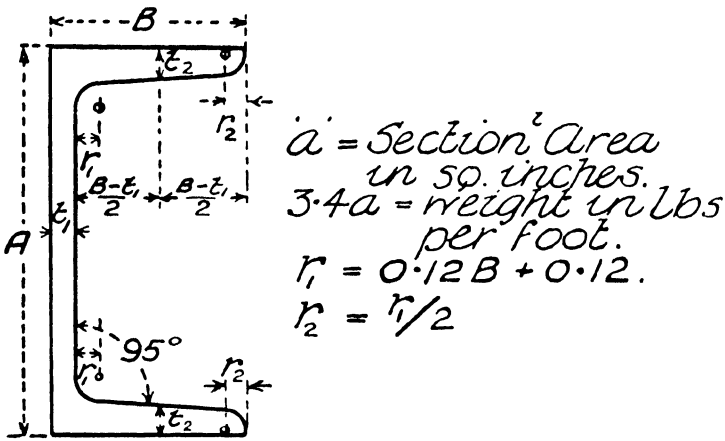


FIG. 22.—BRITISH STANDARD CHANNEL SECTION.

flange thickness is measured half-way between the extreme edge of flange and nearer side of web. In describing a channel, the web depth is given first, then the flange width and weight per foot. A 7″ × 3″ × 14·22 lb. B.S.C. would therefore be 7″ deep overall, with a flange width of 3″, and weight per foot of 14·22 lb.

The smallest B.S.C. is 3″ × 1½″ × 4·60 lb. and the largest 17″ × 4″ × 51·28 lb.

Stock Sizes and 'Extra' Sizes of Channels.—B.S.C.s are usually in stock from 5″ × 2½″ and up, in even lengths up to 36′; sections under 5″ depth, up to 20′ or 30′.

Channels 6″ to 12″ deep, 3″ flange width and under, having a $\frac{5}{16}$″ web thickness and over, are charged an extra on the angle basis price. If the depth is under 6″, or over 12″, an additional extra is levied, also for thicknesses less than $\frac{5}{16}$″. The maximum and minimum lengths are respectively 60′ and 10′, for no extra.

Plates.—The thinnest plate rolled in an ordinary plate mill is a $\frac{1}{4}$″ plate. Thinner plates than these are usually termed *sheets*. The maximum thickness is 2″, without special arrangements being made. Plates are rolled to a maximum area, the area depending on the plate thickness. Corresponding to each thickness there is also a maximum length and a maximum width. Both the maxima cannot be obtained together, so that the maximum width for any given thickness equals the listed maximum area divided by the length required. For example, a $\frac{3}{8}$″ plate has a maximum length of 55′ and a maximum width of 110″, but a maximum area of only 250 sq. ft. A $\frac{3}{4}$″ plate has a maximum length of 60′, maximum width of 156″ and a maximum area of 320 sq. ft. Intermediate thicknesses will have correspondingly intermediate values to those given.

Stock Sizes and Extra Sizes.—$\frac{3}{8}$″ to $\frac{3}{4}$″ plates are usually in stock in various sizes up to 6′ wide by 30′ 6″ long. Maximum stock thickness is usually about $1\frac{1}{4}$″, with a width of 4′ and length of 8′. Plates having thicknesses from $\frac{3}{8}$″ to $1\frac{1}{2}$″ are charged the basis price, those with thicknesses outside these limits have extras. The width to be obtained without extra depends on the plate thickness. For $\frac{1}{4}$″ plate, maximum width = 66″; for $\frac{1}{2}$″ and up, the maximum width = 96″. Long thin plates and narrow plates are subject to extras. Plates weighing more than 4 tons each, or under 4 sq. ft. in area, are rated above the basis price.

Flats.—An inspection of the diagrams, indicating the forms of rolls employed (Fig. 18), will show the difference between the methods employed in rolling plates and flats respectively. In the case of flats, it will be seen that the rolls bear on the edges, and thus have a control on the width. There is a special type of flat known as a *universal plate* or *wide flat*. For long and

narrow details, as in plate girder work, flats are superior to plates. Ordinary flats are obtainable up to a width of $20\frac{1}{4}''$, which size has a minimum thickness of $\frac{5}{16}''$ and a maximum of $\frac{5}{8}''$.

Stock Sizes and Extra Sizes.—The largest usual stock size is $18''$ wide, with a thickness from $\frac{1}{2}''$ to $\frac{3}{4}''$. Widths under $6''$ are usually stocked up to $18'$, and from $30'$ to $40'$ for greater widths. Minimum stock width is $\frac{1}{2}''$, with a thickness $\frac{3}{16}''$ to $\frac{3}{8}''$. Universal plates can be obtained from about $6''$ width to $47''$ width, but the usual stock limits are $14''$ width (thickness $\frac{3}{8}''$–$\frac{5}{8}''$, maximum length $90'$) and $36''$ width (thickness $\frac{1}{4}''$–$\frac{3}{4}''$, maximum length $115'$). Flats over $5''$ width or $\frac{1}{2}''$ thickness are charged an extra over the angle basis price, and there is a special rate for $5''$ width and under. Lengths exceeding $40'$ are subject to an extra.

Rounds and Squares.—Rounds are rolled from $\frac{3}{16}''$ diameter to $12''$ diameter. The stock lengths are up to $14'$ for $\frac{3}{4}''$ diameter and under, up to $24'$ for diameters over $\frac{3}{4}''$. $3''$ to $5\frac{1}{2}''$ represents basis price for $3''$ and up, extras are charged for greater diameters. Squares are rolled in sizes from $\frac{3}{16}''$ side to $8''$ side. Stock lengths and extras are as for rounds.

Special Forms of Beams.—An important example of a beam section which is not included in the British Standard lists is the *broad flange beam.* Messrs. R. A. Skelton roll such beams and their Handbook No. 21A gives full particulars of *Broad Flange Beams, Grey Process.* By the courtesy of Messrs. Skelton a table of section properties of a selection of these beams is given on pages 108 to 111.

Broad flange beams have certain special advantages. They possess the carrying capacity of built-up girders without the disability of riveting. The broad flange provides a suitable wide bearing for walls. A pair of ordinary rolled steel joists of given depth may often, with advantage, be replaced by one single broad flange beam of the same depth. The Grey Process rolls the metal on all faces. Fig. 23 (reproduced by permission from Handbook No. 21A referred to) shows the stages in the production of a 'Broad Flange Beam, Grey Process.' There are three stages, viz. (*a*) rolling the ingot in an

ordinary Blooming Mill into the bloom shape shown in first diagram, (*b*) passing the bloom through a mill with rolls as shown in second diagram, (*c*) finishing off in the Finishing Mill, with rolls as illustrated in the third and fourth diagrams. (The last process involves two sets of rolls which are placed close together in the mill.)

THE GREY PROCESS.

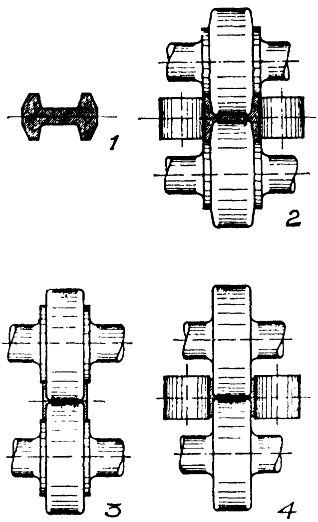


FIG. 23.—ROLLING BROAD FLANGE BEAMS, GREY PROCESS.

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Rivets and Bolts

The plates and sections used in steelwork construction are usually connected together by riveting, or bolting. The welding of structural steelwork is gradually becoming accepted as a standard constructional process, and recognition, in the form of a British Standard Specification (B.S.S. No. 538–1940), has given the science a definite place in the technique of steel construction. The metal arc welding of structural members is dealt with in Chapter XIV.

Riveting and bolting form the subject of a number of clauses

in B.S.S. No. 449 and in the various London Building Act regulations. The London Building Act requires rivets to be used where reasonably practicable. If bolts are used, they must extend through the full thickness of the nuts—which themselves must be secured against the possibility of working loose. The B.S.S. and L.C.C. By-laws (No. 74) require as much as possible of the fabrication to be completed in the works with the use of rivets, or turned bolts of driving fit. *Black bolts* (i.e. bolts as manufactured, and not turned down to precise size) may be used in the circumstances noted later.

Rivets.—The type of steel used in the manufacture of rivets

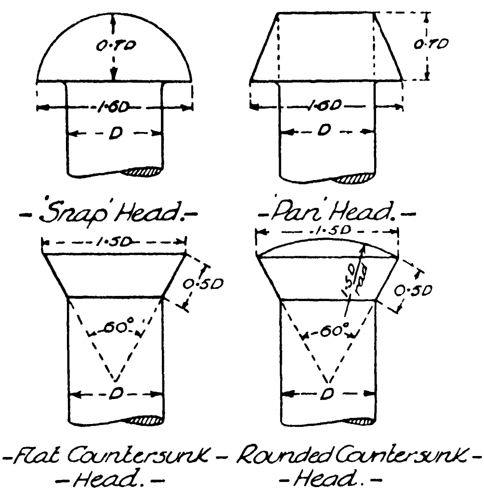


FIG. 24.—FORMS AND PROPORTIONS OF RIVET HEADS.

is that described on page 20, Chapter II. The usual form of rivet head employed in structural steelwork is the *snap* head (Fig. 24). Snap heads and *pan* heads form a projection beyond the plate face, and where this is an objection—as in bearings, where continuity of contact between plate and plate, or between plate and masonry, is necessary—a *countersunk* head is employed. Occasionally snap heads are flattened a little to provide clearance.

B.S.S. No. 153–1933, Part 2, deals with the requirements of

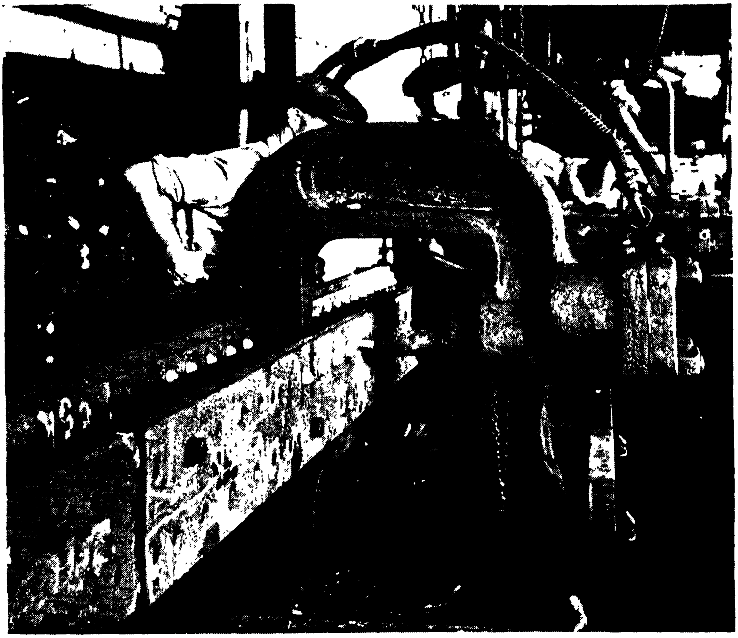
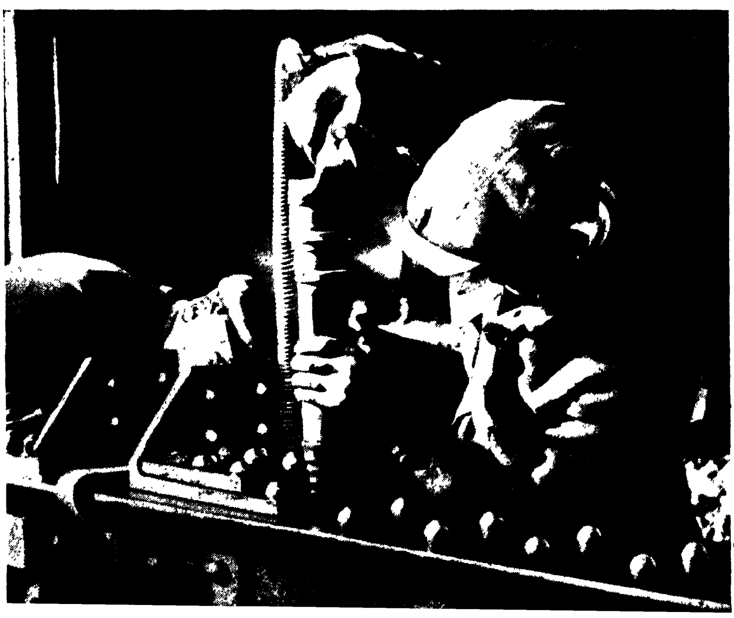


FIG. 25.—HYDRAULIC RIVETING.
(Rivet head about to be formed.)
Reproduced by permission and courtesy of Messrs. Dawnays Ltd.



good riveting. It states that rivet holes should preferably be drilled through the solid metal. In cases where a compound girder or plate girder is built up of several plates and sections the parts should be firmly clamped, or tacked together, with temporary bolts, and the holes drilled through in one operation. This procedure ensures correct alignment of holes. Any burrs formed around the holes by drilling should be removed, and the parts re-bolted together in preparation for final riveting-up. The practice of driving in *drifts* (slightly tapered round bars of iron), in order to effect alignment of holes, is forbidden by the B.S.S., but drifts may be used to position the various parts together.

Methods of Riveting.—The rivet, having been rendered soft by heating, is placed in the rivet hole prepared and *closed*, i.e. the second head is formed. The contraction in length on cooling tends to draw the parts connected closer together. The closing of the rivet may be effected in several ways, but the B.S.S. recommends some form of machine riveting, preferably of the pressure type. In the hydraulic riveter (Fig. 25) water pressure is utilised to force a die on to the soft rivet shank, while the other end of the rivet is held firmly by a stationary die. The water, in some forms of pressure riveters, is replaced by compressed air. Such machines are used in fabrication shops and are suitably installed for rapidly dealing with the various riveting operations which are carried out there. The closing of the rivets by pressure leads to the best results, as the rivet hole becomes compactly filled with the metal. Site riveting, and some shop riveting, is carried out by a pistol-shaped compressed-air machine known as the *Pom-pom* or *Pneumatic Hammer* (Fig. 26). The head is formed in this case by a rapid succession of blows. The rivet is held tightly in position by one of the riveters (usually known as the 'holder-up'), while the other man plays skilfully upon the projecting shank until he has formed—by repeated blows—the necessary rivet head. This form of riveting gives rise to considerable noise which resembles a machine gun in operation, and in some cases, near hospitals, bolting has been resorted to in order to obviate the nuisance. There is a risk of rivet heads

being formed by the ' Pom-pom ' without the holes being completely filled, and shop pressure riveting is to be preferred.

In order that the hot rivet shall easily be placed in position in the rivet hole, it is necessary to make the diameter of the hole bigger than the nominal rivet diameter. The maximum *clearance* (i.e. difference in diameters) permitted by B.S.S. is $\frac{1}{16}''$.

Bolts and Nuts.—All bolts and nuts should be of mild steel of the quality specified for round bars in B.S.S. No. 15. The B.S.S. requires standard Whitworth threads, and the head and

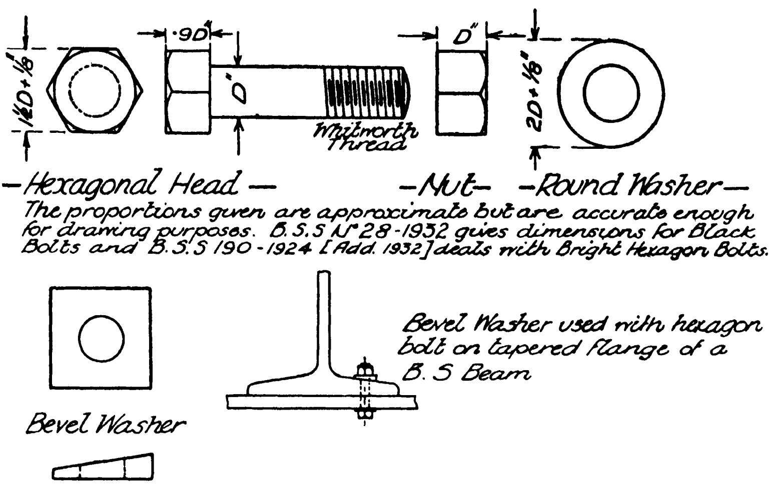


FIG. 27.—HEXAGON BOLTS. ROUND AND BEVEL WASHERS.

nut must be of hexagonal shape (Fig. 27). Tapered washers must be provided for heads and nuts bearing on bevelled surfaces. Washers must be used with all ' turned ' bolts, the washer under the nut being at least $\frac{1}{4}''$ thick.

Two varieties of bolts are used: (i) *turned and fitted* bolts, (ii) *black* bolts.

Turned bolts are carefully turned parallel throughout the length of the barrel. The maximum clearance allowed in the case of turned bolts is ·01". It will be observed, from the list of working stresses given in Chapter II, that a turned and fitted bolt is regarded as being the equal of a shop rivet.

'Black' bolts are not reduced to precise size, and the hole diameter for such bolts can be made $\frac{1}{16}''$ bigger than the nominal bolt diameter. Black bolts are not therefore a tight fit in the hole. The allowable shear stress is consequently not so high as for rivets, or turned bolts. Black bolts are not permissible for all purposes, even with the lower stress value. In shop fabrication L.C.C. regs. permit the use of black bolts for the end cleat connections of secondary floor beams. Such bolts may be used on 'site' for roof-truss work and end connections of secondary floor beams; also for some other field connections, if the shear forces are otherwise resisted. Bolts, turned and black, must have washers under the nuts of such thickness that the thread is clear of the hole in the plate. The shanks must also project at least one full thread beyond the nuts.

The following B.S.S. are referred to in this chapter: No. 4–1932, No. 4A–1934, No. 15–1936, No. 449–1937, No. 153–1933, No. 538–1940, No. 275–1927. The extracts have been made by permission of the British Standards Institution, 28 Victoria Street, London, S.W.1 (see Appendix I), from whom official copies of the specifications may be obtained, price 2*s.* 2*d.* each, post free.

EXERCISES 3

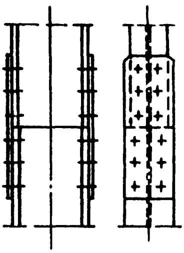
(1) Give a few considerations which influence the choice of a structural section, apart from the question of strength.

(2) What are the trade allowances for structural sections in (*a*) length, and (*b*) weight?

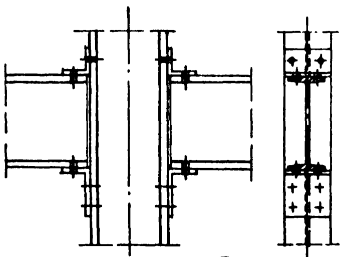
(3) Name the British Standard sections in common use. Which dimension is given first in naming (*a*) a tee-bar, (*b*) a beam?

(4) Which of the standard sections have tapered flanges, and which have not? Give the angle between the component parts in each case, and state where the standard thicknesses are measured.

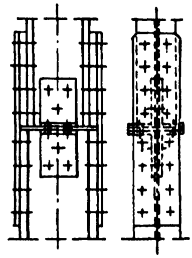
(5) Distinguish between a plate and a sheet, and between a plate and a flat. Give the maximum thickness of plates usually kept in stock.



—Fig. A.—

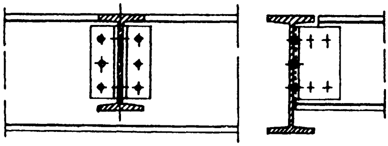


—Fig. B.—



—Fig. C.—

Fig. A. Pillar joint shewing reduction in flange width.
Fig. B. Connection of beam to flange of pillar.
Fig. C. Pillar joint with reduction in flange width and depth of section.



—Fig. D.—

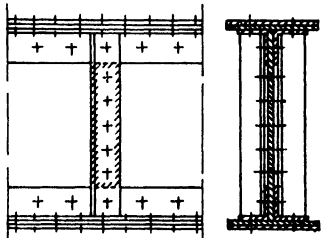
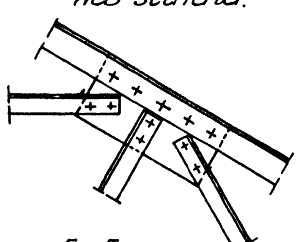


Fig. D. Beam to beam connection. —Fig. E.—
Fig. E. Girder built up of plates and angles with angle web stiffener.



—Fig. F.—

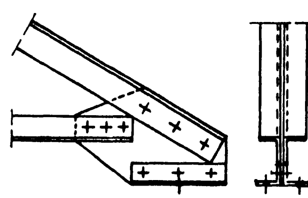
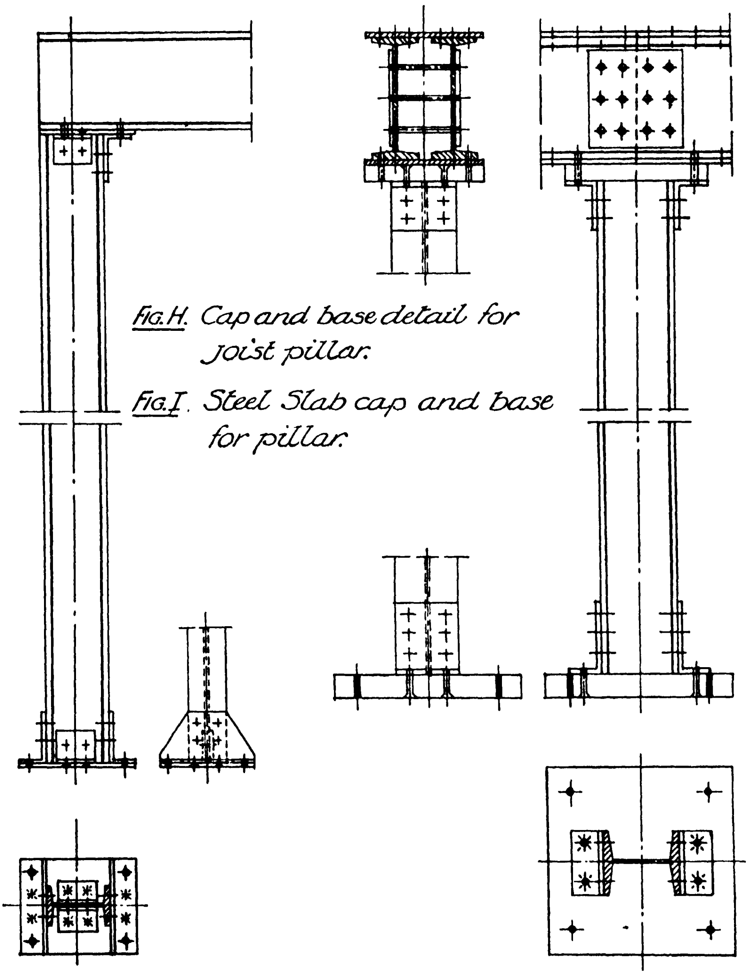


Fig. F. Joint on rafter of a roof truss. —Fig. G.—
Fig. G. Detail of shoe of roof truss.

Plate I(*a*)

Plates I(a) and I(b) reproduced by permission and courtesy of Messrs. Redpath, Brown & Co., Ltd.

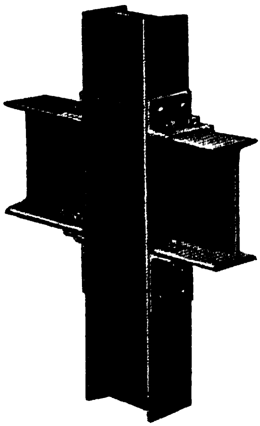


—FIG. H.— —FIG. I.—

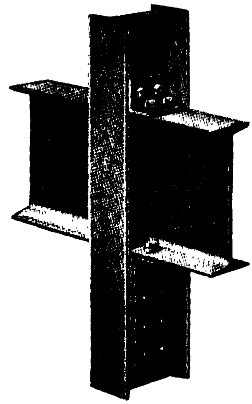
PLATE I(*b*)

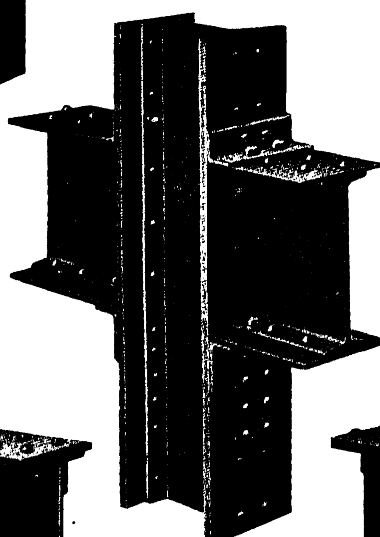
REDPATH, BROWN & CO., LIMITED

CONNECTIONS OF BEAMS TO STANCHIONS



ALL CLEATS BOLTED TO BEAM AND STANCHION EXCEPT SEAT ANGLES TO STANCHIONS





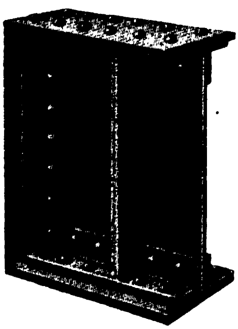


PLATE GIRDERS, END ANGLES AND STIFFENERS

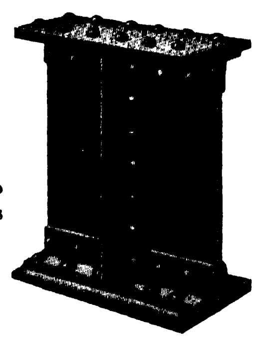
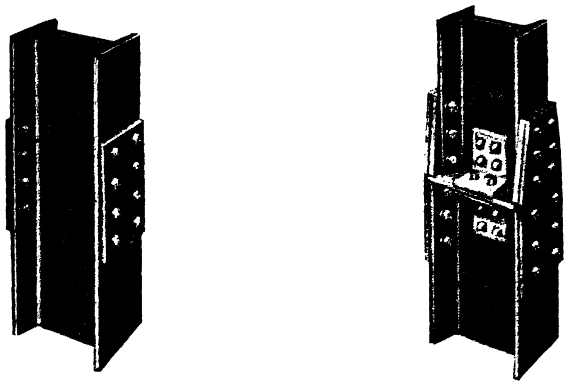


Plate II(*a*)

REDPATH, BROWN & CO., LIMITED

PILLAR JOINTS AT FLOORS: NO BENDING

UNPLATED JOIST STANCHIONS



PLATED JOIST STANCHIONS

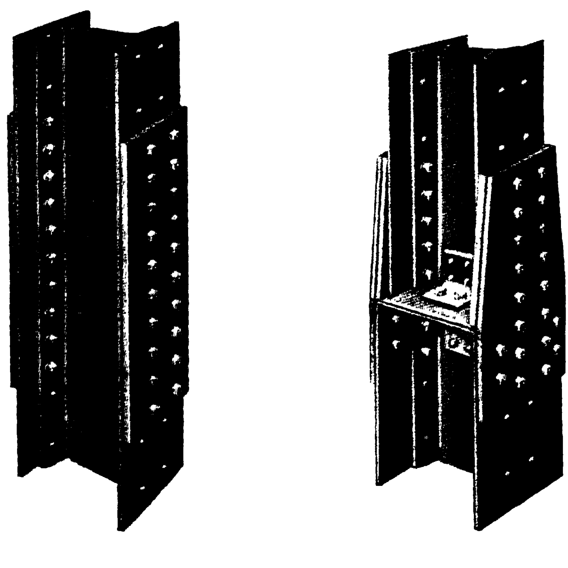


PLATE II(*b*)

Plates II(*a*) *and* II(*b*) *reproduced by permission and courtesy of Messrs. Redpath, Brown & Co., Ltd.*

(6) Name and sketch the forms of rivet heads generally adopted in structural steelwork. Indicate the most commonly used of these forms and give, for this head, the proportions laid down in the B.S.S.

(7) Distinguish between 'black bolts' and 'turned bolts.' Give any L.C.C. regulations you know which affect the use of these bolts.

(8) Give values of working stresses in shear which exemplify the superiority of (*a*) turned bolts over black bolts, (*b*) hydraulic shop riveting over field riveting with the pneumatic hammer.

CHAPTER IV

PRACTICAL DESIGN OF RIVETED AND BOLTED CONNECTIONS

Introduction.—The general principles of design are the same whether the connecting together of the units of steelwork is effected by riveting or by bolting. The working stresses for rivets driven at the works and for tight-fitting bolts being the same, their respective use becomes largely a matter of practical convenience. The lower working stress for site-driven or *field* rivets has already been referred to, and black bolts have a limited application (by regulations) even with their reduced working stress. In the normal steelwork connection a rivet or bolt is called upon to resist 'shear' and *bearing* stresses only. Rivets and bolts may, however, under certain conditions, be designed to resist tension. In the connections dealt with in this chapter the rivets or bolts are not intentionally subjected to tension. The line diagrams given on Plates (*a*) and (*b*) (kindly supplied by Messrs. Redpath, Brown & Co., Ltd.) illustrate a number of types of structural steelwork connections, in which bolts and rivets of various forms are used. The conventional methods adopted to indicate these forms should be carefully noted, particularly those where countersinking is necessary (denoted by asterisk).

Strength of One Rivet (or Bolt)

(1) **Shear Strength.**—According to the type of a given joint, the connecting rivets may be subjected to *single shear* or to *double shear*. Fig. 28 illustrates these two shearing tendencies.

In *single shear* the shearing action is across one cross-sectional plane of the rivet. In *double shear* two such cross-sectional areas are involved.

If d'' = diameter of rivet, the area of metal provided in one cross-sectional area $= \dfrac{\pi d^2}{4}$ sq. ins. Using the symbol f_s for the

working stress in shear (in tons per sq. in.) for the rivet material, the formula for the strength of one rivet in single shear becomes $\frac{\pi d^2}{4} f_s$ tons. The sectional area of metal provided being twice as much, in the case of double shear, as that in the case of single shear, the corresponding expression for the strength of one rivet in double shear is $\frac{2\pi d^2}{4} f_s$ tons. The latter value is

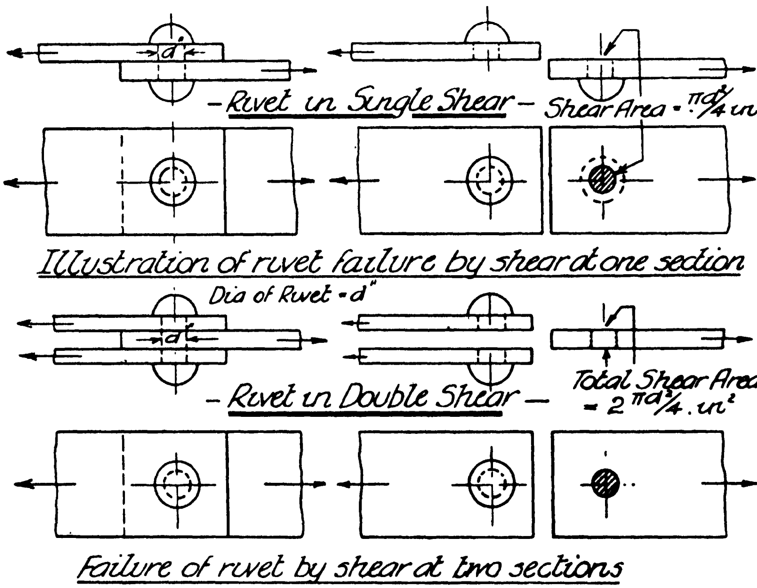


FIG. 28.—SINGLE SHEAR AND DOUBLE SHEAR IN A RIVET OR BOLT.

accepted by most steelwork regulations now (including L.C.C. regs.), but the London Building Act 1930 gives the strength in double shear as being only 1¾ times that in single shear.

(2) **Bearing Strength.**—If we walk on smooth sand the depth of the impression left depends upon the type of shoe worn. Flat-bottomed sand shoes, which provide a large *bearing* area, would cause a shallow depression, but, if shoes with well-defined heels be worn, the impression is much deeper. In the latter case the *intensity of bearing pressure* is higher, owing to the reduction in bearing area. In the same way, the intensity of bearing stress between a plate and a rivet—for a given

applied force in the plate—becomes greater as the bearing area between the two becomes less. With the usual plate thicknesses the bearing strength of a rivet is less than its strength in double shear, but greater than that in single shear.

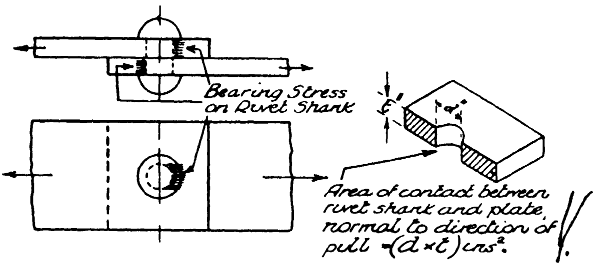


FIG. 29.—BEARING STRESS IN RIVET OR BOLT.

As shown in Fig. 29, the bearing area is taken as *diameter of rivet* $\times$ *plate thickness*, i.e. $d \times t$ sq. ins. If f_b tons per sq. in. be the working stress in 'bearing' for the rivets, the strength of one rivet in bearing will be given by the formula 'dtf_b' tons. We have therefore the following three important formulæ:

$$\left.\begin{array}{lll} \textbf{Single shear (S.S.) strength of one rivet} & = \dfrac{\pi d^2}{4} f_s & \text{tons.} \\ \textbf{Double shear (D.S.)} \quad ,, \quad ,, \quad ,, & = \dfrac{2\pi d^2}{4} f_s & ,, \\ \textbf{Bearing} \quad ,, \quad ,, \quad ,, & = dtf_b & ,, \end{array}\right\}$$

The actual strength, or value, of one rivet in a joint will be the lesser of its shear and bearing values.

The value of d in the formulæ above may be taken to be the diameter of the *finished* rivet, i.e. the actual diameter of the rivet hole.* Engineers often assume d to be the *nominal* rivet diameter (the value taken throughout the calculations in subsequent problems). In the case of countersunk rivets, one half of the depth of the countersink must be omitted, in calculating bearing area.

EXAMPLES

(i) *Calculate the actual value or worth, in tons, of one rivet in the following circumstances: Rivet diameter* $= \frac{3}{4}''$, *plate thickness* $= \frac{1}{2}''$; *the rivets are in double shear and are works driven.*

* Rivet hole diameter always, when calculating loss of plate area.

The working stresses to be used in this case are $f_s = 6$ tons per sq. in.; $f_b = 12$ tons per sq. in.

D.S. value of one rivet $= \frac{2\pi d^2}{4} f_s = \frac{2\pi(\frac{3}{4})^2}{4} \times 6 \text{ tons} = 5{\cdot}3$ tons.

B.V. (bearing value) $= dtf_b = \frac{3}{4} \times \frac{1}{2} \times 12$ tons $= 4{\cdot}5$ tons.

$\therefore$ the actual value of one rivet (V) in this case $= 4{\cdot}5$ tons.

(ii) *Find the maximum safe load, from the point of view of rivet strength, for the joint given in Fig.* 30.

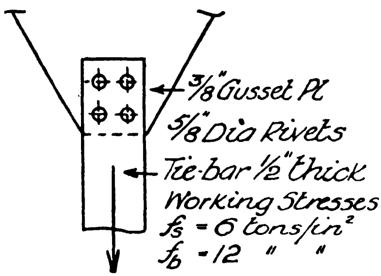


FIG. 30.

In this case the rivets are in single shear and bearing in $\frac{3}{8}''$ plate (the thinner of the two plates concerned).

$$\text{S.S. value of one rivet} = \frac{\pi d^2}{4} f_s = \frac{\pi \times \frac{5}{8}^2}{4} \times 6 \text{ tons}$$
$$= 1{\cdot}84 \text{ tons.}$$
$$\text{B.V. of one rivet} = dtf_b = \frac{5}{8} \times \frac{3}{8} \times 12 \text{ tons}$$
$$= 2{\cdot}81 \text{ tons.}$$
$$\text{Actual value} = 1{\cdot}84 \text{ tons.}$$
$$\text{Rivet strength of joint} = (4 \times 1{\cdot}84) \text{ tons}$$
$$= 7{\cdot}36 \text{ tons.}$$

Table of Rivet and Bolt Strengths

The tables on pages 50 and 51 give the shearing and bearing values for rivets and bolts, for the usual working stresses, and for the plate thicknesses in common use. The figures shown in italics will be found useful in allowing for loss of sectional area, caused by rivet holes in tension members. The rivet hole is assumed to be $\frac{1}{16}''$ greater in diameter than the nominal rivet diameter. Readers will be able to check the rivet values already

calculated, and those in subsequent examples, by reference to these tables. The figure 8 tons/in.² at the head of the tables refers to the tensile value of the steel for which the given rivet stresses are suitable. A high tensile steel will require the rivet material to exhibit higher working stresses in order to maintain equivalent strength.

Rivet Diameter and Plate Thickness.—The choice of a suitable rivet diameter for a given structural connection involves several factors, and it is not possible to lay down any hard and fast rule. As far as possible rivet diameters are kept constant throughout any particular built-up unit. For example, $\frac{3}{4}''$ diameter rivets might be used solely throughout a plate girder, and the rivet positions designed to maintain this uniformity. It is usual to select a rivet size with reference to the plate thicknesses involved. A useful formula can be obtained by equating the shear and bearing strengths of a rivet or bolt.

$$\frac{2\pi d^2}{4} f_s = dtf_b$$

$$\therefore\ d = \frac{4t}{2\pi} \times \frac{f_b}{f_s}. \quad \text{Taking } \frac{f_b}{f_s} = 2,$$

$$d = 1{\cdot}3\, t.$$

Thus for a $\frac{5}{8}''$ plate, d would be $1{\cdot}3 \times \frac{5}{8}'' = \frac{7}{8}''$. The size of rivet to be used in standard cases of compound girders, etc., will be found tabulated in the section books issued by steel firms. The diameters in common use in building work are $\frac{3}{4}''$ and $\frac{7}{8}''$. Heavy engineering work requires 1″ diameter rivets sometimes.

Design of Riveting Detail

The positioning of rivets and rivet lines forms the subject of a number of practical regulations and theoretical considerations. The latter will be taken up later in the book, but it is essential that the reader should early become familiar with the requirements of standard regulations with respect to riveting. B.S.S. 449 and the L C.C. reg's. are in close agreement in regard to the main essentials.

SHEARING & BEARING VALUES FOR BOLTS & RIVETS

8 Tons/inch²

Figures in italic type represent the area to be deducted from any bar for one hole 1/16 inch larger in diameter than bolt or rivet.

Dia of Bolt or Rivet in inches	Area in square inches	Shearing Value @ 6 tons/inch²		Bearing Values @ 12 tons/inch² — Thickness in inches of plate passed through									
		Single Shear	Double Shear	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	7/8
3/8	0·1104	0·66	1·33	1·13 *109*	1·41 *137*	1·69 *164*							
1/2	0 1963	1·18	2·36	1·50 *·141*	1·88 *·176*	2·25 *·211*	2·63 *·246*	3·00 *·281*					
5/8	0·3068	1 84	3·68	1·88 *172*	2·34 *·215*	2·81 *·258*	3·28 *·301*	3·75 *·344*	4·22 *·387*				
3/4	0·4418	2·65	5 30	2·25 *203*	2·81 *254*	3·38 *·305*	3·94 *·355*	4·50 *·406*	5·06 *·457*	5·63 *·508*	6·19 *·559*		
7/8	0 6013	3 61	7 22	2·63 *·234*	3·28 *·293*	3·94 *·352*	4·59 *·410*	5·25 *·469*	5·91 *·527*	6·56 *·586*	7·22 *·645*	7·88 *·703*	
1	0 7854	4 71	9·42	3·00 *·266*	3·75 *·332*	4·50 *·398*	5·25 *·465*	6·00 *·531*	6·75 *·598*	7·50 *·664*	8·25 *·730*	9·00 *·797*	10 50 *·930*

Dia of Rivet in inches	Area in square inches	Shearing Value @ 5 tons/inch²		Bearing Values @ 10 tons/inch² — Thickness in inches of plate passed through									
		Single Shear	Double Shear	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	7/8
3/8	0 1104	0·55	1 10	0 94 *109*	1 17 *·137*	1·41 *·164*							
1/2	0·1963	0·98	1·96	1·25 *141*	1·56 *·176*	1·88 *·211*	2·19 *·246*	2·50 *·281*					
5/8	0 3068	1·53	3·07	1·56 *172*	1·95 *·215*	2·34 *·258*	2·73 *·301*	3·13 *·344*	3·52 *·387*				
3/4	0·4418	2·21	4·42	1 88 *·203*	2·34 *·254*	2·81 *·305*	3·28 *·355*	3·75 *·406*	4·22 *·457*	4·69 *·508*	5·16 *·559*		
7/8	0·6013	3·01	6·01	2·19 *·234*	2·73 *·293*	3·28 *·352*	3·83 *·410*	4·38 *·469*	4·92 *·527*	5·47 *·586*	6·02 *·645*	6·56 *·703*	
1	0·7854	3·93	7 85	2·50 *·266*	3·13 *·332*	3·75 *·398*	4·38 *·465*	5·00 *·531*	5·63 *·598*	6·25 *·664*	6·88 *·730*	7·50 *·797*	8·75 *·930*

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8 Tons/inch²

SHEARING & BEARING VALUES FOR BOLTS & RIVETS

Figures in italic type represent the area to be deducted from any bar for one hole $\frac{1}{16}$ inch larger in diameter than bolt or rivet.

Dia. of Bolt in inches	Area in square inches	Shearing Value @ 4 tons/inch²		Bearing Values @ 8 tons/inch² — Thickness in inches of plate passed through									
		Single Shear	Double Shear	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{7}{8}$
$\frac{3}{8}$	0·1104	0·44	0·88	0·75 *·109*	0·94 *·137*	1·13 *·164*							
$\frac{1}{2}$	0·1963	0·79	1·57	1·00 *·141*	1·25 *176*	1·50 *·211*	1·75 *·246*	2·00 *·281*					
$\frac{5}{8}$	0 3068	1·23	2·45	1·25 *·172*	1·56 *·215*	1·88 *·258*	2·19 *·301*	2·50 *·344*	2·81 *·387*				
$\frac{3}{4}$	0·4418	1·77	3·53	1·50 *·203*	1·88 *·254*	2·25 *·305*	2·63 *·355*	3·00 *·406*	3·38 *·457*	3·75 *·508*	4·13 *·559*		
$\frac{7}{8}$	0·6013	2·41	4·81	1·75 *·234*	2·19 *·293*	2·63 *·352*	3·06 *·410*	3·50 *·469*	3·94 *·527*	4·38 *·586*	4·81 *·645*	5·25 *·703*	
1	0·7854	3·14	6·28	2·00 *·266*	2·50 *·332*	3 00 *·398*	3·50 *·465*	4·00 *·531*	4·50 *·598*	5·00 *·664*	5·50 *·730*	6·00 *·797*	7·00 *·930*

Maximum working stresses on bolts and rivets as prescribed in the British Standard Specification (No. 449—1937) for the Use of Structural Steel in Building, and in the London County Council By-laws.

For parts in shear

	Tons/inch²
On shop rivets and tight fitting turned bolts	**6**
On field rivets	**5**
On black bolts, where permissible	**4**

NOTE.—The strength of rivets and bolts in double shear may be taken as twice that for single shear

For parts in bearing.

On shop rivets and tight fitting turned bolts	**12**
On field rivets	**10**
On black bolts, where permissible	**8**

of the British Steelwork Association.

Minimum Pitch of Rivets.—*The distance between centres of rivets shall not be less than three times the diameter of the rivet.*

Maximum Pitch of Rivets.—*The straight-line pitch in the direction of stress in riveted girders, columns or other members shall not exceed the following values :*

For parts in tension, 16 times the thickness of the thinnest outside plate or angle, with a maximum pitch of 8″.

For parts in compression, 16 *times the thickness of the thinnest outside plate or angle, with a maximum pitch of* 6″.

L.C.C. regs. and B.S.S. give the following variations : *Where two rows of staggered rivets occur in one flange of a single angle* (as in 5″ × 5″ or 6″ × 6″Ls) *the straight-line pitch in the direction of stress shall not exceed* $1\frac{1}{2}$ *times the above.* This will apply to angles in tension or compression. Tacking rivets, i.e. rivets merely used for tacking flange plates together, and not subjected to calculated stress (see Chapter IX) may be spaced farther apart.

Applying these rules to a $\frac{3}{4}''$ diameter rivet, the minimum pitch equals $3 \times \frac{3}{4}'' = 2\frac{1}{4}''$. Similarly for a $\frac{7}{8}''$ diameter rivet the minimum pitch $= 3 \times \frac{7}{8}'' = 2\frac{5}{8}''$. If a plated member have an outside plate $\frac{1}{2}''$ thick, or an angle thickness $\frac{1}{2}''$, the maximum pitch (for rivets in a single row in one angle flange) in the case of the tension flange $= 16 \times \frac{1}{2}'' = 8''$ (the maximum allowed in any case). For the compression flange it would be 6″ as this is less than $16 \times \frac{1}{2}''$.

Edge Distance of Rivets.—L.C.C. By-laws give the minimum distance in the form of a rule : *The distance from the edge of a rivet hole or bolt hole to the edge of a plate, bar or member shall not be less than the diameter of the rivet or bolt.* Allowing the usual $\frac{1}{16}''$ clearance in rivet holes, the minimum distance of rivet centre to plate edge is $d'' + \frac{1}{2}(d'' + \frac{1}{16}'')$ or, practically, $1\frac{1}{2}\,d''$—a rule commonly quoted.*

Riveting for Built-up Girders.—The L.C.C. By-laws give a few practical regulations respecting the edge distance of rivet lines for flange plates. These regulations are important, as they influence the choice of flange plate width in plated joists and

* Exactly $1\frac{1}{2}\,d''$ if d'' = finished rivet diameter.

girders. Fig. 31 shows two cases: (A) *where one flange plate is used,* (B) *where two or more flange plates are used.*

The maximum distance x'' permitted is measured in each case from the centre line of the rivets—connecting the flange plates to the web construction—which lie nearest the plate edge. For a single plate the maximum distance is '9 times' the plate thickness, and, for two or more plates, it is '12 times' the thickness of the thinnest outside plate.

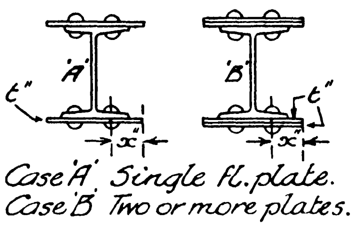


FIG. 31.—EDGE DISTANCE OF RIVETS.

If, in the case of two or more plates, the edge distance referred to exceed 9 times the thickness of thinnest outside plate, tacking rivets must be employed to hold together the plates between the edge of the main angles and the plate edge The spacing of such rivets does not involve the type of calculation given later in the book for angle rivets, but their pitch must not exceed 24 times the thickness of the thinnest outside plate or 12", whichever is the lesser.

Principles of Design of Riveted and Bolted Joints

The principles of design will be illustrated by consideration of some of the common forms of joints used in structural steelwork.

Joint in a Tie-bar.—The plates may be joined together in one of the three ways illustrated in Fig. 32. The third method shown is the best, as methods (*a*) and (*b*) have a tendency to subject the rivets to tension.

In (*a*) and (*b*) the rivets are in single shear and bearing, and in (*c*) in double shear and bearing.

Tie-bar joints have to be designed to resist liability to failure in the following ways:

(i) *By the failure of the rivets.*—If n rivets be provided

altogether in a lap joint, or to each side of the butt in a butt joint, and V tons be the value of one rivet, the rivet strength of the joint will be nV tons.

(ii) *By the tearing of the tie-bar across a section weakened by one or more rivet holes.*

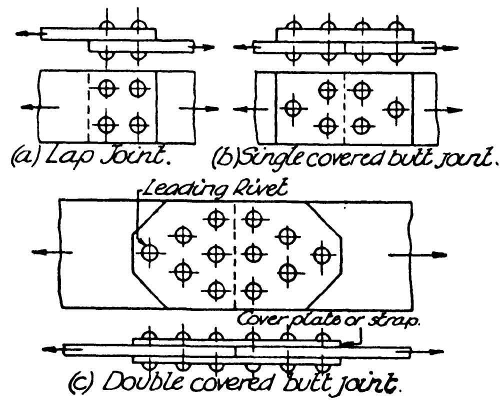


FIG. 32.—FORMS OF JOINTS IN TENSION MEMBERS.

Consider section 1 in Fig. 33. The effective solid width of the tie-bar = $(x - d)''$. The net area of metal provided = $x - d)t$ sq. ins. If f_t tons/in.^{2} = the working tensile stress

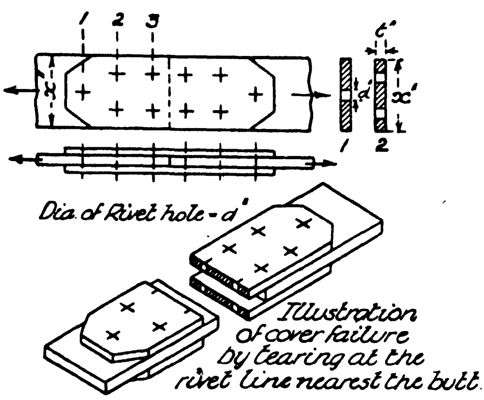


FIG. 33.—DOUBLE-COVERED BUTT JOINT IN TENSION MEMBER.

in the plate, the safe load for section 1 = $(x - d)tf_t$ tons. (Note the contracted form used for expressing the stress unit.)

Similarly the net sectional area of plate at section 2 is

$(x - 2d)t$ sq. ins., giving a strength—from the point of view of tearing only—of $(x - 2d)tf_t$ tons. But the rivet situated at section 1 acts as a peg and would have to be got rid of before failure at section 2 could actually take place. The actual strength at section 2 is therefore $[(x - 2d)tf_t + V]$ tons. In the same way the strength for section 3 would be $[(x - 2d)tf_t + 3V]$ tons. The reason for the adoption of the *leading rivet* form of rivet arrangement will now be apparent. Where a serious deduction of metal is made by rivet holes—as at section 2—compensation is afforded by the strength of one rivet, so that one hole only is made without alternative strength being supplied.

(iii) *By failure of the covers (in a butt joint).*—If the covers failed at section 3 (see Fig. 33) the joint would fail without any assistance from the rivets. The cover plate strength of the joint illustrated is given, therefore, by the expression $[(x - 2d) \times 2Tf_t]$ tons where T″ is the thickness of one cover. In a double-covered butt joint the thickness of each cover should be about $\frac{5}{8} \times$ the plate thickness.

(iv) *By the rivets being placed too near the edge of a plate.*—The tendency to split, or shear out, the intervening piece of metal between the rivet and plate edge is guarded by the rules already given for minimum edge distance.

EXAMPLE. *A tie member in a frame has to transmit an axial dead load of 32 tons. The plate thickness is to be $\frac{5}{8}''$ and the rivet diameter $\frac{7}{8}''$. Design the general joint details and evaluate the percentage efficiency of the connection. Working stresses $f_s = 6$ tons/in.², $f_b = 12$ tons/in.², $f_t = 8$ tons/in.². The joint is to be a double-covered butt joint.*

$$\text{D.S. value of one rivet} = \frac{2\pi d^2}{4} f_s = \frac{2 \times \pi \times (\frac{7}{8})^2}{4} \times 6 \text{ tons}$$
$$= 7{\cdot}2 \text{ tons.}$$

$$\text{B.V. of one rivet} = dtf_b = (\tfrac{7}{8} \times \tfrac{5}{8} \times 12) \text{ tons} = 6{\cdot}56 \text{ tons.}$$
$$\therefore V = 6{\cdot}56 \text{ tons.}$$

Number of rivets required each side of butt

$$= \frac{32}{6{\cdot}56} = 5.$$

The rivets are arranged as in Fig. 34.

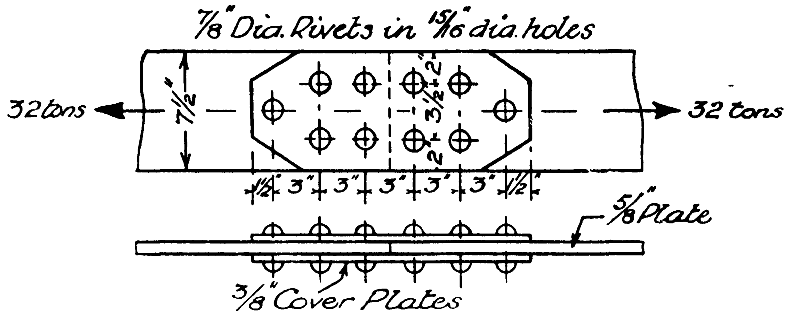


FIG. 34.—JOINT IN THE MEMBER.

Width of tie.

Section 1. $(x - d)tf_t = 32$ tons.

Allowing $\frac{1}{16}''$ clearance for rivet hole,

$$(x - \cdot 94)\tfrac{5}{8} \times 8 = 32.$$
$$(x - \cdot 94) = 6\cdot 4.$$
$$x = 7\cdot 34'', \text{ say } 7\tfrac{1}{2}''.$$

Section 2. $(x - 2d)tf_t + V = 32.$

$$(x - 1\cdot 88)\tfrac{5}{8} \times 8 + 6\cdot 56 = 32.$$
$$(x - 1\cdot 88) = \frac{25\cdot 44}{5} = 5\cdot 09.$$
$$x = 6\cdot 97''.$$

Section 3 will be stronger than section 2.

$\therefore$ necessary width of tie $= 7\frac{1}{2}''$.

Thickness of covers.

Section 3. $(x - 2d)2Tf_t = 32.$

$$(7\cdot 5 - 1\cdot 88)2T \times 8 = 32.$$
$$T = \tfrac{3}{8}''.$$

Practical rule: $T = \frac{5}{8}t = \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}''$, say $\frac{3}{8}''$.

Efficiency of Joint.—The efficiency of a joint is the ratio of its actual strength to the strength of the solid plate outside the joint.

$$\text{Percentage efficiency} = \frac{\text{Strength of joint}}{\text{Strength of solid plate}} \times 100.$$

A low efficiency would mean that the stress in the tie-bar outside the joint was considerably below the economic working value. Taking the joint designed, the rivet strength is $5 \times 6\cdot 56$ tons $= 32\cdot 8$ tons. The tearing strength is governed

by section 1 and equals $(7 \cdot 5 - \cdot 94) \times \frac{5}{8} \times 8$ tons $= 32 \cdot 8$ tons.

Cover-plate strength $= (7 \cdot 5 - 1 \cdot 88)\, 2 \times \frac{3}{8} \times 8$ tons $= 33 \cdot 72$ tons.

Actual safe load for joint $= 32 \cdot 8$ tons, i.e. the smallest of its various strengths.

Strength of solid plate outside joint $= xtf_t$ tons

$$= (7\tfrac{1}{2} \times \tfrac{5}{8} \times 8) \text{ tons} = 37 \cdot 5 \text{ tons.}$$

$$\%\ \textit{efficiency} = \frac{32 \cdot 8}{37 \cdot 5} \times 100 = 87 \cdot 5.$$

Connection of Beam to Stanchion.—Fig. B, Plate I(*a*), shows a typical connection of a beam to the flange of a stanchion. It will be observed that the seating angle cleat is riveted to the stanchion in the shop, and that site bolts or rivets effect the remaining connection. The top cleat is useful for erection purposes, but the additional strength it provides is not added in, when computing the strength of the connection.

A similar type of connection, involving similar calculations, is used to connect a beam to the web of a stanchion. When sufficient rivets cannot be provided in a seating cleat, web cleats are employed, as indicated in Fig. 35.

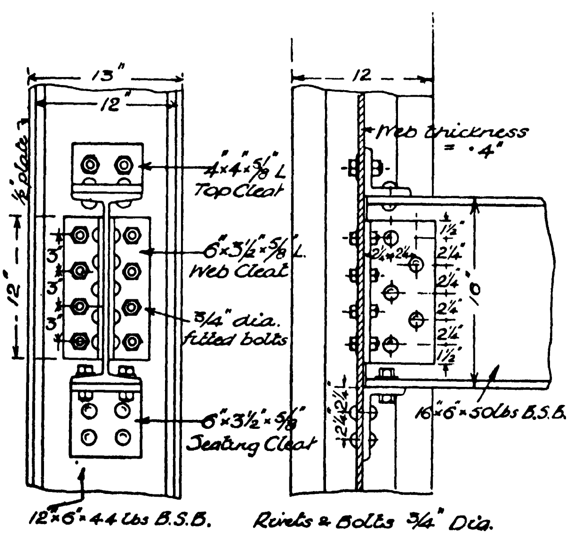


FIG. 35.—CONNECTION OF BEAM TO WEB OF STANCHION.

Taking the example shown, we have the following strength values:

Web cleat bolts. The bolts are in single shear and bearing in either ·4″ plate thickness or $\frac{5}{8}$″ (angle) thickness.

$$\text{S.S. value of one } \tfrac{3}{4}'' \text{ diameter bolt} = \frac{\pi d^2}{4} f_s$$

$$= \left(\frac{\pi \times \frac{3}{4}^2}{4} \times 6\right) \text{ tons} = 2{\cdot}65 \text{ tons.}$$

$$\text{B.V. in } {\cdot}4'' \text{ plate} = dtf_b = (\tfrac{3}{4} \times {\cdot}4 \times 12) \text{ tons}$$

$$= 3{\cdot}6 \text{ tons.}$$

$$\therefore \text{Value of one bolt} = 2{\cdot}65 \text{ tons.}$$

$$\text{For 8 bolts the safe load} = 8 \times 2{\cdot}65 \text{ tons}$$

$$= 21{\cdot}2 \text{ tons.}$$

Seating angle cleat rivets. The value of one rivet = 2·65 tons (as before).

Strength for 4 rivets = 4 × 2·65 tons = 10·6 tons.

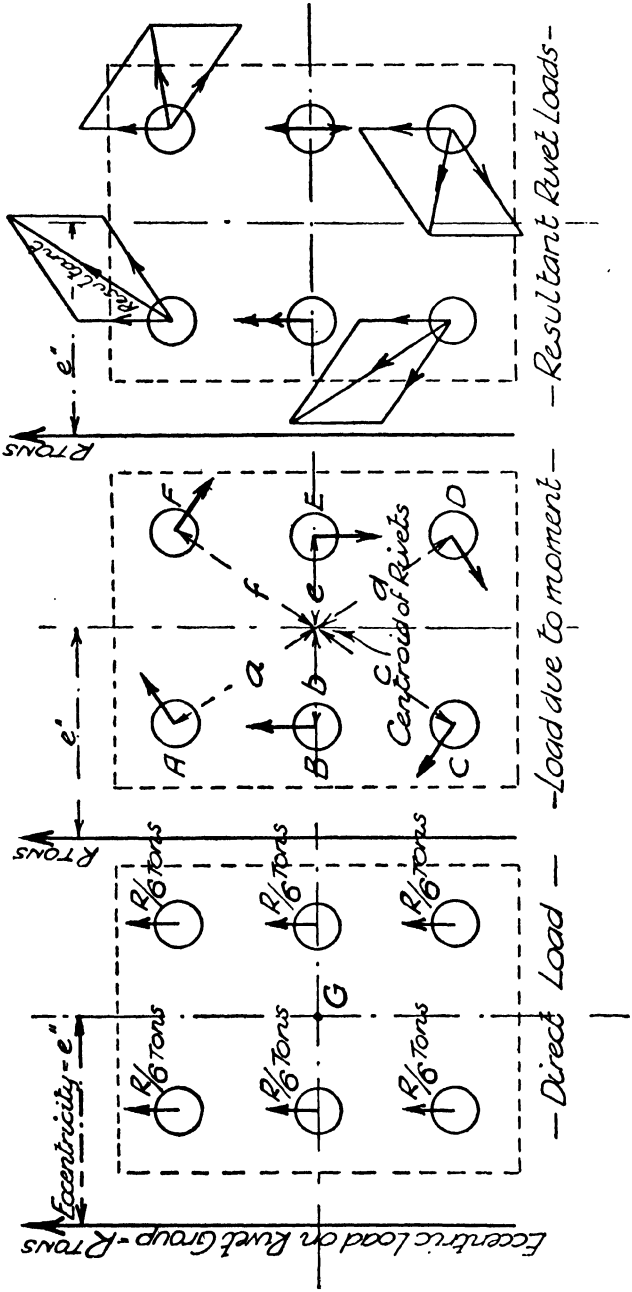
Total safe load for the connection from the point of view of the bolts and rivets (in bottom cleat) = (21·2 + 10·6) tons = 31·8 tons.

The riveting of a web cleat to the web of a beam requires calculations of a special form, illustrated in Fig. 36.

Connection of Beam to Beam.—The end connections of beams are usually of a standard design and the detail for any particular standard beam size will be found in the section books issued by steel firms. With the beam size is given a minimum span, so that the greatest load safely carried by the beam will not result in a reaction at the end exceeding the safe strength of the joint.

Fig. 37 gives a standard web cleat end connection for a 13″ × 5″ × 35 lb. B.S.B. Before calculating the maximum safe load for this connection, the effect of rotation on groups of rivets will have to be considered.

Eccentric Loading of Rivet Groups.—The theory of eccentricity of load application in members is considered in Chapter XI. We may assume here that the eccentric load R tons in Fig. 36 has two effects: (i) a tendency to push all the rivets in a direction vertically upwards, and (ii) a rotary effect, tending



36.—Rivet Group subjected to Eccentric Load.

to turn the rivet group round the centroid G of the rivets as centre. The vertical load per rivet

$$= \frac{\text{R tons}}{\text{Number of rivets}}$$

$$= \frac{\text{R}}{6} \text{ tons, in the example given.}$$

Let L tons be the load on rivet A due to the rotation effect. Rivet B, not being so far from G, will have a smaller load—in proportion to its distance. A list of loads may therefore be compiled as follows:

Load on rivet A = L tons.

„ „ „ $\text{B} = \text{L} \times \frac{b}{a}$ tons.

„ „ „ $\text{C} = \text{L} \times \frac{c}{a}$ tons, and so on.

The moments of these loads about the centroid G will be respectively $(\text{L} \times a)$ tons ins., $\left(\text{L} \times \frac{b}{a} \times b\right)$ tons ins., etc. The sum of all these moments must equal the applied turning moment.

$$\therefore \text{R} \times e = \text{L}a + \text{L}\frac{b^2}{a} + \text{L}\frac{c^2}{a} + \text{etc.}$$

$$= \frac{\text{L}}{a}(a^2 + b^2 + c^2 + \text{etc.})$$

$$= \frac{\text{L}}{a}(\Sigma\, a^2),$$ where Σa^2 (*sigma* a^2) means the sum of all such quantities as a^2.

This equation enables L to be found, from which the load, *due to the turning moment*, can be deduced for any other rivet. The two loads, ' direct ' and that due to turning moment, for any given rivet, are combined by the parallelogram of forces. The resultant load must not exceed the safe load for the rivet, from the point of view of bearing and shear.

EXAMPLE. *Find, by means of the foregoing theoretical principles, the safe load for the beam connection given in Fig.* 37.

The eccentricity of loading is, in this case, the distance from the web face of main beam to the centroid of the rivets connecting the angle cleat to the web of the secondary beam.

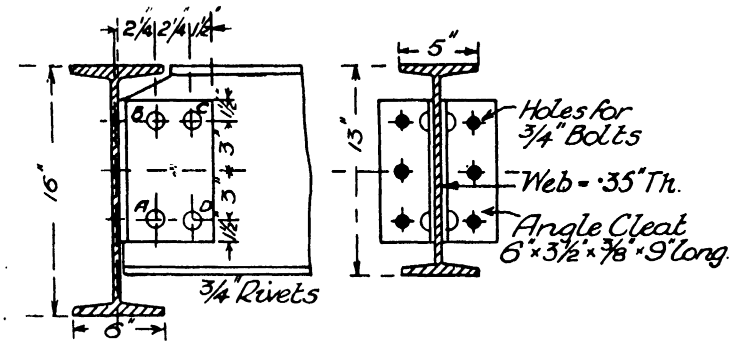


FIG. 37.—WEB CLEAT END CONNECTION FOR BEAM.

Assume a load of 1 ton to be the reaction load applied to the connection (Fig. 38). Direct load carried by one rivet $= \frac{1 \text{ ton}}{4} = \cdot 25$ tons. The turning moment about the centroid of the rivet group $= 1$ ton $\times 3\frac{3}{8}'' = 3\cdot375$ tons ins.

Let L tons be the load on rivet A.

In the example $a = b = c = d = \sqrt{1\frac{1}{8}^2 + 3^2} = 3\cdot2''$.

$$R \times e = \frac{L}{a}(\Sigma a^2).$$

$$\therefore 3\cdot375 = \frac{L}{3\cdot2}(4 \times 3\cdot2^2) = 12\cdot8L.$$

$$\therefore L = \cdot264 \text{ tons}.$$

As $a = b = c = d$ the same load will be applied to each rivet.

The maximum resultant load carried by any rivet in the group $= \cdot42$ tons.

The value of one rivet in the connection is the lesser of its double shear value and bearing value in the thickness of the secondary beam web, i.e. in $\cdot35''$ plate thickness.

D.S. value of one $\frac{3}{4}''$ shop rivet $= 5\cdot3$ tons.

B.V. in $\cdot35''$ plate $= \frac{3}{4} \times \cdot35'' \times 12$ tons $= 3\cdot15$ tons.

$\therefore V = 3\cdot15$ tons.

The reaction load transmitted to the connection can therefore be increased from 1 ton to $\frac{3\cdot15}{\cdot42}$ tons $= 7\cdot5$ tons, which is

the safe maximum load for the connection on the basis of the stated theory.

The connection must now be tested from the point of view of the bolts in the web of the main beam.

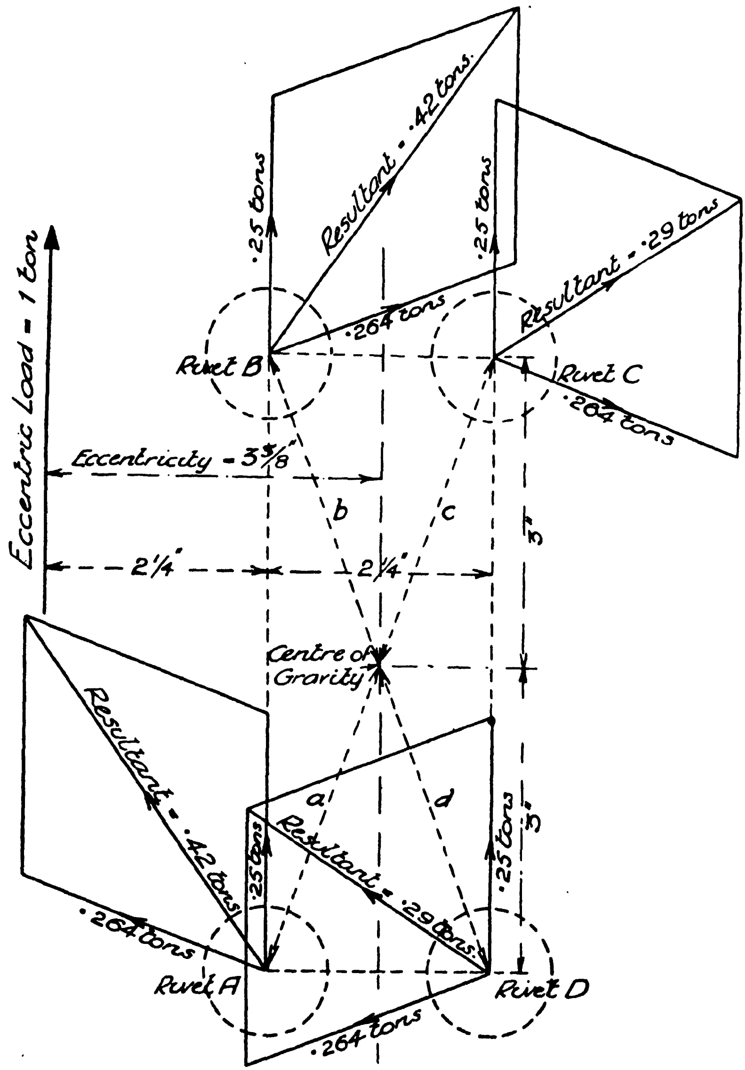


FIG. 38.—RIVETS IN BEAM CONNECTION CARRYING ECCENTRIC REACTION.

There is a tendency for some bolts in such a connection to have tensile stress developed in them, in addition to shear. A good deal of experimental work is being carried out at the present time on the end connections of beams. It is the practice of some designers to calculate the strength of the bolts on the basis of a low working stress in shear—about 4 tons per inch. Applying this to the given example, the safe load per bolt $= \frac{\pi d^2}{4} f_s$ tons $= \frac{\pi \times \frac{3}{4}^2}{4} \times 4$ tons $= 1{\cdot}76$ tons.

For 6 bolts the safe reaction load would thus be $6 \times 1{\cdot}76$ tons $= 10{\cdot}56$ tons. This value more nearly agrees with the strength of the connection, as given in 'section books,' than the value of 7·5 tons, obtained on the 'rotational' theory for the rivets in the secondary beam web.

Top flange cleats are sometimes used, but, as in beam to stanchion connections, no addition to strength is attributed to these. Seating brackets are employed for heavy reaction loads.

Fish-plated Beam Connections.—Two lengths of standard beams, which are in alignment and butt together over a support, are connected by fish-plates. The connections are standardised and will be found in section books. A typical fish-plated connection is given in Fig. I, Plate I(*b*).

There are a number of types of joints in steelwork construction, other than those already dealt with. Stanchion lengths are connected, flange and web joints have to be made in plate girder construction, and so on. Such joints involve theory not yet considered, and their design will be taken up in later chapters.

Exercises 4

(1) Find the strength of one $\frac{3}{4}''$ diameter rivet in single shear, and one $\frac{7}{8}''$ diameter rivet in double shear, using a working stress in shear of 6 tons per sq. in.

(2) Calculate the value in tons of one rivet in the following circumstances: Rivet diameter $= \frac{3}{4}''$; plate thickness $= \frac{5}{8}''$; $f_s = 5$ tons/in.^{2}; $f_b = 10$ tons/in.^{2}. Rivet is in D.S.

(3) For the connection given in Fig. 39, calculate

(*a*) Safe load for section 1.

(*b*) „ „ „ „ 2.

(*c*) „ „ „ rivets.

Hence determine the safe value of L and the percentage efficiency of the joint.

(f_s = 6 tons/in.^{2} ; f_b = 12 tons/in.^{2} ; f_t = 8 tons/in.^{2}.)

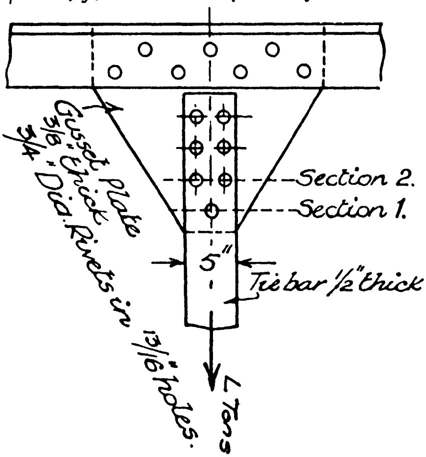


FIG. 39.—CONNECTION OF TIE-BAR TO FRAME.

(4) Obtain the safe maximum reaction load for the 15" × 6" B.S.B. shown in Fig. 40. Use the working stress values appropriate to the case. Angle bracket thickness = ½".

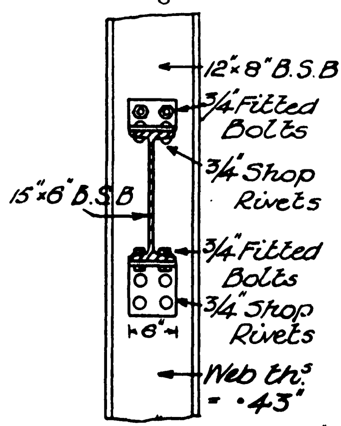


FIG. 40.—BEAM CONNECTION TO WEB OF STANCHION.

(5) Two $\frac{1}{2}''$ plates are to be connected by a double-covered butt joint. The load to be transmitted through the connection is 18 tons. The rivets are to be $\frac{5}{8}''$ diameter in $\frac{11}{16}''$ diameter holes. Taking the working stresses given in question 3 and adopting a leading rivet in the arrangement of rivets, find the number of rivets required, the necessary width of the

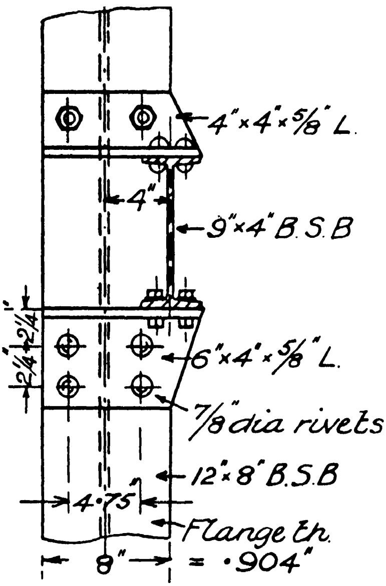


FIG. 41.—ECCENTRIC CONNECTION OF BEAM AND STANCHION.

plate, and a suitable cover thickness. Evaluate the percentage efficiency of the joint.

(6) Fig. 41 gives details of an eccentrically loaded connection. Assuming the 9" × 4" B.S.B. to transmit a load of 6 tons to the seating angle cleat, find the maximum load carried by one $\frac{7}{8}''$ diameter rivet, and test for safety.

CHAPTER V

THEORY OF BEAM DESIGN

Bending Moment and Moment of Resistance

THE subject of beam design may be divided into two parts, (i) the consideration of the effects which the external loads carried have on beams and (ii) the design of beam sections, and details, to resist these effects. The experimental model shown in Fig. 42 illustrates the nature of the forces acting across a vertical section of a loaded beam. The portion ABCD of an

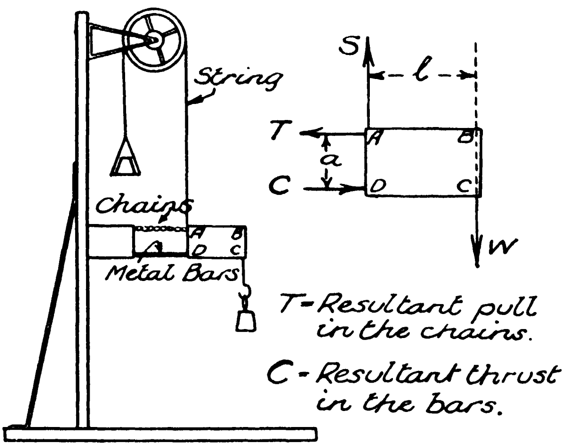


FIG. 42.—EXPERIMENTAL BEAM MODEL.

originally solid cantilever is assumed to have been cut off from the remainder at a section AD, and to have been removed to the position shown. In order to maintain this portion in equilibrium, it is necessary to introduce at the face AD certain forces, which in the model are supplied by (i) the horizontal pull in the chains, (ii) the horizontal thrust in the metal bars and (iii) the vertical pull in the string. Removal of either of these forces results in collapse of the cantilever, so that they are independently necessary for equilibrium.

Applying the laws of equilibrium to the detached portion ABCD we get:

For horizontal equilibrium **T** = **C**
For vertical equilibrium **S** = **W**.

The forces T and C constitute a couple of anticlockwise moment T (or C) $\times$ *a*. Similarly forces S and W form a couple of clockwise moment W (or S) $\times$ *l*. These two couples must have equal moment for equilibrium, therefore

$$\mathbf{T}\ (\text{or}\ \mathbf{C}) \times a = \mathbf{W}\ (\text{or}\ \mathbf{S}) \times l.$$

In a practical beam T represents the resultant of all the little fibre pulls exerted across the section AD, by the beam to the left of AD. C is, in the same way, the resultant force of all the thrusts exerted on AD. The relative positions of T and C are, of course, reversed in a simply supported beam.

W stands for the resultant vertical force (which may be compounded of forces acting vertically upwards or downwards) of all the forces acting to the right of the section AD. S represents the resistance the beam section at AD offers to vertical shear.

The moment, W $\times$ *l*, of the couple tending to bend the beam is termed the *bending moment* at the section AD. The moment, T (or C) $\times$ *a*, which resists the bending of the beam, is termed the *moment of resistance* of the section AD.

When a beam has deflected to its position of equilibrium, the *bending moment* at every beam section will be equalled by the *moment of resistance* of the section. This is a very important result, and will be often used in later chapters.

The force W tending to produce vertical sliding, or shear, at AD is the *shear force* at the section.

We may now express these facts in the form of three important definitions, assuming the usual case of a horizontal beam with vertical loads.

Bending Moment (B.M.).—The bending moment at any given section of a beam is the resultant moment, about that section, of all the external forces acting to one side of the section.

Shear Force (S.F.).—The shear force at any given section

of a beam is the resultant vertical force of all the external forces acting to one side of the section.

Moment of Resistance (M.R.).—The moment of resistance of a beam section is the moment of the couple which is set up at the section by the longitudinal forces created in the beam by its deflection.

Bending Moment and Shear Force

When computing values of these quantities it should be noted that:

(i) In finding B.M. values, it does not matter which side of the given section is taken, but only one side must be considered.

(ii) In finding S.F. values, **the actual positions of the loads do not matter,** provided only those loads to one side of the section be taken.

EXAMPLE. *Determine the B.M. and S.F. values for the section* XX *in the beam shown in Fig.* 43.

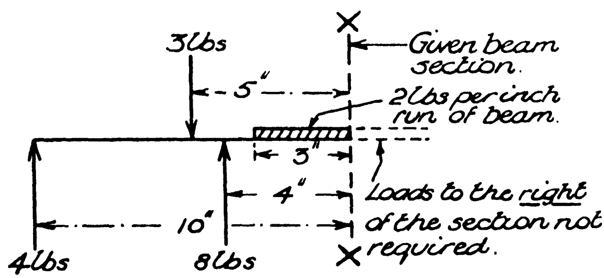


FIG. 43.—BEAM WITH GENERAL LOAD SYSTEM.

Resultant moment about section.

$= [(4 \times 10) + (8 \times 4) - (3 \times 5) - (6 \times 1\frac{1}{2})]$ lb. ins.

$= (40 + 32 - 15 - 9)$ lb. ins. $= 48$ lb. ins.

This is the value of the 'bending moment' at the section. It is a clockwise moment tending to bend the beam so that it is concave upwards. The unit 'lb. ins.' is sometimes written as 'in. lb.'

Resultant vertical load at section

$$= (4 + 8 - 3 - 6) \text{ lb.} = 3 \text{ lb.}$$

This is the value of the shear force at the section, and the tendency is for the left portion of the beam to move vertically upwards, at section XX, with respect to the right portion.

It will be necessary to distinguish between bending moments which, respectively, produce *concave* and *convex* bending in a beam. Also the two possible types of shearing must be capable of identification. Distinction is made in both cases by the use of positive and negative symbols. The convention of

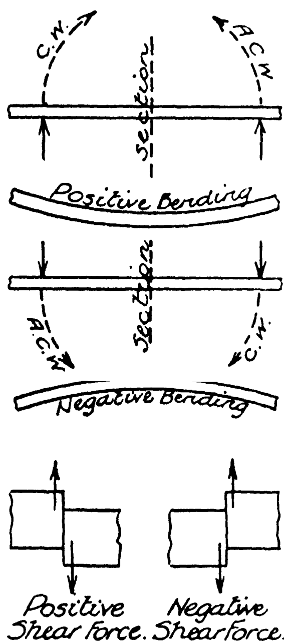


FIG. 44.—CONVENTION OF SIGNS FOR BENDING MOMENTS AND SHEAR FORCE.

signs adopted in the subsequent calculations is illustrated in Fig. 44.

Certain standard cases of beams and loading will now be considered.

(a) Cantilever with Single Concentrated Load W at the Free End

To the right of the given typical section XX, Fig. 45, there is only one load, viz. W, and it is at a distance x from the section.

The B.M. at the section is therefore Wx. We may write this as follows: $\text{B.M.}_x = Wx$.

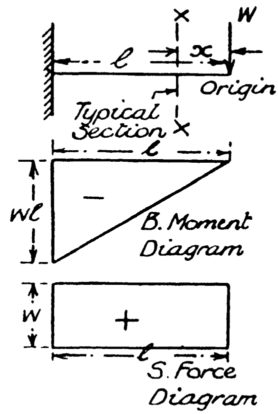


FIG. 45.—CANTILEVER WITH CONCENTRATED END LOAD.

x may have any value from 0 to l, and a graph can be drawn showing the change in value of the B.M. as x varies.

$$\text{When } x = 0, \text{ B.M.} = W \times 0 = 0.$$
$$\text{When } x = l, \text{ B.M.} = W \times l = Wl.$$

Such a graph, or diagram, exhibiting the value of the B.M. for all points of the beam span, is known as a *bending moment diagram.* It is not necessary, in most cases, actually to plot a large number of points, as in graph construction. The form of the B.M. expression for the typical section is a clue to the geometrical form of the diagram, and geometrical means may be used to construct it. The expression ' Wx ' is of the first degree in x and leads to a **straight-line diagram.**

Shear force diagrams are constructed on the same general principles.

$$\text{S.F.}_x = \text{resultant vertical load to right of section}$$
$$= W.$$

This value is independent of x, so that the graph is a **horizontal straight line.**

It will be noted that, according to the convention of signs adopted, the B.M. is negative and the S.F. positive, in this case.

(b) Cantilever with Several Concentrated Loads

The loading on a beam may be disintegrated, as convenient, for the calculation of B.M. (or S.F.) values. The net B.M. (or S.F.) value at any given section will then be the algebraic sum of the various separately calculated values. The principle of addition and subtraction may also be applied to *diagrams* of B.M. (and S.F.).

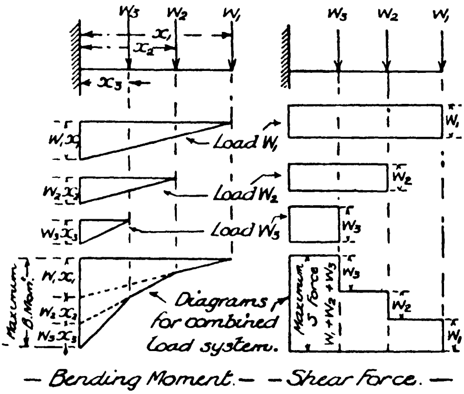


FIG. 46.—SUMMATION OF COMPONENT DIAGRAMS.

In Fig. 46 is shown the building up of the final B.M. and S.F. diagrams from the component diagrams. In practice, the final diagrams are drawn directly.

EXAMPLE. *A cantilever carries the load system given in Fig. 47. Construct the B.M. and S.F. diagrams, and compute the B.M. and S.F. values, respectively, for a section 4 ft. from the free end.*

B.M. at support due to 8 cwts. load = (8 × 8) cwts. ft.
= 64 c.f. (–).

B.M. at support due to 12 cwts. load = (12 × 5) c.f.
= 60 c.f. (–).

Total B.M. at support = (64 + 60) c.f. = 124 c.f.

B.M. at given section AA = [(8 × 4) + (12 × 1)] c.f.
= (32 + 12) c.f. = 44 c.f. (–).

S.F. at section AA = total load to right of section
= (12 + 8) cwts. = 20 cwts. (+).

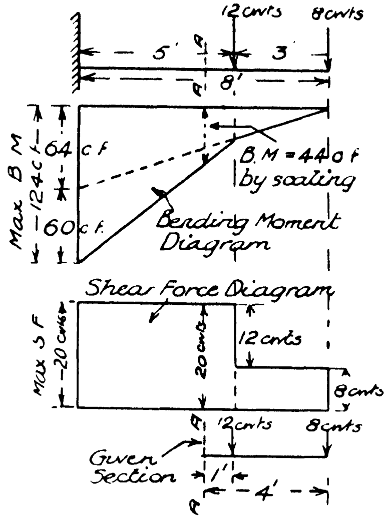


FIG. 47.—CANTILEVER WITH CONCENTRATED LOADS.

The diagrams may now be constructed as shown in the figure. The reader is recommended to draw out these diagrams (and those in subsequent worked examples) to suitable scales. Suggested scales: $1'' = 2'$ (for span); $1'' = 40$ c.f. (for B.M. values); $1'' = 10$ cwts. (for S.F. values).

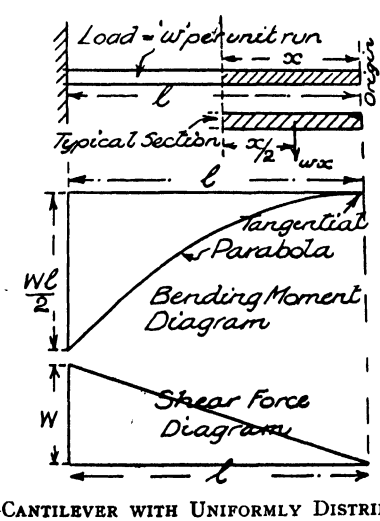


FIG. 48.—CANTILEVER WITH UNIFORMLY DISTRIBUTED LOAD.

(c) Cantilever with Uniformly Distributed Load

The total load on the hatched position of the cantilever (Fig. 48), to the right of the given typical section $= w$ per unit run $\times\ x$ units of length $= wx$. The centre of gravity of this load is $\frac{x}{2}$ from the section, hence the moment of the load about the section $= wx \times \frac{x}{2} = \frac{wx^2}{2}$. $\text{B.M.}_x = \frac{wx^2}{2}(-)$.

If this expression were plotted for different values of x, the curve obtained would be a 'parabola,' tangential to the beam at the free end. The reader is referred to Appendix III for the method of constructing a parabola, and for other useful properties of this important curve.

When $x = 0$, B.M. $= \frac{wx^2}{2} = \frac{w \times 0^2}{2} = 0$.

When $x = l$, B.M. $= \frac{wx^2}{2} = \frac{w \times l^2}{2} = \frac{wl^2}{2}$.

If W $=$ total load on cantilever, W $= wl$. B.M. at support, i.e. the **maximum value of the B.M.,** $= \frac{wl^2}{2} = \frac{Wl}{2}$.

$\text{S.F.}_x =$ total load to right of section

$= wx$.

The S.F. diagram is therefore linear.

When $x = 0$, S.F. $= w \times 0 = 0$.

When $x = l$, S.F. $= w \times l = wl =$ W (positive).

$\therefore$ **S.F. maximum $=$ W.**

EXAMPLE. *A balcony is carried by B.S.B.s placed at 2′ 6″ centres (Fig. 49). The beams project 4 ft. from a wall. The total depth of the concrete floor, including floor finish, is 9″. Construct the B.M. and S.F. diagrams for one beam, assuming a super load of 100 lb./sq. ft. on the balcony.*

Area of floor carried by one joist $= 4' \times 2{\cdot}5' = 10$ ft.^{2}.

Volume of concrete $= (10 \text{ ft.}^2 \times \frac{9''}{12})$ cu. ft. $= 7{\cdot}5$ cu. ft.

Total weight at 130 lb./cu. ft. $= (7{\cdot}5 \times 130)$ lb. $= 975$ lb.

Total dead load including steel beam, say 1000 lb.

Super. load per beam $= (10 \times 100)$ lb. $= 1000$ lb.

Total uniformly distributed load per beam $= 2000$ lb.

B.M. maximum $= \frac{Wl}{2} = \frac{2000 \times 4}{2}$ lb. ft. $= 4000$ lb. ft.

S.F. maximum $= W = 2000$ lb.

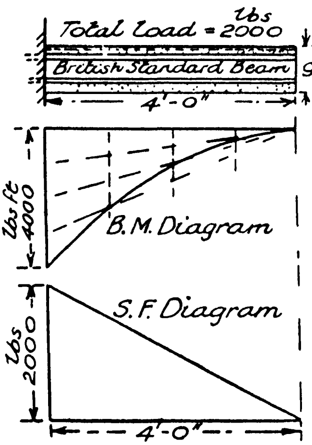


FIG. 49.

The diagrams may be conveniently drawn to the following scales: 1" = 1 ft.; $\frac{3}{4}$" = 1000 lb. ft.; 1" = 1000 lb.

(d) Simply Supported Beam with a Single Concentrated Load

In the case of beams, the first step is to calculate the two support reactions. By taking moments about point B (Fig. 50) we get $R_A \times l = W \times b$. $\therefore R_A = \frac{Wb}{l}$.

Similarly, $R_B = \frac{Wa}{l}$. There is only one load to the left of the typical section indicated, viz. the reaction at A $\left(= \frac{Wb}{l}\right)$.

$B.M._x = \frac{Wb}{l} \times x = \frac{Wbx}{l}$. This value indicates that the B.M. graph from A to C will be a straight line.

When $x = 0$, B.M. $= \frac{Wb}{l} \times 0 = 0$.

When $x = a$, B.M. $= \frac{Wb}{l} \times a = \frac{Wab}{l}$.

We cannot put x greater than a in this expression, but similar reasoning will show that the B.M. gradually increases from zero at the point B to the value $\frac{Wab}{l}$ at the point C.

Maximum B.M. for this case is therefore $\frac{Wab}{l}$.

$$\begin{aligned} S.F._x &= \text{total load to left of given section} \\ &= \frac{Wb}{l}. \end{aligned}$$

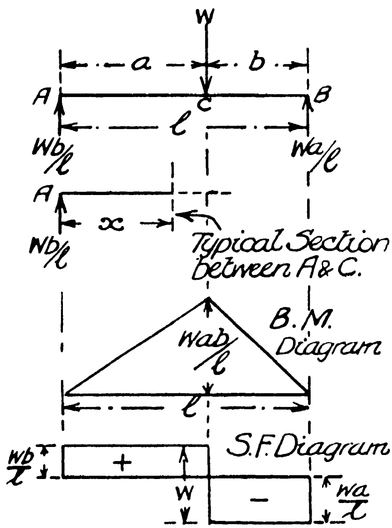


FIG. 50.—SIMPLY SUPPORTED BEAM WITH SINGLE CONCENTRATED LOAD.

For any beam section to the right of point C, the total load to the *left* of the section would clearly be

$$\frac{Wb}{l} - W = \frac{Wb}{a+b} - W = \frac{Wb - Wa - Wb}{a+b} = -\frac{Wa}{l},$$

i.e. the right end reaction with a negative sign.

In this type of beam we get, therefore, positive B.M.s and both positive and negative S.F.s.

If W is at the centre of the beam, $a = b = \frac{l}{2}$.

B.M. maximum becomes $\frac{W \times \frac{l}{2} \times \frac{l}{2}}{l} = \frac{\mathbf{W}\boldsymbol{l}}{\mathbf{4}}$.

S.F. maximum $= \pm \frac{W}{2}$. The values for B.M. maximum and S.F. maximum in this case should be memorised.

In the calculation of B.M. and S.F. values for beams simply supported at the ends, the following concise statements will be found useful:

B.M. = 'Reaction moment -- load moments,' the reactions and loads being taken to one side of the section, and moments taken about the section.

S.F. = 'Left end reaction — sum of loads up to section.' If the left side of section be taken always for S.F., the danger of incorrect sign of S.F. will be avoided.

(e) Simply Supported Beam with Several Concentrated Loads

Fig. 51 shows the derivation of the B.M. and S.F. diagrams for this case from the component diagrams for the separate

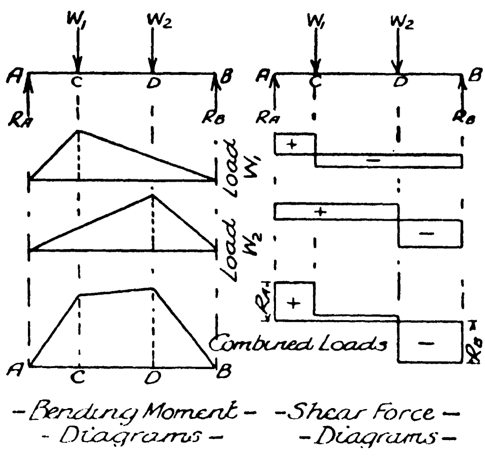


FIG. 51.—COMPOSITION OF B.M. AND S.F. DIAGRAMS.

loads. It will be seen that we need only calculate the values of the B.M.s at C and D to complete the B.M. diagram. The S.F. diagram is a stepped diagram, the vertical drop at C representing the load W_1, and that at D the load W_2.

EXAMPLE. *In Fig. 52 there is shown a simply supported*

beam carrying three concentrated loads. Calculate the values of B.M. and S.F. respectively for a section 6 ft. from the left end. Construct the B.M. and S.F. diagrams for the beam.

$$R_A \times 10 = (6 \times 7) + (4 \times 5) + (2 \times 2)$$
$$10\,R_A = 42 + 20 + 4 = 66$$
$$R_A = 6{\cdot}6 \text{ cwts.}$$
$$R_B \times 10 = (2 \times 8) + (4 \times 5) + (6 \times 3)$$
$$10\,R_B = 16 + 20 + 18 = 54$$
$$R_B = 5{\cdot}4 \text{ cwts.}$$

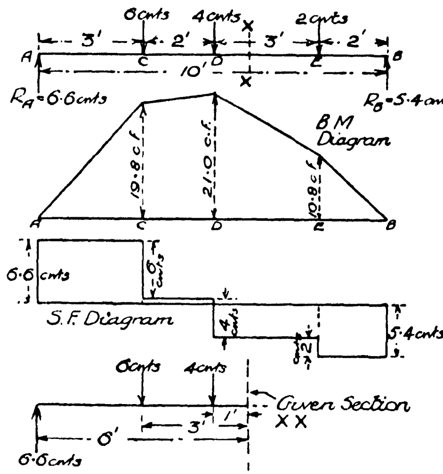


FIG. 52.

$R_A + R_B = (6{\cdot}6 + 5{\cdot}4)$ cwts. $= 12$ cwts., which agrees with the total load on the beam.

$\text{B.M.}_C = R_A \times 3 = (6{\cdot}6 \times 3)$ cwts. ft. $= 19{\cdot}8$ c.f.

$\text{B.M.}_D =$ Reaction moment — load moment

$= (6{\cdot}6 \times 5) - (6 \times 2)$ c.f. $= 21{\cdot}0$ c.f.

B.M._E (taking loads to right of section for convenience) $= R_B \times 2 = (5{\cdot}4 \times 2)$ c.f. $= 10{\cdot}8$ c.f.

B.M. at the given section XX

$= (6{\cdot}6 \times 6) - (6 \times 3) - (4 \times 1)$ c.f.

$= (39{\cdot}6 - 18 - 4)$ c.f. $= 17{\cdot}6$ c.f.

S.F. at given section = Left end reaction — sum of loads up to section

$= (6{\cdot}6 - 6 - 4)$ cwts. $= - 3{\cdot}4$ cwts.

The reader should check these given section values by taking loads to the *right of the section*, and also by scaling diagrams of B.M. and S.F. for the beam.

Suitable scales: $1'' = 2$ ft.; $1'' = 8$ c.f.; $1'' = 4$ cwts.

(f) Simply Supported Beam with Uniformly Distributed Load

The total load on the beam (Fig. 53) $= wl$, so that each reaction $= \frac{wl}{2}$. Considering the B.M. at the typical section and taking loads to the left of the section, we get

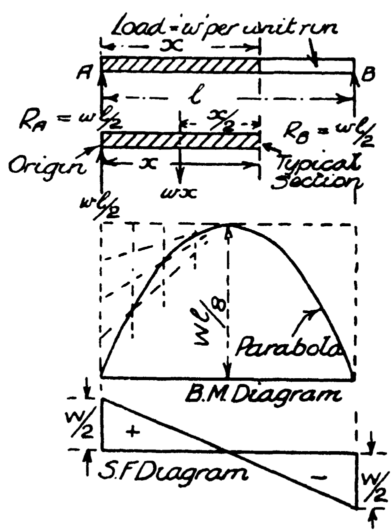


FIG. 53.—SIMPLY SUPPORTED BEAM WITH UNIFORMLY DISTRIBUTED LOAD.

$$\begin{aligned} \text{B.M.}_x &= \text{Reaction moment} - \text{load moment} \\ &= \frac{wl}{2} \times x - wx \times \frac{x}{2} \\ &= \frac{wlx}{2} - \frac{wx^2}{2}. \end{aligned}$$

This expression would give a parabola if plotted.

$$\text{When } x = 0, \text{ B.M.} = \frac{wl}{2} \times 0 - \frac{w}{2} \times (0)^2 = 0.$$

$$\text{When } x = \frac{l}{2}, \text{ B.M.} = \frac{wl}{2} \times \frac{l}{2} - \frac{w}{2} \times \left(\frac{l}{2}\right)^2$$

$$= \frac{wl^2}{4} - \frac{wl^2}{8} = \frac{wl^2}{8}.$$

Inserting W (the total load) for wl, the B.M. at centre of beam $= \frac{Wl}{8}$.

At B, $x = l$ and B.M. $= \frac{wl}{2} \times l - \frac{w}{2} \times l^2 = 0$, as we would expect.

The B.M. is clearly a maximum at the beam centre, but readers familiar with the Calculus will be able to show that the expression $\frac{wlx}{2} - \frac{wx^2}{2}$ has a maximum value for $x = \frac{l}{2}$.

S.F.$_{x}$ = Reaction — total load up to section

$$= \frac{wl}{2} - wx.$$

This indicates uniform variation for the shear force, as x is in the first degree.

$$\text{When } x = 0, \text{ S.F.} = \frac{wl}{2} - w \times 0 = \frac{wl}{2} = \frac{W}{2}.$$

$$\text{When } x = \frac{l}{2}, \text{ S.F.} = \frac{wl}{2} - \frac{wl}{2} = 0.$$

$$\text{When } x = l, \text{ S.F.} = \frac{wl}{2} - wl = -\frac{wl}{2} = -\frac{W}{2}.$$

For this case, therefore, **B.M. maximum** $= \frac{\mathbf{W}l}{\mathbf{8}}$ and **S.F. maximum** $= \pm \frac{\mathbf{W}}{\mathbf{2}}$.

The formulæ given for B.M. maximum and S.F. maximum are very important and are frequently required in the design of beams.

EXAMPLE. *Draw the B.M. and S.F. diagrams for one of the steel floor beams given in Fig.* 54. *Find the B.M. and S.F. values respectively, for the section of a beam,* 8 *ft. from the left end. Total load (including weight of floor) to be taken as* 90 *lb. per sq. ft.*

Area supported by one joist $= 10' \times 12' = 120$ ft.^{2}

Uniformly distributed load carried by one beam $= (120 \times 90)$ lb. $= 10800$ lb.

$$\text{B.M. maximum} = \frac{Wl}{8} = \frac{10800 \times 12}{8} \text{ lb. ft.} = 16200 \text{ lb. ft.}$$

$$\text{S.F. maximum} = \pm \frac{W}{2} = \pm \frac{10800}{2} \text{ lb.} = \pm 5400 \text{ lb.}$$

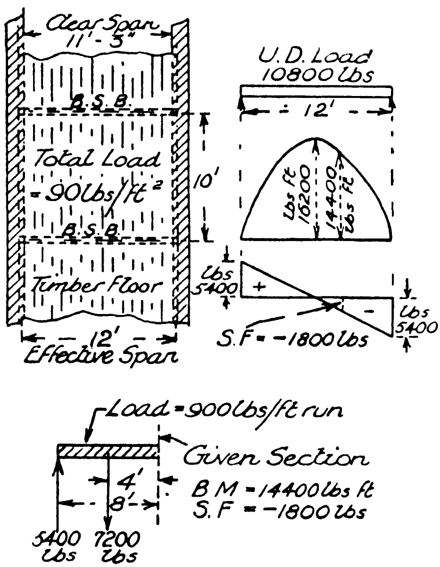


FIG. 54.

$$\text{Load carried per foot length of beam} = \frac{10800}{12} \text{ lb.}$$
$$= 900 \text{ lb.}$$

B.M. at given section = Reaction moment − load moment

$= [(5400 \times 8) - (900 \times 8 \times 4)]$ lb. ft.

$= (43200 - 28800)$ lb. ft.

$= 14400$ lb. ft.

S.F. at given section = Reaction — load up to section

$= 5400$ lb. $- 7200$ lb.

$= - 1800$ lb.

Suggested scales for diagrams:

$1'' = 2'$; $1'' = 4000$ lb. ft. ; $1'' = 4000$ lb.

(g) Beam Overhanging its Supports and Carrying a System of Concentrated Loads

It will now be appreciated that **a system of concentrated loads will always give a B.M. diagram composed of straight lines,** so that B.M. values at load points only are required to be calculated.

S.F. diagrams for such a system will always be **stepped diagrams, with a step, up or down at a load point, representing the value of the given load.**

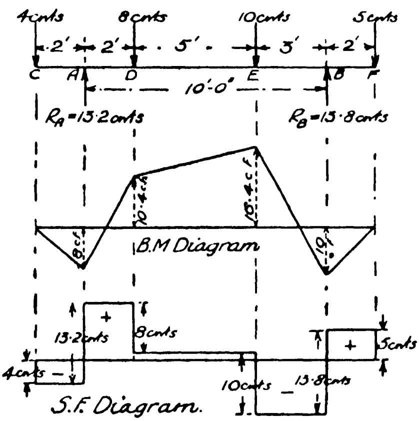


FIG. 55.—OVERHANGING BEAM WITH CONCENTRATED LOADS.

EXAMPLE. *Construct the B.M. and S.F. diagrams for the case given in Fig.* 55.

Taking moments about B, to find R_A, we get

$$(R_A \times 10) + (5 \times 2) = (4 \times 12) + (8 \times 8) + (10 \times 3)$$
$$10\,R_A + 10 = 48 + 64 + 30 = 142$$
$$10\,R_A = 142 - 10 = 132. \quad \therefore R_A = 13{\cdot}2 \text{ cwts.}$$

Moments about A :

$$(R_B \times 10) + (4 \times 2) = (5 \times 12) + (10 \times 7) + (8 \times 2)$$
$$10\,R_B + 8 = 60 + 70 + 16 = 146$$
$$10\,R_B = 146 - 8 = 138. \quad \therefore R_B = 13{\cdot}8 \text{ cwts.}$$

$B.M._C = 0$ (there is no load to left of section).

$B.M._A = (4 \times 2)$ c.f. $= 8$ c.f. (negative).

B.M._D = Reaction moment — load moment
= [(13·2 × 2) − (4 × 4)] c.f.
= (26·4 − 16) c.f. = 10·4 c.f. (positive).

B.M._E = [(13·2 × 7) − (8 × 5) − (4 × 9)] c.f.
= (92·4 − 40 − 36) c.f.
= 16·4 c.f. (positive).

Alternatively, *taking loads to right of section at E*:

B.M._E = [(13·8 × 3) − (5 × 5)] c.f. = (41·4 − 25) c.f.
= 16·4 c.f.

B.M._B = (5 × 2) c.f. = 10 c.f. (negative).

B.M._F = 0.

Suggested scales for diagrams:

$\frac{3}{8}''$ = 1 ft.; 1″ = 8 c.f.; 1″ = 8 cwts.

(h) Beam Overhanging its Supports and Carrying a Uniformly Distributed Load

The form of the B.M. diagram will be understood from an inspection of Fig. 56, in which it is built up from component diagrams which are already familiar to the reader.

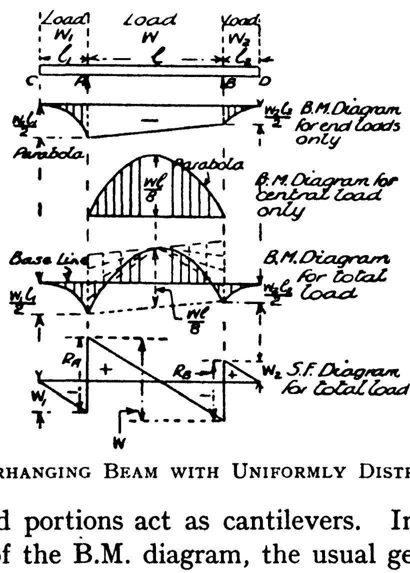


FIG. 56.—OVERHANGING BEAM WITH UNIFORMLY DISTRIBUTED LOAD.

The two end portions act as cantilevers. In the practical construction of the B.M. diagram, the usual geometrical construction for a parabola—somewhat modified as shown—can be used for the central span. The S.F. diagram presents no

difficulty, if it is borne in mind that a uniformly distributed load causes a uniform slope in the S.F. diagram.

EXAMPLE. *A steel and concrete floor is carried by B.S.B.s which rest on pillars in the manner shown in Fig. 57. The total load transmitted to one B.S.B. is 1 ton per foot run. Construct the B.M. and S.F. diagrams for a beam.*

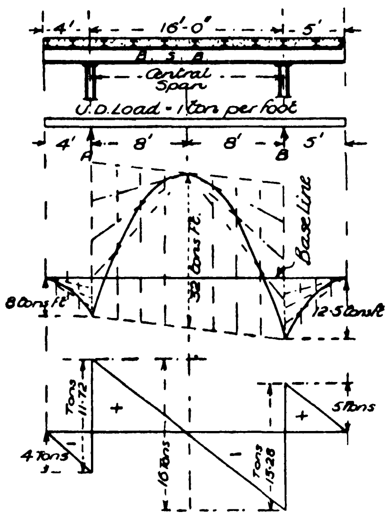


FIG. 57.

Portion of beam from left end to A.

$$\text{B.M. at A} = \frac{Wl}{2} = \left(4 \times \frac{4}{2}\right) \text{ tons ft.} = 8 \text{ tons ft.}$$

Portion of beam from B to right end.

$$\text{B.M. at B} = \frac{Wl}{2} = \left(5 \times \frac{5}{2}\right) \text{ tons ft.} = 12{\cdot}5 \text{ tons ft.}$$

Both these will be negative bending moments.

B.M. maximum for central portion AB, regarded temporarily as a simple beam of length AB,

$$= \frac{Wl}{8} = \frac{16 \times 16}{8} \text{ tons ft.} = 32 \text{ tons ft.}$$

The B.M. diagram is constructed as shown. For the S.F. diagram we require to calculate the two support reactions.

$R_A \times 16 = 25$ tons $\times 7{\cdot}5$ ft. (the c.g. of the whole load is $7{\cdot}5$ ft. from B).

$$R_A = \frac{25 \times 7{\cdot}5}{16} \text{ tons} = 11{\cdot}72 \text{ tons.}$$

$$R_B \times 16 = (25 \times 8{\cdot}5).$$

$$R_B = \frac{25 \times 8{\cdot}5}{16} \text{ tons} = 13{\cdot}28 \text{ tons.}$$

(Check $R_A + R_B = 25$ tons.)

Between the left end of the beam and A the diagram slopes uniformly to a negative value of 4 tons at A. There is then a sudden vertical jump of 11·72 tons, owing to the reaction at A. Between A and B the total fall = 16 tons. The diagram finally ends with zero value at the extreme right end of the beam.

Two or more Load Systems

The self-weight of a beam always involves a distributed load, in addition to any other loads carried. The weight of the beam itself may be small in comparison with the super. loading and able to be neglected in calculations. To obtain the net B.M. (or S.F.) diagrams for two or more simple load systems, the component diagrams should be drawn out to the same scale and arranged one beneath the other. Vertical lines—drawn at suitable intervals in the span—to cut the series of diagrams will give corresponding ordinates in the separate diagrams. These ordinates algebraically added are then plotted on a new base line to give the final net diagram.

The load systems dealt with in this chapter represent those commonly occurring in practical problems. For the treatment of other load types, the reader is referred to books on the theory of structures.

Exercises 5

(1) Distinguish between the terms 'bending moment' and 'moment of resistance,' and explain the meaning of the term 'shear force.' State the regions of a simply supported beam for which the B.M. and S.F. are, respectively, of relatively greater importance.

(2) A cantilever projects 5′ horizontally from a wall. It carries a load of 2 cwts. at its free end, and 4 cwts. at 3′ from the free end. Find the B.M. and S.F. values for the centre of the cantilever, and draw the B.M. and S.F. diagrams for the given loads.

(3) A tank is supported outside a building by two B.S.B.s which are fixed in the wall and extend 4′ horizontally therefrom. The tank when full weighs 2 tons, the load being equally divided between, and uniformly distributed along, the two beams. Draw the B.M. and S.F. diagrams for one beam, and find the B.M. and S.F. respectively to which each beam is subjected at 1 foot from the wall.

(4) A simply supported beam of 10′ span carries the following concentrated load system :

4 cwts. at 2′ from left end of beam.
8 " " 5′ " " " "
8 " " 9′ " " " "

Calculate the B.M. and S.F. values for a section 4′ from the left end and check the values obtained by the construction of the B.M. and S.F. diagrams for the beam.

(5) A room, 18′ × 16′, has a B.S.B. parallel to the longer side and at mid-width of floor, supporting the pitch-pine beams of which the floor is composed. The total load carried by the floor, including the weight of the floor itself, is 150 lb. per sq. ft. The floor beams are spaced at 12″ centres.

(*a*) Construct the B.M. and S.F. diagrams for one of the timber beams, and (*b*) determine the maximum B.M. in the steel beam, allowing 400 lb. for its self-weight.

(6) A lintol of 6′ effective span carries a wall of uniform height and thickness. The thickness is 1′ 6″ and the average density of the wall material is 112 lb. per cu. ft. Calculate the maximum height of the wall so that the B.M., due to the weight of the wall, does not exceed 2·25 tons ft. Sketch the B.M. and S.F. diagrams for the lintol, inserting thereon all important values.

(7) A beam, resting on two walls, A and B, 16′ apart, carries four lifting appliances which may cause the loads given in

diagram (Fig. 58). Assuming the four loads to be simultaneously applied, draw the B.M. and S.F. diagrams for the beam for these loads.

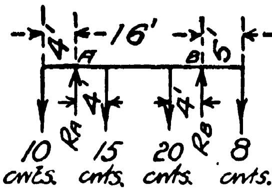


FIG. 58.

(8) Draw the B.M. and S.F. diagrams for the self-weight of the beam referred to in question 7, assuming it to weigh 18 lb. per foot run.

CHAPTER VI

DESIGN OF SIMPLE BEAMS. MOMENT OF RESISTANCE

Assumptions in the Beam Theory

THE theory which is involved in the derivation of the formula for the *moment of resistance* of a beam section is based upon certain assumptions.

(i) The beam section, for the theory to apply strictly, should have a vertical axis of symmetry, in order to ensure the deflection of the beam being in a vertical plane.

(ii) *Simple bending* is assumed. This is the type of bending produced by the application of pure couples at the ends of the beam. Such bending does not involve the beam in deflection due to shear strain (which is, however, relatively small in any practical case) and is exemplified by the portion AB of the beam shown in Fig. 59.

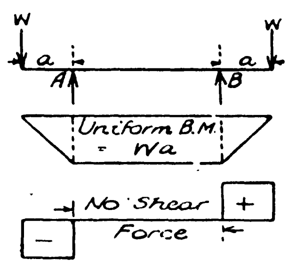


FIG. 59.—EXAMPLE OF SIMPLE BENDING.

(iii) Vertical sections of the beam before bending are assumed to remain plane after bending. In Fig. 61 the plane sections AB and CD remain plane in the positions A′B′ and C′D′ respectively.

(iv) The stress in any given beam fibre is assumed to be proportional to its strain, i.e. Hooke's law is assumed to hold for the beam material. Young's modulus (E) is assumed to have the same value throughout the beam.

It will be instructive, before taking the general theory, to study the applications of these assumptions to the case of a beam having a rectangular cross-section.

Rectangular Beams

The beam shown in Fig. 60 will have its upper fibres in compressive strain (and hence compressive stress) and its lower fibres in tensile strain (and hence tensile stress). It will be clear that the biggest strains will occur at the top and bottom of the beam respectively. These strains will decrease in value as we proceed inwards from the extreme fibres. At some level

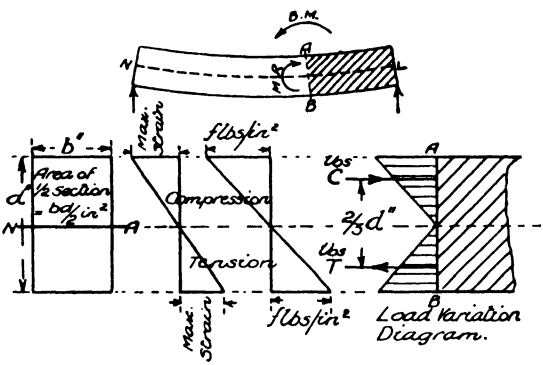


FIG. 60.—MOMENT OF RESISTANCE OF A RECTANGULAR BEAM SECTION.

in the beam the strains will have completely vanished. Throughout a bent beam there is a layer of material which undergoes no strain and, therefore, is in an unstressed condition. This extremely thin layer is termed the *neutral layer* (N.L.) and the straight line, in which this layer cuts any given beam cross-section, is known as the *neutral axis* (N.A.) of that section. The N.A. of a rectangular beam section will be at mid-depth.

From assumption (iii) it may be concluded that the strain in the beam fibres will increase uniformly as we proceed, up and down, from the N.A. to the extreme beam fibres. As stress and strain are assumed to vary in proportion, the stress variation will also be of a uniformly changing character. Further, in the case being considered, little horizontal strips of beam cross-section will be of constant width so that the load dis-

tribution (as a result of the stress) will also be of the same uniformly varying type. The diagrams in Fig. 60 will now be clear to the reader.

The two like parallel systems composed, respectively, of the numerous little thrusts and pulls on section AB, will each have a resultant with a definite line of action. These two resultants (C and T) form the forces in the couple resisting bending, and the distance between their lines of action is the 'arm' of the couple.

Value of C (or T). C will be found by multiplying the average stress in the top half-section by the area of this portion. Taking the symbols and units given in Fig. 60,

$$C = \frac{f}{2} \text{ lb. per in.}^2 \times \frac{bd}{2} \text{ ins.}^2$$

$$= \frac{fbd}{4} \text{ lb.}$$

$$\text{Similarly } T = \frac{fbd}{4} \text{ lb.}$$

Value of the 'arm' of the couple. C will act through the c.g. of the upper load triangle and T through that of the lower. The distance between their lines of action will therefore be $\left[d - \left(\frac{1}{3} \text{ of } \frac{d}{2}\right) - \left(\frac{1}{3} \text{ of } \frac{d}{2}\right)\right] \text{ ins.} = \frac{2}{3}d \text{ ins.}$

Moment of couple.

$$\text{Moment} = \text{Force} \times \text{arm}$$

$$= \frac{fbd}{4} \text{ lb.} \times \frac{2}{3}d \text{ ins.}$$

$$= \frac{fbd^2}{6} \text{ lb. ins.}$$

This is the 'moment of resistance' and equals the 'bending moment' at the section. Writing 'M' for 'M.R.' or 'B.M.', the formula becomes (without reference to any special system of units)

$$M = \frac{fbd^2}{6}.$$

EXAMPLES

(1) *A timber beam is 3" wide, 6" deep and has an effective span of 10'. It carries a total U.D. load of 1200 lb. Calculate the maximum stress in the timber.*

$$\text{B.M. maximum} = \frac{Wl}{8} = \frac{1200 \times 10 \times 12}{8} = 18000 \text{ lb. ins.}$$

(Note that the span must always be in inches, as the stress is expressed in inch units.)

$$M = \frac{fbd^2}{6}$$

$$18000 = \frac{f \times 3 \times 6 \times 6}{6}$$

$$f = 1000 \text{ lb./in.}^2$$

(2) *Find the maximum safe central load for a pitch-pine beam, 4" wide and 12" deep, if the effective span (i.e. centre to centre of bearings) is 15'. The working stress may be taken as 1200 lb./in.²*.

Let W lb. = safe central load, neglecting weight of beam.

$$\text{B.M. maximum} = \frac{Wl}{4} = \frac{W \times 15 \times 12}{4} \text{ lb. ins.}$$

$$= 45\,W \text{ lb. ins.}$$

$$M = \frac{fbd^2}{6}$$

$$45\,W = \frac{1200 \times 4 \times 12 \times 12}{6}$$

$$W = 2560 \text{ lb.}$$

(3) *A floor is composed of timber joists 2" wide, 7" deep and 11' effective span. It is to carry a total load (including the weight of the floor) of 100 lb. per sq. ft. Adopting a working stress of 1300 lb. per sq. in., determine the maximum spacing for the joists.*

Let x ft. = spacing.

Area of floor carried by one joist = $(11 \times x)$ ft.²

$= 11x$ ft.².

Load carried by one joist = $(11x \times 100)$ lb.

$$\text{B.M. maximum in joist} = \frac{Wl}{8} = \frac{1100x \times 11 \times 12}{8} \text{ lb. ins.}$$

$$= 18150x \text{ lb. ins.}$$

$$M = \frac{fbd^2}{6}$$

$$18150x = \frac{1300 \times 2 \times 7 \times 7}{6}$$

$$x = 1{\cdot}17' \ (= \text{say, } 14'').$$

General Theory of Bending

Fig. 61 shows a portion of a beam in the unbent and in the bent conditions. AB and CD are two vertical cross-sections, assumed so close together that the portion of beam between them may be regarded as bending to the arc of a circle.

EF is the part of the neutral layer intercepted between the sections. GH represents a typical layer of material at a distance y from the neutral axis. R is the radius of

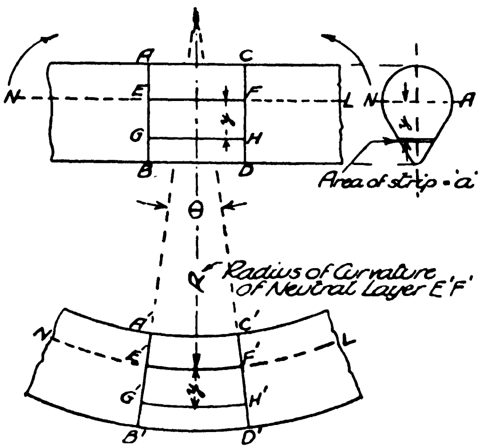


FIG. 61.—DIAGRAM ILLUSTRATING THE THEORY OF BENDING.

curvature of the portion of the neutral layer, in the bent beam. The following are the steps in the development of the theory:

(1) Determination of the strain in layer G′H′ by principles of geometry.

(2) Evaluation of the stress in this layer by means of Young's modulus.

(3) Determination of the load carried by the little strip of cross-section at distance y from the N.A.

(4) Computation of the moment this load has about the N.A. and, by summation, the total moment of all such strip loads.

Step 1. Extension in layer G′H′ = G′H′ − GH.

$$\text{Strain in layer } G'H' = \frac{\text{Extension}}{\text{Original length}}$$

$$= \frac{G'H' - GH}{GH}$$

But GH = EF and EF = E′F′ (being on the unstrained layer).

$\therefore$ Strain in layer $G'H' = \dfrac{G'H' - E'F'}{E'F'}$.

Expressing these distances in terms of R and θ (the angle in radians contained by B′A′ and D′C′) we have :

$$\text{Strain in layer } G'H' = \frac{(R + y)\,\theta - R\theta}{R\theta} = \frac{y}{R}.$$

Step 2. $\dfrac{\text{Stress in } G'H'}{\text{Strain in } G'H'} = E.$

$$\therefore \text{Stress in layer} = E \times \text{strain} = \frac{Ey}{R}.$$

$$\text{If } f = \text{the stress, } f = \frac{Ey}{R}.$$

Step 3. If a = the area of the cross-sectional strip, the load carried = stress $\times$ area

$$= \frac{Ey}{R} \times a = \frac{E}{R} \times ay.$$

Step 4. Moment of the load on this strip about N.A.

$$= \text{Load} \times \text{distance}$$

$$= \left(\frac{E}{R} \times ay\right) \times y = \frac{E}{R} \times ay^2.$$

The total ‘moment of resistance’ of the beam section is made up of all such moments as this.

$$\text{Total Moment of Resistance} = \Sigma \frac{E}{R} \times ay^2$$

$$= \frac{E}{R} \times \Sigma ay^2.$$

Σay^2 (*sigma* ay^2) is a geometrical property of the beam section, with reference to the axis N.A. It is termed the *moment of inertia* of the beam section, and is denoted by the letter I.

Writing M for ‘moment of resistance,’

$$M = \frac{E}{R} \times I.$$

$$\text{But } f = \frac{Ey}{R} \text{ or } \frac{E}{R} = \frac{f}{y} \text{ (step 2).}$$

$$\therefore M = \frac{fI}{y}.$$

This is the important formula for finding the moment of resistance of a beam section. Writing the results found as a continued ratio we get the complete expression for the theory.

$$\frac{E}{R} = \frac{M}{I} = \frac{f}{y}$$

In using this expression f will normally represent a maximum stress, so that y, in that case, will be the distance from the neutral axis to an extreme fibre, top or bottom of the section, as the case may be.

M will usually be a 'bending moment,' such as $\frac{Wl}{4}$, $\frac{Wl}{8}$, etc.

Position of the Neutral Axis.—In the theory we found that the strip load was given by the expression $\frac{E}{R} \times ay$.

As long as y is measured downwards all these strip loads will represent tension. If we put y negative, i.e. measured the distance upwards from N.A., the load would be compression.

$\Sigma\frac{E}{R} \times ay$ or $\frac{E}{R}\ \Sigma ay$ (since E and R are constants) will therefore represent a summation of a large number of positive and negative quantities. But, as the total compressive force = the total tensile force (being forces in a couple), $\frac{E}{R}\ \Sigma ay$ must = 0.

This, i.e. $\Sigma ay = 0$, means that the axis, from which y is measured, passes through the centre of gravity of the section. **The neutral axis of a beam section therefore passes through its centre of gravity.**

Moment of Inertia

This is one of three very important *properties of section.* It is a geometrical property of the shape and size of the beam section, and has reference to an axis. The material of the beam has nothing to do with its value.

In Fig. 62(a) we have an extremely small area 'a' at a distance y from an axis XX. The product ($a \times y^2$) is termed the *moment of inertia* of the area 'a' about the axis XX.

$$I_{XX} = ay^2.$$

In Fig. 62(b) there is shown an area divided up into very small elements a_1, a_2, etc., at distances y_1, y_2, etc., from

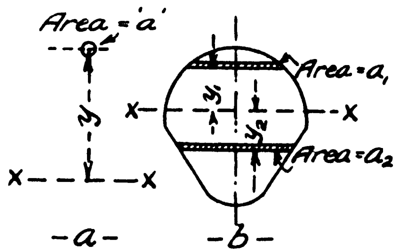


FIG. 62.

an axis XX. The sum of all such products as $a_1 y_1^2$, $a_2 y_2^2$, etc., will be the *moment of inertia* of the given area about the axis XX.

$$I_{XX} = a_1 y_1^2 + a_2 y_2^2 + \text{etc.}$$
$$= \Sigma a y^2.$$

In the beam theory, the little strip of beam cross-section had to be extremely small, so that the stress could be regarded as being of the value f all over it. In computing I values, we will necessarily be approximate if we divide the given section into areas of finite size, but we can reduce the error by taking reasonably narrow strips.

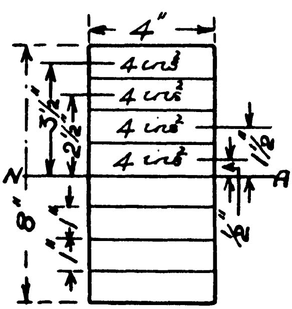


FIG. 63.—APPROXIMATE I_{NA} FOR A RECTANGLE.

EXAMPLE. *Fig.* 63 *shows a rectangle,* 4" *wide and* 8" *deep, divided into* 8 *strips. Find an approximate value for* I_{NA}.

I_{NA} for top strip $= ay^2 = 4$ sq. ins. $\times (3\frac{1}{2}$ ins.$)^2 = 49$ ins.^{4}
,, ,, 2nd ,, $= ay^2 = 4$ sq. ins. $\times (2\frac{1}{2}$ ins.$)^2 = 25$,,
,, ,, 3rd ,, $= ay^2 = 4$ sq. ins. $\times (1\frac{1}{2}$ ins.$)^2 = 9$,,
,, ,, 4th ,, $= ay^2 = 4$ sq. ins. $\times (\frac{1}{2}$ in.$)^2 = 1$,,

Total I_{NA} for $\frac{1}{2}$ section $= 84$ ins.^{4}

I_{NA} for whole section $= 168$ ins.^{4}

The reader should note carefully the unit in which I values are expressed.

If the strips were taken $\frac{1}{2}''$ deep, instead of $1''$, the answer would be slightly higher, and if taken extremely thin the value would be $170 \cdot 66$ ins.^{4}

I_{NA} for a Rectangular Section.—To obtain a correct value for I_{NA}, the methods of calculus must be employed. Readers unfamiliar with the Calculus should omit the work immediately following, and accept the formula given for this case.

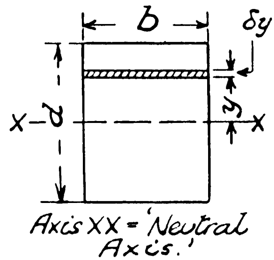


FIG. 64.

$$I_{XX} = \Sigma ay^2 \text{ (Fig. 64).}$$

Consider top half-section only.

$$I_{XX} = \int_{y=0}^{y=\frac{d}{2}} ay^2 = \int_{y=0}^{y=\frac{d}{2}} b\delta y \times y^2$$

$$= b\int_{y=0}^{y=\frac{d}{2}} y^2\,\delta y. \quad = b\left[\frac{y^3}{3}\right]_{y=0}^{y=\frac{d}{2}} = \frac{bd^3}{24}.$$

$$I_{XX} \text{ for whole section} = 2 \times \frac{bd^3}{24} = \frac{bd^3}{12}.$$

Applying this formula to the example worked previously,

$$I_{NA} = \frac{bd^3}{12} = \frac{4 \times 8 \times 8 \times 8}{12} \text{ ins.}^4 = 170 \cdot 66 \text{ ins.}^4$$

$I_{maximum}$ and $I_{minimum}$.—In dealing with the strength of beams the computation of I is made about the appropriate neutral axis of the section. If we calculated the various I values for all the possible axes *passing through the c.g. of the section,* we would find that the biggest value, and the least value, were associated with the axes of symmetry of the section—assuming such axes existed. It is supposed in the beam theory that there is at least a vertical axis of symmetry, and in this case, if the horizontal axis through the c.g. is not an axis of symmetry, it still becomes the axis for either $I_{maximum}$ or $I_{minimum}$. The 'properties of section' for certain standard beams given on pages 104 to 107 (published by kind permission of the British Steelwork Association) illustrate these two I values, $I_{maximum}$ being required for ordinary beam calculations.

EXAMPLE. *$I_{max.}$ for a 12″ × 6″ × 44 lb. B.S.B. is given in the table referred to as 316·76 ins.*^{4}

Calculate the safe U.D. load for this section for an effective span of 16′, using a working stress of 8 tons/ins.^{2}

$$M = \frac{fI}{y}.$$

$$\frac{Wl}{8} = \frac{fI}{y} = \frac{8 \times 316{\cdot}76}{6} \quad (y = \tfrac{12''}{2} = 6'').$$

$$\frac{W \times 16 \times 12}{8} = \frac{8 \times 316{\cdot}76}{6}$$

$$W = 17{\cdot}5 \text{ tons.} \quad \text{(Check from table of safe loads.)}$$

Addition and Subtraction of I Values.—Moments of inertia of component areas about any given axis may be directly added, or subtracted, to obtain the final result for the compound area—**but only provided each component I value has reference to the one common given axis.** In ordinary beam problems this axis will be the neutral axis of the beam section.

EXAMPLE. *Find the value of I_{XX} for the section given in Fig.* 65.

We first treat the section as a solid rectangle 8″ wide and 12″ deep.

$$I_{XX} = \frac{bd^3}{12} = \frac{8 \times 12 \times 12 \times 12}{12} \text{ ins.}^4 = 1152 \text{ ins.}^4$$

To get the net section we have to subtract the two rectangles lying, respectively, to either side of the web (shown hatched).

$$I_{XX} \text{ for these two rectangles} = 2\left[\frac{bd^3}{12}\right]$$

$$= 2\left[\frac{3{\cdot}75 \times 10^3}{12}\right] \text{ ins.}^4 = 625 \text{ ins.}^4.$$

Net I_{XX} for section $= (1152 - 625)$ ins.$^4 = 527$ ins.^{4}.

It is useful to remember a formula for such a section, viz. :

$$I_{XX} = \frac{BD^3}{12} - \frac{bd^3}{12}. \quad [b = B - \text{web thickness.}]$$

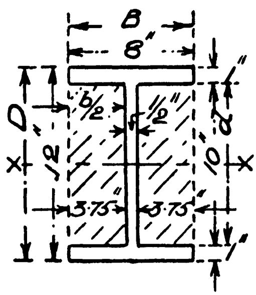


FIG. 65.

Economy of B.S.B. Type of Section.—Beams of rectangular section are not economical of material, as the parts of the section situated near the neutral axis are only stressed to low values (Fig. 66). The small loads carried here are further

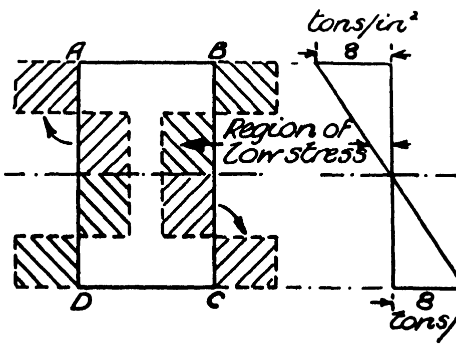


FIG. 66.—DERIVATION OF FLANGED SECTION FROM RECTANGULAR SECTION OF SAME AREA.

subject to the disadvantage of having a small arm for their resistance moment. In the B.S.B. form of section a fair proportion of the steel is placed in such a position as to become highly stressed, and the corresponding loads have a bigger arm of resistance moment.

EXAMPLE. *Fig. 67 shows three similar plates welded together*

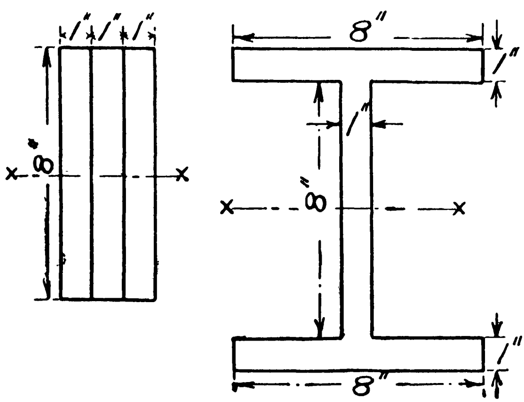


FIG. 67.

to form a beam section in two different ways. Compare the safe U.D. loads for these two beams, for an effective span of 12'.

(*a*) $M = \frac{fbd^2}{6}$

$$\frac{W \times 12 \times 12}{8} = \frac{8 \times 3 \times 8 \times 8}{6} \quad (f = 8 \text{ tons/in.}^2)$$

$$W = 14{\cdot}2 \text{ tons.}$$

(*b*) $M = \frac{fI}{y}$

$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12} = \left(\frac{8 \times 10^3}{12} - \frac{7 \times 8^3}{12}\right) \text{ins.}^4$$

$$= 368 \text{ ins.}^4$$

$$\frac{W \times 12 \times 12}{8} = \frac{8 \times 368}{5}$$

$$W = 32{\cdot}7 \text{ tons.}$$

Both beams have the same weight of steel, but the B.S.B. type can support a far greater load.

Section Modulus

In the expression $\frac{fI}{y}$ we have two symbols, I and y, which represent geometrical properties associated with the beam section. For any given geometrical figure, or standard beam section, they can be separately computed. It is convenient to merge these two section properties into one single property, in the form of their ratio $\frac{I}{y}$. To this property is given the name *section modulus.* The symbol used for this composite property is usually Z. The units for Z will be 'ins.^{3}' being $\frac{\text{ins.}^4}{\text{ins.}}$. Corresponding to $I_{maximum}$ and $I_{minimum}$ there will be $Z_{maximum}$ and $Z_{minimum}$.

$$Z = \frac{I}{y}.$$

In most structural sections, as in Fig. 68(a), the neutral axis is also an axis of symmetry, so that y is half the overall depth of the section. In this case:

$$Z = \frac{\text{Moment of Inertia (about N.A.)}}{\frac{1}{2}\text{ overall depth of section}}.$$

In the event of the distances from the neutral axis to the extreme fibres at the top and bottom of the section, respect-

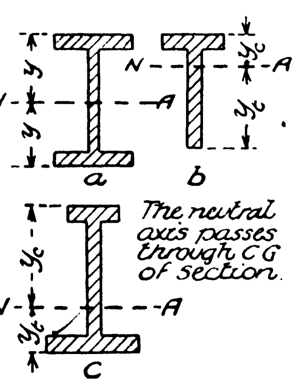


FIG. 68.

ively, not being equal, as in Fig. 68(b) and (c), there will be two Z values for this axis. In a beam composed of material

like steel, which is equally reliable in tension and compression, the lower of these two possible values must be used for calculation of strength. Where the working stresses are not equal in tension and compression (as in cast iron), the safe load for the beam must be separately calculated from the two values—Z_t (tension) and Z_c (compression)—using the corresponding working stress values. The latter case is not common, owing to the extensive modern use of rolled steel sections.

Values of Z for the recently revised list of B.S.B.s will be found on pages 104–107.

The formula $M = \frac{fI}{y}$ may now be written $\mathbf{M = \mathit{f}Z}$.

In the form required for design purposes, the expression for Z becomes $Z = \frac{M}{f}$, or, for a beam of constant section throughout its length, we may write

$$\textbf{Necessary section modulus for beam section} = \frac{\textbf{Maximum B.M. in tons ins.}}{\textbf{Working stress (tons/in.}^2\textbf{)}}.$$

If L = span in feet for a simply supported beam with a U.D. load = W tons, and if 8 tons/in.² be taken as the working stress,

$$\frac{W \times L \times 12}{8} = 8 \times Z.$$

$\therefore\ Z = \frac{WL}{8} \times \frac{12}{8} =$ B.M. in tons ft. $\times\ 1\frac{1}{2}$, a rule often used by designers for the given case.

$$\text{Also } Z = \frac{W \times L \times 12}{64} = \frac{W \times L}{5{\cdot}33}$$

$$= \frac{\text{Load in tons} \times \text{span in feet}}{5{\cdot}33}.$$

Both these rules are employed in the calculations given in Chapter XVI.

EXAMPLES

(In the following examples, the self-weights of the steel beams are not taken into account. In practice an allowance may be made, if required—usually the self-weight for an uncased simple beam is small compared with the load carried.)

(1) *A 9″ × 4″ × 21 lb. B.S.B. has a value of* Z_{XX} *equal to 18·03 ins.*^{3} *(see tables). Calculate the safe U.D. load for this beam, for an effective span of 12′. Use a working stress of 8 tons/in.*^{2}.

(The reader will note that the XX axis is always at right angles to—and the YY axis always parallel to—the direction of the web, in standard sections.)

$$M = fZ.$$

$$\frac{Wl}{8} = fZ.$$

$$\frac{W \times 12 \times 12}{8} = 8 \times 18{\cdot}03. \quad \therefore W = 8 \text{ tons.}$$

This value may be checked by inspection of the table of safe distributed loads given on page 106. The zig-zag lines on this table will be explained later.

(2) *A steel joist of 10′ effective span is required to carry a central load of 2·9 tons. Select a suitable joist from the section tables.*

$$M = fZ.$$

$$\frac{Wl}{4} = fZ.$$

$$\frac{2{\cdot}9 \times 10 \times 12}{4} = 8Z. \quad \therefore Z = \frac{2{\cdot}9 \times 10 \times 12}{4 \times 8} \text{ ins.}^3$$

$$= 10{\cdot}88 \text{ ins.}^3.$$

The nearest value given in the tables, under the heading *Moduli of Section* (axis XX), and not less than the required figure, is 11·29 ins.^{3}, which corresponds to a 7″ × 4″ × 16 lb. B.S.B. The tabular load for this section, for 10′ span, is given as 6 tons (U.D.). This is equivalent to 3 tons as a central load.

(3) *An opening 11′ 9″ in the clear is to be made in a well-bonded brick wall 13½″ thick. The lintol beam for the opening, in addition to carrying the brickwork loading, has to support floor joists which transmit a uniform load of total value 6 tons. Taking the detail given in Fig. 69, design the lintol.*

Provided the brickwork is well bonded and extends at least half the span to each side of the opening, it is usual to take the brickwork loading on a lintol as that contained in an equilateral triangle standing on the effective span as base. This load is

then treated as a uniformly distributed (U.D.) load. Any loads immediately above the opening, such as floor or roof loads, must be treated as additional load.

In the example, allowing 9″ bearing, the effective span = 12′ 6″.

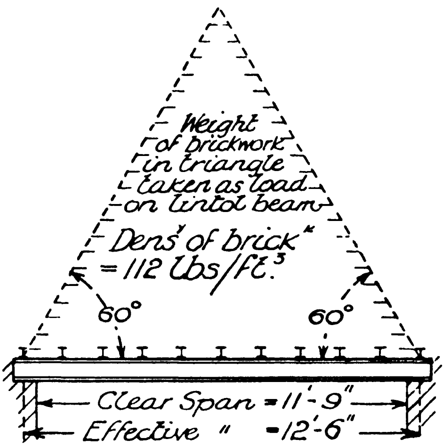


FIG. 69.—BEAM CARRYING A BRICK WALL.

The weight of brickwork in the equilateral triangle

$$= \left[\frac{12{\cdot}5}{2} \times \left(\frac{\sqrt{3}}{2} \times 12{\cdot}5\right) \times \frac{13{\cdot}5}{12} \times 112\right] \text{lb.}$$

$= 3{\cdot}8$ tons.

Total U.D. load $= (3{\cdot}8 + 6)$ tons $= 9{\cdot}8$ tons.

$$M = fZ.$$

$$\frac{9{\cdot}8 \times 12{\cdot}5 \times 12}{8} = 8 \times Z.$$

$$Z = \frac{9{\cdot}8 \times 12{\cdot}5 \times 12}{8 \times 8} \text{ ins.}^3 = 22{\cdot}97 \text{ ins.}^3.$$

Assuming two B.S.B.s, each should have a Z value of at least 11·49 ins.³

Two 8″ × 4″ × 18 lb. B.S.B.s would provide (see tables) 2 × 13·91 ins.³ = 27·82 ins.³, hence these will be suitable. (A 6″ × 4½″ would be too shallow to use.)

(4) *Fig. 70 shows a floor supported by a plate girder, with B.S.B.s as secondary beams. The floor is constructed of concrete with fillers. Assuming the inclusive floor load at 170 lb.*

per sq. ft. find the necessary section modulus for one of the B.S.B.s, and also for the girder (allowing $1\frac{1}{4}$ *tons for its self-weight).*

Area of floor supported by one B.S.B. $= 10' \times 15' = 150 \, ft.^2$.

Total load carried $= (150 \times 170)$ lb. $= 11{\cdot}38$ tons.

$$\frac{Wl}{8} = fZ.$$

$$\frac{11{\cdot}38 \times 15 \times 12}{8} = 8 \times Z. \quad \therefore Z = \frac{11{\cdot}38 \times 15 \times 12}{8 \times 8} \text{ ins.}^3$$

$$= 32 \text{ ins.}^3$$

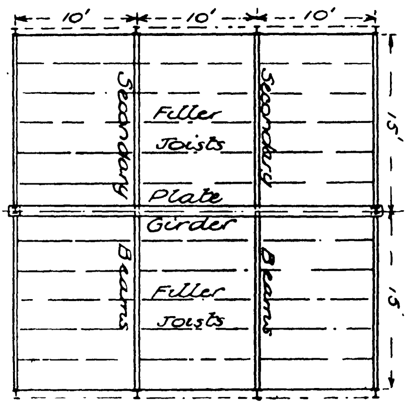


FIG. 70.—STEEL FLOOR FRAME.

A $12'' \times 5'' \times 32$ lb. B.S.B. has a section modulus of $36{\cdot}84$ ins.^{3}, hence will be suitable. The Maximum B.M. for the reaction loads due to the secondary beams in the case of the plate girder will be :

$$(11{\cdot}38 \text{ tons} \times 15') - (11{\cdot}38 \text{ tons} \times 5')$$

$$= 113{\cdot}8 \text{ tons ft.}$$

$$= 1365{\cdot}6 \text{ tons ins.}$$

Due to self-weight of girder the B.M. maximum will be

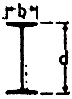
$$\frac{Wl}{8} = \frac{1{\cdot}25 \times 30 \times 12}{8} \text{ tons ins.} = 56{\cdot}25 \text{ tons ins.}$$

Total B.M. maximum $= 1421{\cdot}85$ tons ins.

$$Z = \frac{M}{f} = \frac{1421{\cdot}85}{8} \text{ ins.}^3$$

(assuming a working stress of 8 tons/in.^{2} for both flanges).

(*Continued on p.* 108)



JOISTS

Safe Distributed Loads, in Tons

8

Tons/Inch²

Size d × b inches	SPANS IN FEET													
	10	12	14	16	18	20	22	24	26	28	30	32	36	40
24 × 7½	112	93·8	80·4	70·3	62·5	56·2	51·1	46·9	43·2	40·2	37·5	35·1	31·2	28·1
22 × 7	81·2	67·7	58·0	50·8	45·1	40·6	36·9	33·8	31·2	29·0	27·0	25·4	22·5	20·3
20 × 7½	89·2	74·3	63·7	55·7	49·5	44·6	40·5	37·1	34·3	31·8	29·7	27·8	24·7	22·3
20 × 6½	65·3	54·4	46·7	40·8	36·3	32·6	29·7	27·2	25·1	23·3	21·7	20·4	18·1	16·3
18 × 8	76·5	63·8	54·6	47·8	42·5	38·2	34·8	31·9	29·4	27·3	25·5	23·9	21·2	17·2
18 × 7	68·2	56·8	48·7	42·6	37·8	34·1	31·0	28·4	26·2	24·3	22·7	21·3	18·9	15·3
18 × 6	49·8	41·5	35·6	31·1	27·7	24·9	22·6	20·7	19·1	17·8	16·6	15·5	13·8	11·2
16 × 8	64·9	54·1	46·3	40·5	36·0	32·4	29·5	27·0	24·9	23·1	21·6	20·2	16·0	12·9
$16 \times 6_H$	48·3	40·2	34·5	30·2	26·8	24·1	21·9	20·1	18·5	17·2	16·1	15·1	11·9	9·6
$16 \times 6_L$	41·2	34·3	29·4	25·7	22·8	20·6	18·7	17·1	15·8	14·7	13·7	12·8	10·1	8·2
15 × 6	34·9	29·1	24·9	21·8	19·4	17·4	15·9	14·5	13·4	12·4	11·6	10·2	8·0	
15 × 5	30·4	25·3	21·7	19·0	16·9	15·2	13·8	12·6	11·7	10·8	10·1	8·9	7·0	
14 × 8	53·7	44·7	38·3	33·5	29·8	26·8	24·4	22·3	20·6	19·1	16·7	14·7		
$14 \times 6_H$	40·6	33·8	29·0	25·3	22·5	20·3	18·4	16·9	15·6	14·5	12·6	11·1		
$14 \times 6_L$	33·7	28·0	24·0	21·0	18·7	16·8	15·3	14·0	12·9	12·0	10·4	9·2		
13 × 5	23·2	19·3	16·6	14·5	12·9	11·6	10·5	9·6	8·9	7·7	6·7			
12 × 8	43·3	36·1	30·9	27·0	24·0	21·6	19·7	18·0	15·3	13·2				
$12 \times 6_H$	33·4	27·8	23·8	20·8	18·5	16·7	15·1	13·9	11·8	10·2				
$12 \times 6_L$	28·1	23·4	20·1	17·5	15·6	14·0	12·7	11·7	9·9	8·6				
12 × 5	19·6	16·3	14·0	12·2	10·9	9·8	8·9	8·1	6·9	6·0				

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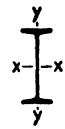
8

Tons/Inch²

JOISTS

Dimensions and Properties



Size d × b inches	Weight per ft. in lbs.	Area in sq. ins.	Standard Thicknesses Web	Standard Thicknesses Flange	Moments of Inertia Axis x—x Max.	Moments of Inertia Axis y—y Min.	Moduli of Section Axis x—x Max.	Moduli of Section Axis y—y Min.	Safe Distributed Load on 1 foot Span	Deflection Coefficient
24 × 7½	95	27·94	·57	1·011	2533·04	62·54	211·09	16·68	1125·8	·000769
22 × 7	75	22·06	·50	·834	1676·80	41·07	152·44	11·73	813·0	·000839
20 × 7½	89	26·19	·60	1·010	1672·85	62·54	167·29	16·68	892·2	·000923
20 × 6½	65	19·12	·45	·820	1226·17	32·56	122·62	10·02	654·0	·000923
18 × 8	80	23·53	·50	·950	1292·07	69·43	143·56	17·36	765·7	·001026
18 × 7	75	22·09	·55	·928	1151·18	46·56	127·91	13·30	682·2	·001026
18 × 6	55	16·18	·42	·757	841·76	23·64	93·53	7·88	498·8	·001026
16 × 8	75	22·06	·48	·938	973·91	68·30	121·74	17·08	649·3	·001154
16 × 6	62	18·21	·55	·847	725·05	27·14	90·63	9·05	483·4	·001154
16 × 6	50	14·71	·40	·726	618·09	22·47	77·26	7·49	412·1	·001154
15 × 6	45	13·24	·38	·655	491·91	19·87	65·59	6·62	349·8	·001231
15 × 5	42	12·36	·42	·647	428·49	11·81	57·13	4·72	304·7	·001231
14 × 8	70	20·59	·46	·920	705·58	66·67	100·80	16·67	537·6	·001319
14 × 6	57	16·78	·50	·873	533·34	27·94	76·19	9·31	406·3	·001319
14 × 6	46	13·59	·40	·698	442·57	21·45	63·22	7·15	337·2	·001319
13 × 5	35	10·30	·35	·604	283·51	10·82	43·62	4·33	232·6	·001420
12 × 8	65	19·12	·43	·904	487·77	65·18	81·30	16·30	433·6	·001538
12 × 6	54	15·89	·50	·883	375·77	28·28	62·63	9·43	334·0	·001538
12 × 6	44	13·00	·40	·717	316·76	22·12	52·79	7·37	281·5	·001538
12 × 5	32	9·45	·35	·550	221·07	9·69	36·84	3·88	196·5	·001538

of the British Steelwork Association.

JOISTS

Safe Distributed Loads, in Tons

8 Tons/inch²

Size d×b inches	SPANS IN FEET													
	3	4	5	6	7	8	9	10	11	12	14	16	18	20
10 ×8							34·2	30·7	27·9	25·6	21·9	19·2	17·1	15·3
10 ×6					31·2	27·3	24·2	21·8	19·8	18·2	15·6	13·6	12·1	10·9
10 ×5			31·2	26·0	22·2	19·5	17·3	15·6	14·1	12·9	11·1	9·7	8·6	7·8
10 ×4½			26·1	21·7	18·6	16·3	14·4	13·0	11·8	10·8	9·3	8·1	7·2	6·5
9 ×7						30·8	27·4	24·6	22·4	20·5	17·6	15·4	13·7	11·1
9 ×4		24·0	19·2	16·0	13·7	12·0	10·6	9·6	8·7	8·0	6·8	6·0	5·3	4·3
8 ×6				25·5	21·9	19·1	17·0	15·3	13·9	12·7	10·9	9·5	7·5	6·1
8 ×5			23·9	19·9	17·0	14·9	13·2	11·9	10·8	9·9	8·5	7·4	5·9	4·7
8 ×4		18·5	14·8	12·3	10·5	9·2	8·2	7·4	6·7	6·1	5·2	4·6	3·6	2·9
7 ×4		15·0	12·0	10·0	8·6	7·5	6·6	6·0	5·4	5·0	4·3	3·2	2·6	
6 ×5		19·4	15·5	12·9	11·0	9·7	8·6	7·7	7·0	6·4	4·7	3·6		
6 ×4½	20·5	15·4	12·3	10·2	8·8	7·7	6·8	6·1	5·6	5·1	3·7	2·8		
6 ×3	12·4	9·3	7·4	6·2	5·3	4·6	4·1	3·7	3·3	3·1	2·2	1·7		
5 ×4½		13·3	10·6	8·8	7·6	6·6	5·9	5·3	4·4	3·7				
5 ×3	9·7	7·2	5·8	4·8	4·1	3·6	3·2	2·9	2·4	2·0				
4¾×1¾	5·0	3·7	3·0	2·5	2·1	1·8	1·6	1·4	1·1					
4 ×3	6·9	6·1	4·1	3·4	2·9	2·5	2·0	1·6						
4 ×1¾	3·2	2·4	1·9	1·6	1·3	1·2	·96	·78						
3 ×3	4·5	3·3	2·7	2·2	1·6	1·2								
3 ×1½	1·9	1·4	1·1	·98	·72	·55								

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8

Tons/inch²

JOISTS

Dimensions and Properties

Size d × b inches	Weight per ft. in lbs.	Area in sq. ins.	Standard Thicknesses		Moments of Inertia		Moduli of Section		Safe Distributed Load on 1 foot Span	Deflection Coefficient
			Web	Flange	Axis x—x Max.	Axis y—y Min.	Axis x—x Max.	Axis y—y Min.		
10 × 8	55	16·18	·40	·783	288·69	54·74	57·74	13·69	307·9	·001846
10 × 6	40	11·77	·36	·709	204·80	21·76	40·96	7·25	218·5	·001846
10 × 5	30	8·85	·36	·552	146·23	9·73	29·25	3·89	156·0	·001846
10 × 4½	25	7·35	·30	·505	122·34	6·49	24·47	2·88	130·5	·001846
9 × 7	50	14·71	·40	·825	208·13	40·17	46·25	11·48	246·7	·002051
9 × 4	21	6·18	·30	·457	81·13	4·15	18·03	2·07	96·2	·002051
8 × 6	35	10·30	·35	·648	115·06	19·54	28·76	6·51	153·4	·002308
8 × 5	28	8·28	·35	·575	89·69	10·19	22·42	4·08	119·6	·002308
8 × 4	18	5·30	·28	·398	55·63	3·51	13·91	1·75	74·2	·002308
7 × 4	16	4·75	·25	·387	39·51	3·37	11·29	1·69	60·2	·002637
6 × 5	25	7·37	·41	·520	43·69	9·10	14·56	3·64	77·7	·003077
6 × 4½	20	5·89	·37	·431	34·71	5·40	11·57	2·40	61·7	·003077
6 × 3	12	3·53	·23	·377	20·99	1·46	7·00	·97	37·3	·003077
5 × 4½	20	5·88	·29	·513	25·03	6·59	10·01	2·93	53·4	·003692
5 × 3	11	3·26	·22	·376	13·68	1·45	5·47	·97	29·2	·003692
4¾ × 1¾	6·5	1·91	·18	·325	6·73	·26	2·83	·30	15·1	·003887
4 × 3	10	2·94	·24	·347	7·79	1·33	3·89	·88	20·7	·004615
4 × 1¾	5	1·47	·17	·239	3·66	·19	1·83	·21	9·76	·004615
3 × 3	8·5	2·52	·20	·332	3·81	1·25	2·54	·83	13·5	·006154
3 × 1½	4	1·18	·16	·249	1·66	·13	1·11	·17	5·92	·006154

of the British Steelwork Association.

B.F. BEAMS, GREY PROCESS: AS GIRDERS.

SAFE DISTRIBUTED LOADS, WITH DEFLECTIONS: 8 TONS STRESS.

Nominal Size. d × b	Weight per Foot.	Delivery.	Maximum Distributed Load.	Moment of Resistance.	6 ft. Safe Load.	6 ft. Defl'n.	7 ft. Safe Load.	7 ft. Defl'n.	8 ft. Safe Load.	8 ft. Defl'n.	9 ft. Safe Load.	9 ft. Defl'n.	10 ft. Safe Load.	10 ft. Defl'n.	12 ft. Safe Load.	12 ft. Defl'n.
Ins.	Lb.		Tons.	In Tns	Tons.	Ins.	Tons.	Ins.	Tons.	Ins.	Tons.	Ins.	Tons.	Ins	Tons.	Ins.
	11·0		5·9	34·0	3 8	·18	3 2	·24	2·8	·32	2·5	·40	2·3	·50	1·9	·72
	14·2		6·3	46·1	5·1	·17	4·4	·23	3 8	·30	3 4	·38	3·1	·47	2·6	·67
4 × 4	14·8	b	8 2	46·6	5 2	·17	4·4	·23	3·9	·30	3·5	·38	3·1	·47	2·6	·67
	23·2		13·8	74 6	8·3	·15	7·1	·21	6·2	·27	5 5	·34	5·0	·42	4·1	·60
	13·2		7·2	51·2	5·7	·15	4·9	·20	4·3	·26	3 8	·33	3 4	·41	2·8	·59
	17·0		7·5	69·6	7·7	·14	6·6	·19	5·8	·25	5 2	·32	4 6	·39	3·9	·56
5 × 5	17·8	b	9 8	69 6	7 7	·14	6 6	·19	5·8	·25	5 2	·32	4·6	·39	3 9	·56
	27 9		16 2	111	12	·13	11	·17	9 3	23	8 2	·29	7 4	·36	6 2	·51
	16 4		9 2	74 6	8·3	·13	7·1	·17	6 2	·23	5 5	·29	5 0	·36	4·1	51
	21·1		7 9	103	...	...	..	...	...	...	7 6	·27	6·9	·33	5 7	·48
5½ × 5½	23·4	ab	13 7	106	12	·12	10	·16	8 8	·21	7·8	·27	7 0	·33	5·9	·48
	47·9		32 6	224	25	·10	21	·16	19	·18	17	·23	15	·29	12	·41
	17 6		9 9	87 2	9 7	·12	8·3	·16	7 3	21	6·5	·27	5 8	·33	4 8	·47
	22 8		9 0	120	...	...	...	...	...	...	8 9	·25	8 0	·31	6 7	·45
6 × 6	24·9	ab	14 7	123	14	·11	12	·15	10	·20	9 1	·25	8 2	·31	6 8	·45
	51 3		34·5	258	29	·10	25	13	22	17	19	·22	17	27	14	·39
	20 0		11 3	103	11	·11	9 8	·15	8 6	20	7 6	·25	6 9	31	5 7	·45
	26 3		10 1	147		. .	. .	.	. .	.	...	. .	9 8	29	8 2	·42
6½ × 6½	30·8	b	17 6	161	...	...	15	14	13	·19	12	·24	11	·29	8 9	·42
	56·0		36 1	298	33	09	28	·13	25	16	22	·21	20	·26	17	·37
	24·8		14 1	148				.	12	·17	. .	...	9·9	·27	8·2	·39
	31·9		12·5	202		...	.	. .		. .		...	..	...	11	·37
7 × 7	34 7	ab	19·9	208	.	...	20	·13	17	·17	15	·21	14	·26	12	·37
	63·0		40·1	383		. .	36	·11	32	·16	28	·19	26	·23	21	·33
	30·1		16·8	199	...		.	. .	17	·16	.	. .	13	·25	11	·35
	38 0		15·1	269		. .	..	...	.	...	...	...	...	...	15	·34
8 × 8	43·6	ab	24 6	280	. .	. .		.	24	·15	22	·19	19	·23	16	·34
	71·6		43 6	484	..	...	. .	.	40	·14	36	·17	32	·21	27	·31
	34·5		19·3	256	..	...	...	.	..	...	...	...	17	·22	14	·32
	44·6		18·0	349	..	. .	.	. .	...	.	...	...	...	...	...	...
8½ × 8½	48·0	ab	27·0	358	.	. .	..	. .	. .	...	...	...	24	·21	20	·31
	78·8		47·6	593	...	...	...	...	...	...	...	...	40	·20	33	·28
	40 9		22 3	330	...	...	...	...	...	...	...	...	22	·21	18	·30
	51 9		21 2	444	...	. .	. .	...	...	...	...	...	...	...	...	...
9½ × 9½	58·7	b	32 5	475	...	...	...	...	...	...	...	...	32	·20	26	·28
	92 2		54 9	754	...	...	...	...	...	...	...	...	50	·18	42	·26

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Necessary section modulus for girder $= 177{\cdot}73$ ins.^{3} ✓

(The question of encasement of steelwork is considered in Chapter XIII.)

Deflection.—Beams have to possess *stiffness* as well as strength. In choosing a beam section for a given maximum U.D. load and span—by means of the tables of safe loads—

B.F. BEAMS, GREY PROCESS: AS GIRDERS.
SAFE DISTRIBUTED LOADS, WITH DEFLECTIONS: 8 TONS STRESS.

14 ft.		16 ft.		18 ft.		20 ft.		22 ft.		24 ft.		26 ft.		28 ft		30 ft.		Nominal Depth.
Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n	Safe Load	Defl'n.	
Tons.	Ins.	Tons.	Ins.	Tons	Ins.	Tons.	Ins.	Tons.	Ins.	Tons.	Ins.	Tons.	Ins.	Tons.	Ins.	Tons	Ins.	Ins.
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	..	
...	...	...	...	...	...	...	...	...	...	...	...	..	...	...	...	...	...	
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	4
2 0	·92	..	...	...	...	...	...	...	...	...	...	...	...	...	..	...	.	
2·4	·80	...	...	...	...	...	...	...	...	...	...	...	...	...	..	...	..	
3 3	·77	...	...	...	...	...	...	...	...	...	..	...	...	...	...	..	..	
3·3	·77	...	...	...	...	...	...	...	..	...	..	...	...	...	...	..	..	5
5 3	·70	...	...	...	...	...	...	...	...	...	..	...	...	...	...	...	..	
3 6	·70	3·1	·91	...	...	...	...	...	...	...	..	...	...	...	...	.	...	
4 9	·65	4 3	·86	...	...	...	...	...	...	...	...	...	...	...	...	...	..	
5 0	·65	4·4	·86	...	...	...	...	...	...	..	..	...	...	...	...	...	...	5½
11	·56	9·3	·73	...	...	...	...	...	...	..	...	...	...	...	...	...	..	
4 2	·65	3·6	·84	...	...	...	...	...	...	...	..	...	...	...	...	..	...	
5·7	·61	5·0	·80	...	...	...	...	...	...	...	...	...	...	...	...	...	..	
5 9	·61	5·1	·80	...	...	...	...	...	...	...	...	...	...	..	...	...	..	6
12	·53	11	·69	9·6	·87	8 6	1 1	...	...	...	...	...	...	..	...	...	...	
4 9	·61	4 3	·80	3 8	1·0	...	...	...	...	..	...	...	...	.	...	...	..	
7·0	·57	6·1	·75	5·5	·95	...	...	...	...	...	.	...	...	...	..	..		
7·7	·57	6·7	·75	6·0	·95	...	...	...	...	...	.	..	...	...	...	...		6½
14	·50	12	·66	11	·83	9 9	1 0	..	..	..	..	..	...	...	...	..	.	
7·1	·53	6·2	·70	5 5	·88	4 9	1 1	..	..	...	...	..	...	...	...	...	..	
9·6	·51	8·4	·67	7·5	·84	6·7	1 0	...	...	..	...	...	...	...	...	...	.	
9 9	·51	8·7	·67	7·7	·84	6·9	1·0	..	..	...	...	...	...	...	...	...	..	7
18	·46	16	·59	14	·75	13	93	12	1 1	...	...	...	...	...	...	...	...	
9 5	·48	8·3	·63	7·4	·80	6·6	98	6 0	1·2	...	...	..	...	...	...	...	...	
13	·46	11	60	10	·76	9·0	94	8·1	1·1	...	...	...	...	...	...	...	...	
14	·46	12	·60	11	·76	9·7	94	8·8	1·1	...	...	...	..	...	...	...	...	8
23	·42	20	·55	18	·69	16	85	15	1 0	13	1·2	...	...	...	...	...	...	
12	·44	11	·57	9·5	·72	8 5	89	7 8	1·1	7·1	1·3	...	...	...	..	...	...	
17	·42	14	·55	13	·69	12	·85	11	1·0	9·7	1·2	...	...	...	...	...	...	
17	·42	15	·55	13	·69	12	·85	11	1·0	9·9	1·2	...	...	...	...	...	...	8½
28	·38	25	·50	22	·63	20	·78	18	·94	16	1·1	15	1 3	...	...	...	...	
16	·40	14	·53	12	·66	11	·82	10	·99	9 2	1·2	8·5	1·4	...	...	...	...	
21	·38	18	·50	16	·63	15	·78	13	·94	12	1·1	11	1·3	...	...	...	...	
23	·38	20	·50	18	·63	16	·78	14	·94	13	1·1	12	1·3	...	...	...	..	9½
36	·35	31	·46	28	·58	25	·72	23	·87	21	1 0	19	1·2	18	1·4	...	..	

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the choice should be made so that the safe load falls to the left of the zig-zag black line, shown on the section tables. The subject of deflection of beams is considered in detail in Chapter VIII.

The' L.C.C. By-laws (Clause 84) state that the span of any beam (except a filler floor beam) shall not exceed 24 times

B.F. BEAMS, GREY PROCESS: AS GIRDERS.
SAFE DISTRIBUTED LOADS, WITH DEFLECTIONS: 8 TONS STRESS.

Nominal Size. d × b	Weight per Foot.	Delivery.	Maximum Distributed Load.	Moment of Resistance.	12 ft.		14 ft.		16 ft.		18 ft.		20 ft.		22 ft.	
					Safe Load.	Defl'n	Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n.
Ins.	Lb.		Tons.		Tons.	Ins.	Tons.	Ins.	Tons.	Ins.	Tons.	Ins.	Tons.	Ins.	Tons.	Ins.
	44·2		23·3	374	21	·28	18	·38	16	·50	14	·64	12	·79	11	·95
	55 6		22·8	497	...	...	...	...	21	·48	18	·61	17	·75	15	·91
10 × 10	61·1	ab	33·8	519	29	·27	25	·37	22	·48	19	·61	17	·75	16	·91
	103		61·3	880	49	·25	42	·34	37	·44	33	·55	29	·68	27	·83
	46·0		24·3	407	23	·27	19	·37	17	·48	15	·61	14	·75	12	·91
	59·5		24·6	553	...	...	...	...	23	·46	21	·58	18	·72	17	·87
10½ × 10½	63·6	b	35·2	566	31	·26	27	·35	24	·46	21	·58	19	·72	17	·87
	116		71·7	1032	57	23	49	·32	43	·42	38	·53	34	·65	31	·79
	51·4		26·9	488	...	..	23	·34	20	·45	18	·57	16	·70	15	·85
	67·7		27·3	679	...	...	...	...	..	...	25	·54	23	·67	21	·81
11 × 11	75·7	ab	41·4	722	40	·24	34	·33	30	·43	27	·54	24	·67	22	·81
	135		81·1	1296	72	·22	62	·30	54	·39	48	·49	43	·61	39	·73
	58·9		31·0	606	...	..	29	·32	25	·41	22	·52	20	·65	18	·78
	76·4		31·2	824	...	...	...	...	...	...	31	·51	28	·63	25	·76
12 × 12	81·2	ab	44·4	840	...	...	40	·31	35	·40	31	·51	28	·63	25	·76
	158		96·3	1648	92	·20	78	·27	69	·36	61	·45	55	·56	50	·68
	65·8		35·8	715	...	...	34	·30	30	·39	26	·49	24	·61	22	·74
	81·4		35 3	928	...	...	...	...	...	...	34	·47	31	59	28	·71
12½ × 12	90·3	b	51·4	984	...	...	47	·29	41	·38	36	·47	33	·59	30	·71
	166		102	1832	102	·19	87	·26	76	·34	68	·43	61	53	56	·64
	70·7		40·6	816	...	..	39	·28	34	·36	30	·46	27	·57	25	·69
	86·2		39·6	1040	...	...	...	...	...	...	39	·45	35	·55	32	·67
13½ × 12	91·6	b	54·6	1064	...	...	51	·27	44	·35	39	·45	35	·55	32	·67
	168		108	1976	...	..	94	·24	82	·32	73	·40	66	·50	60	·60
	75·7		44·9	912	...	...	43	·26	38	·35	34	·44	30	·54	28	·65
	91·3		44·2	1160	...	...	...	...	...	...	43	·42	39	·52	35	·63
14 × 12	101	ab	62·3	1224	...	...	58	·26	51	·33	45	·42	41	·52	37	·63
	170		112	2080	...	...	99	·23	87	·31	77	·39	69	·48	63	·58
	80·6		50·2	1032	..	...	49	·25	43	32	38	·41	34	51	31	·61
	96·3		49·1	1280	...	...	...	...	...	...	47	·40	43	·49	39	·60
15 × 12	102	c	65·8	1312	...	...	62	·24	55	·32	49	·40	44	·49	40	·60
	172		118	2224	...	...	106	·22	93	·29	82	·37	74	46	67	·55
	84·9		52·6	1136	...	...	...	...	47	·31	42	39	38	48	34	58
	101		54·2	1408	...	..	..	...	..	...	52	·38	47	47	43	57
16 × 12	110	b	69·3	1480	...	...	...	...	62	·30	55	·38	49	·47	45	·57
	172		117	2328	...	...	111	·21	97	·28	86	·36	78	44	71	·53

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its depth, unless the calculated deflection of the beam is less than one three-hundred-and-twenty-fifth part of the span.

Shear in Webs.—The webs of rolled steel sections may be stressed up to 5 tons per sq. in. of gross section, provided there is no danger of buckling. The question of shear stress in beams is considered in Chapter IX.

B.F. BEAMS, GREY PROCESS: AS GIRDERS.
SAFE DISTRIBUTED LOADS, WITH DEFLECTIONS: 8 TONS STRESS.

24 ft.		26 ft.		28 ft.		30 ft.		32 ft.		36 ft.		40 ft.		44 ft.		48 ft.		52 ft.		Nominal Depth.
Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n.	Safe Load.	Defl'n.	
Tons.	Ins.	Tons.	Ins.	Tons	Ins.	Tons	Ins.	Tons.	Ins.	Tons	Ins.	Tons	Ins.	Tons.	Ins.	Tons	Ins.	Tons.	Ins.	Ins.
10	1·1	9·6	1·3	8·9	1·5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
14	1·1	13	1·3	12	1·5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
14	1·1	13	1·3	12	1·5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	10
24	·99	23	1·2	21	1·3	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
11	1·1	10	1·3	9·7	1·5	9·0	1·8	...	...	...	...	...	...	...	...	...	...	...	...	
15	1·0	14	1·2	13	1·4	12	1·6	...	...	...	...	...	...	...	...	...	...	...	...	
16	1·0	14	1·2	13	1·4	13	1·6	...	...	...	...	...	...	...	...	...	...	...	...	10½
29	·94	26	1·1	25	1·3	23	1·5	22	1·7	...	...	...	...	...	...	...	...	...	...	
14	1·0	13	1·2	12	1·4	11	1·6	10	1·7	...	...	...	...	...	...	...	...	...	...	
19	·97	17	1·1	16	1·3	15	1·5	14	1·7	...	...	...	...	...	...	...	...	...	...	
20	·97	18	1·1	17	1·3	16	1·5	15	1·7	...	...	...	...	...	...	...	...	...	...	11
36	·87	33	1·0	31	1·2	29	1·4	27	1·5	24	2·0	...	...	...	...	...	...	...	...	
17	·93	16	1·1	14	1·3	13	1·5	13	1·7	11	2·1	...	...	...	...	...	...	...	...	
23	·90	21	1·1	20	1·2	18	1·4	17	1·6	15	2·0	...	...	...	...	...	...	...	...	
23	·90	21	1·1	20	1·2	19	1·4	17	1·6	16	2·0	...	...	...	...	...	...	...	...	12
46	·80	42	·94	39	1·1	37	1·3	34	1·4	31	1·8	27	2·2	...	...	...	...	...	...	
20	·88	18	1·0	17	1·2	16	1·4	15	1·6	13	2·0	12	2·4	...	...	...	...	...	...	
26	·84	24	·99	22	1·1	21	1·3	19	1·5	17	1·9	15	2·3	...	...	...	...	...	...	
27	·84	25	·99	23	1·1	22	1·3	20	1·5	18	1·9	16	2·3	...	...	...	...	...	...	12½
51	·76	47	·89	44	1·0	41	1·2	38	1·3	34	1·7	31	2·1	28	2·5	...	...	...	...	
23	·82	21	·96	19	1·1	18	1·3	17	1·5	15	1·8	14	2·3	12	2·7	...	...	...	...	
29	·79	27	·93	25	1·1	23	1·2	22	1·4	19	1·8	17	2·2	16	2·7	...	...	...	...	
30	·79	27	·93	25	1·1	24	1·2	22	1·4	20	1·8	18	2·2	16	2·7	...	...	...	...	13½
55	·72	51	·84	47	·98	44	1·1	41	1·3	37	1·6	33	2·0	30	2·4	27	2·9	...	...	
25	·78	23	·91	22	1·1	20	1·2	19	1·4	17	1·7	15	2·2	14	2·6	...	...	...	...	
32	·75	30	·88	28	1·0	26	1·2	24	1·3	21	1·7	19	2·1	18	2·5	...	...	...	...	
34	·75	31	·88	29	1·0	27	1·2	25	1·3	23	1·7	20	2·1	19	2·5	...	...	...	...	14
58	·69	53	·81	50	·94	46	1·1	43	1·2	39	1·6	35	1·9	32	2·3	29	2·8	27	3·2	
29	·73	26	·86	25	·99	23	1·1	21	1·3	19	1·6	17	2·0	16	2·4	14	2·9	...	...	
36	·71	33	·83	30	·97	28	1·1	27	1·3	24	1·6	21	2·0	19	2·4	18	2·8	...	...	
36	·71	34	·83	31	·97	29	1·1	27	1·3	24	1·6	22	2·0	20	2·4	18	2·8	...	...	15
62	·66	57	·77	53	·89	49	1·0	46	1·2	41	1·5	37	1·8	34	2·2	31	2·6	29	3·1	
32	·70	29	·82	27	·95	25	1·1	24	1·2	21	1·6	19	1·9	17	2·3	16	2·8	15	3·3	
39	·68	36	·79	34	·92	31	1·1	29	1·2	26	1·5	23	1·9	21	2·3	20	2·7	18	3·2	
41	·68	38	·79	35	·92	33	1·1	31	1·2	27	1·5	25	1·9	22	2·3	21	2·7	19	3·2	16
65	·63	60	·74	55	·86	52	·99	48	1·1	43	1·4	39	1·8	35	2·1	32	2·5	30	3·0	

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Use of Safe Load Tables.—The reader is recommended to check a number of the safe loads given in the various tables in order to acquire confidence in the use of section properties and beam formulæ.

EXERCISES 6

(Where necessary the section tables on pages 104 to 107 must be consulted. These tables may also be used for checking numerical answers. The self-weight of beams may be neglected.)

(1) Calculate the safe U.D. load for a $14'' \times 8'' \times 70$ lb. B.S.B. for an effective span of 20′. Working stress = 8 tons/in.^{2}.

(2) Select a suitable standard section for a simply supported beam which has to carry a central load of 12 tons, the effective span being 18′. Maximum stress not to exceed 8 tons/in.^{2}.

(3) A $9'' \times 4'' \times 21$ lb. B.S.B. ($Z = 18$ ins.^{3}) carries a U.D. load of 6 tons. If the span is 16′, calculate the maximum steel fibre stress.

(4) A beam of 12′ span carries two concentrated loads: 2 tons at 3′ from the left support, and 4 tons at 4′ from the right support. Assuming a maximum stress of $7\frac{1}{2}$ tons/in.^{2}, find the necessary section modulus for the beam.

(5) Fig. 71 shows a lintol composed of two standard channels, with concrete filling. The effective span is 10′. Calculate the

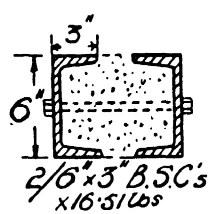


FIG. 71.

safe total U.D. load for the lintol, from the point of view of the channels alone.

I_{XX} for one B.S.C. = 26·28 ins.^{4}.
Maximum stress = 8 tons/in.^{2}.

(6) Select suitable steel sections for the floor given in Fig. 54. The timber beams are of 10′ span, and spaced at 14″ centres. Calculate suitable dimensions for their breadth and depth. Assume a working stress of 1000 lb./in.^{2} in the timber.

CHAPTER VII

PROPERTIES OF COMPOUND BEAM SECTIONS

Parallel Translation of Areas

THE compounding of several smaller areas into one composite section, or the disintegration of a complex figure into a number of simpler diagrams, are methods sometimes employed in dealing with the computation of section properties. The principle employed is known as the *parallel translation of areas.*

If an element of area a, situate at a distance y from a given axis, be moved parallel to the axis, its moment of inertia about that axis will retain the value ay^2 for all positions of the area. Areas of finite size may be regarded as being composed of a very large number of such elemental areas, so that we may move any given section—in whole or part—parallel to an axis, without altering its I value about the axis. Fig. 72

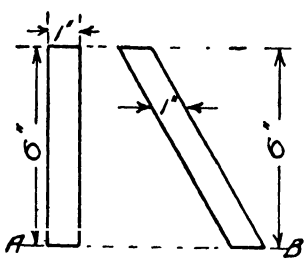


FIG. 72.

illustrates the equality of I value for any horizontal axis, such as AB, of a rectangle and a parallelogram. In this case, rectangles, 1" wide and of very small depth, have been moved horizontally to form the parallelogram.

$$I_{AB} \text{ in each case} = \frac{1 \times 6^3}{3} \text{ ins.}^4 = 72 \text{ ins.}^4 \text{ (see p. 115).}$$

In Fig. 73 the principle is applied to a number of type sections. The diagrams indicate methods of dealing with a half-trough section.

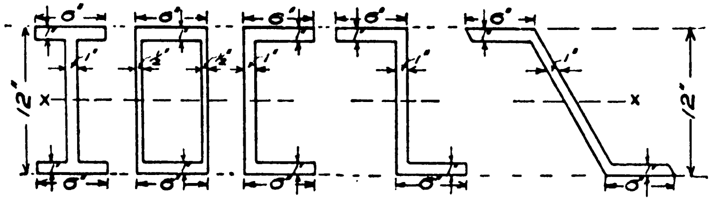


FIG. 73.—PRINCIPLE OF PARALLEL TRANSLATION OF AREAS.

I_{XX} for each of the sections given

$$= \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \left[\frac{6 \times 12^3}{12} - \frac{5 \times 10^3}{12}\right] \text{ins.}^4 = 447{\cdot}3 \text{ ins.}^4.$$

Principle of Parallel Axes

In Fig. 74 the axis AB is assumed to be the axis about which the moment of inertia of the given section is required.

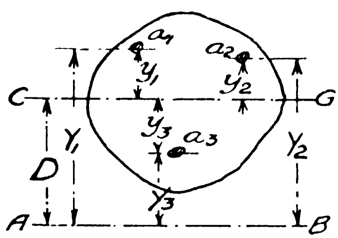


FIG. 74.—PRINCIPLE OF PARALLEL AXES.

CG is an axis parallel to AB, passing through the centre of gravity of the section. The distance between the two axes = D, and A = the total area of the section.

Taking a_1, a_2 and a_3 as typical small component areas of the section, we get :

$$I_{AB} \text{ (by definition)} = a_1Y_1^2 + a_2Y_2^2 + a_3Y_3^2 + \text{etc.}$$
$$= a_1 (y_1 + D)^2 + a_2 (y_2 + D)^2 + a_3(D - y_3)^2 + \text{etc.}$$
$$= a_1y_1^2 + 2ay_1D + a_1D^2 + a_2y_2^2 + 2a_2y_2D + a_2D^2$$
$$+ a_3D^2 - 2a_3y_3D + a_3y_3^2 + \text{etc.}$$
$$= [a_1y_1^2 + a_2y_2^2 + a_3y_3^2 + \text{etc.}]$$
$$+ [a_1D^2 + a_2D^2 + a_3D^2 + \text{etc.}]$$
$$+ [2a_1y_1D + 2a_2y_2D - 2a_3y_3D + \text{etc.}]$$

The expression in the first bracket gives I_{CG}. The second bracket may be written $D^2[a_1 + a_2 + a_3 + \text{etc.}]$, i.e. $= D^2 \times A$. The third bracket $= 2D[a_1y_1 + a_2y_2 - a_3y_3 + \text{etc.}]$. As each term in the latter expression represents an area-moment about an axis passing through the c.g. of the section, the algebraic sum of all the terms will be zero. We may write therefore

$$\mathbf{I_{AB} = I_{CG} + AD^2.}$$

In a practical example, I_{CG} will usually be obtained by means of a standard formula.

EXAMPLES

(1) *Find I_{AB} for the rectangle given in Fig. 75.*

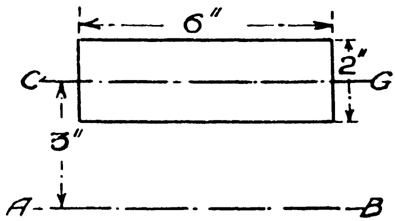


FIG. 75.

$$I_{CG} = \frac{bd^3}{12} = \frac{6 \times 2^3}{12} \text{ ins.}^4 = 4 \text{ ins.}^4.$$

$$A = 6'' \times 2'' = 12 \text{ ins.}^2.$$

$$I_{AB} = I_{CG} + AD^2$$

$$= [4 + 12 \times 3^2] \text{ ins.}^4 = 112 \text{ ins.}^4.$$

(2) *Given the formula $\frac{bd^3}{12}$ for I for an axis at mid-depth of a rectangular section, find the formula for I about the base.*

In this case $I_{CG} = \frac{bd^3}{12}$.

$$A = b \times d.$$

$$D = \frac{d}{2}.$$

$$I_{\text{base}} = I_{CG} + AD^2 = \frac{bd^3}{12} + (b \times d) \times \left(\frac{d}{2}\right)^2$$

$$= \frac{bd^3}{12} + \frac{bd^3}{4} = \frac{bd^3}{3}.$$

This formula is a very useful one, and worth remembering.

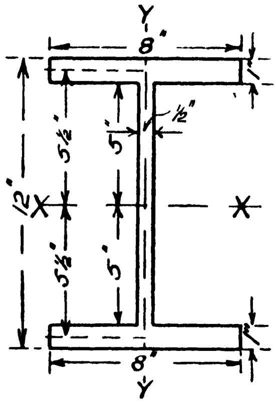


FIG. 76.—R.S.J. SECTION.

(3) *Find the value of I_{XX} for the section given in Fig. 76.*

I_{XX} for top flange $= I_{CG} + AD^2$

$$= \frac{8 \times 1^3}{12} + (8 \times 1) \times 5\tfrac{1}{2}^2$$

$$= \left[\frac{2}{3} + 242\right] \text{ ins.}^4$$

$$= 242{\cdot}66 \text{ ins.}^4.$$

I_{XX} for bottom flange $= 242{\cdot}66$ ins.^{4}.

I_{XX} for web (taken as a rectangle with axis at mid-depth)

$$= \frac{bd^3}{12} = \left[\frac{1}{2} \times \frac{10^3}{12}\right] \text{ ins.}^4 = 41{\cdot}66 \text{ ins.}^4.$$

$$\text{Total } I_{XX} = 527 \text{ ins.}^4.$$

This value may be checked by the formula already given for this case:

$$I_{XX} = \frac{BD^3}{12} - \frac{bd^3}{12} = \left[\frac{8 \times 12^3}{12} - \frac{7{\cdot}5 \times 10^3}{12}\right] \text{ ins.}^4$$

$$= 527 \text{ ins.}^4.$$

To find I_{YY}, in this case, regard the section as being made up of three rectangles.

$$I_{YY} = 2\left[\frac{1 \times 8^3}{12}\right] + \frac{10 \times \frac{1}{2}^3}{12} \text{ ins.}^4$$

$$= 85{\cdot}43 \text{ ins.}^4.$$

Z_{XX} for the section $= \dfrac{I_{XX}}{y} = \dfrac{527}{6}$ ins.^{3} $= 87{\cdot}8$ ins.^{3}.

Z_{YY} for the section $= \dfrac{I_{YY}}{y} = \dfrac{85{\cdot}43}{4}$ ins.^{3} $= 21{\cdot}36$ ins.^{3}.

(4) *Obtain I_{XX} for the channel section given in Fig. 77.*
Method (a) : by subtraction of I values.

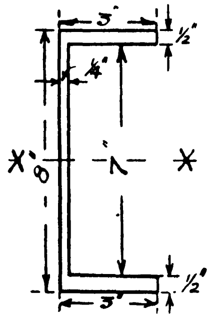


FIG. 77.—CHANNEL SECTION.

$$I_{XX} = \left[\frac{3 \times 8^3}{12} - \frac{2{\cdot}75 \times 7^3}{12}\right] \text{ins.}^4$$
$$= [128 - 78{\cdot}61] \text{ ins.}^4$$
$$= 49{\cdot}39 \text{ ins.}^4.$$

Method (b) : by employing the principle of parallel axes.
Top and bottom flanges :

$$I_{XX} = 2\left[\frac{3 \times \frac{1}{2}^3}{12} + (3 \times \tfrac{1}{2}) \times 3{\cdot}75^2\right] \text{ins.}^4$$
$$= 42{\cdot}24 \text{ ins.}^4.$$

$$\text{Web : } I_{XX} = \frac{bd^3}{12} = \frac{\frac{1}{4} \times 7^3}{12} \text{ ins.}^4 = 7{\cdot}15 \text{ ins.}^4.$$

$$\text{Total } I_{XX} = 49{\cdot}39 \text{ ins.}^4.$$

(5) *Calculate the safe total uniformly distributed load, for an effective span of 8′, for the T-section shown in Fig. 78. Working stress = 8 tons/ins.²*

We must first find the position of the neutral axis of the section.

Taking moments about the base of the section,

$$[(8 \times 1) + (6 \times 1)] \times \bar{y} = [8 \times 6{\cdot}5] + [6 \times 3]$$
$$14\bar{y} = 70$$
$$\bar{y} = 5''.$$

$$I_{XX} \text{ for flange} = I_{CG} + AD^2$$
$$= \left[\frac{8 \times 1^3}{12} + (8 \times 1) \times 1\tfrac{1}{2}^2\right] \text{ins.}^4 = 18{\cdot}66 \text{ ins.}^4$$

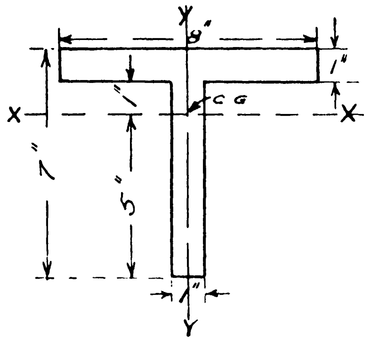


FIG. 78.—T-SECTION.

I_{XX} for web—treated as being composed of two rectangles, with respective axes at the base $\left(\frac{bd^3}{3}\right)$

$$= \left[\frac{1 \times 1^3}{3} + \frac{1 \times 5^3}{3}\right] = 42 \text{ ins.}^4.$$

$$\text{Total } I_{XX} = 60{\cdot}66 \text{ ins.}^4.$$

$$Z_{XX} = \frac{I}{y} = \frac{60{\cdot}66}{5} \text{ ins.}^3 = 12{\cdot}13 \text{ ins.}^3.$$

($y = 5''$, as this is the greater value).

$$\frac{W \times l}{8} = fZ.$$

$$\frac{W \times 8 \times 12}{8} = 8 \times 12{\cdot}13.$$

$$W = 8{\cdot}08 \text{ tons.}$$

(6) *Find I_{XX} and I_{YY} for the angle section given in Fig.* 79.
Determination of c.g. position :

$$[(3 \times \tfrac{1}{2}) + (4 \times \tfrac{1}{2})]\,\bar{x} = (1\tfrac{1}{2} \times \tfrac{1}{4}) + (2 \times 2)$$
$$3\tfrac{1}{2}\bar{x} = 4\tfrac{3}{8}$$
$$\bar{x} = 1\tfrac{1}{4}''$$
$$3\tfrac{1}{2}\bar{y} = (1\tfrac{1}{2} \times 2) + (2 \times \tfrac{1}{4})$$
$$\bar{y} = 1''.$$

I_{XX} for portion beneath the XX axis, treated as the difference of two rectangles,

$$= \left[\frac{4 \times 1^3}{3} - \frac{3\tfrac{1}{2} \times \tfrac{1}{2}^3}{3}\right] \text{ins.}^4 = 1{\cdot}19 \text{ ins.}^4$$

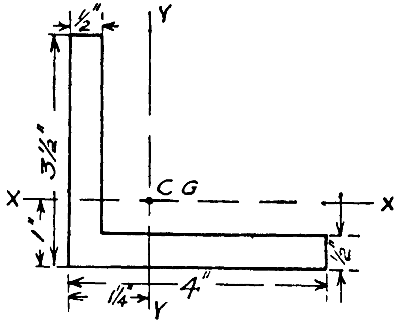


FIG. 79.—ANGLE SECTION.

I_{XX} for portion above XX axis

$$= \frac{bd^3}{3} = \frac{\frac{1}{2} \times 2\frac{1}{2}^3}{3} \text{ ins.}^4 = 2{\cdot}6 \text{ ins.}^4$$

Total $I_{XX} = 3{\cdot}79$ ins.^{4}.

I_{YY} for portion to left of YY axis

$$= \left[\frac{3\frac{1}{2} \times 1\frac{1}{4}^3}{3} - \frac{3 \times \frac{3}{4}^3}{3}\right] \text{ ins.}^4 = 1{\cdot}854 \text{ ins.}^4.$$

I_{YY} for portion to right of YY axis

$$= \frac{\frac{1}{2} \times 2\frac{3}{4}^3}{3} \text{ ins.}^4 = 3{\cdot}466 \text{ ins.}^4.$$

Total $I_{YY} = 5{\cdot}32$ ins.^{4}.

[The XX axis is parallel to shorter leg in B.S. sections.]

Tabular Method for Unsymmetrical Sections

Beam sections, for which the neutral axis is not an axis of symmetry, may be conveniently dealt with by means of a table. The value of I is first determined for *an axis at the base.* The table is arranged in a form suitable for slide rule use.

EXAMPLE. *Obtain I_{XX} for the girder section given in Fig.* 80.

The values of D, shown in the table in Fig. 81, are the respective distances from the centres of gravity of the component areas to the axis AB, as indicated in Fig. 80.

Taking the top flange we have :

$$I_{CG} = \frac{bd^3}{12} = \frac{3 \times 1^3}{12} \text{ ins.}^4 = {\cdot}25 \text{ ins.}^4$$

A $=$ area $= 3'' \times 1'' = 3$ ins.^{2}.

D $=$ distance from its c.g. to axis AB $= 11''$.

The values for the web and bottom flange are similarly obtained.

$$I_{AB} = 757 \text{ ins.}^4$$

$$\bar{y} = \frac{\Sigma AD}{\Sigma A} = \frac{96}{24} \text{ ins.} = 4 \text{ ins.}$$

$$I_{AB} = I_{XX} + AD^2 \text{ or } I_{XX} = I_{AB} - AD^2$$
$$= I_{AB} - A\bar{y}^2$$
$$\therefore\ I_{XX} = [757 - 24 \times 4^2] \text{ ins.}^4$$
$$= 373 \text{ ins.}^4$$

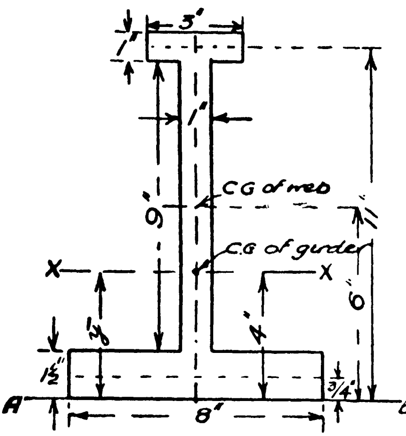


FIG. 80.—UNEQUAL FLANGED SECTION.

The tabular method is useful when a beam section is rendered unsymmetrical by the deduction of rivet holes in one flange only. The section is then treated as gross in the first instance,

INCH UNITS

Part of Section	I_{co}	A	D	D^2	AD	AD^2	$I_{AB} = I_{co} + AD^2$
Top Flange	·25	3	11	121	33	363	363·25
Web	60·75	9	6	36	54	324	384·75
Bot Flange	2·25	12	·75	·5625	9	6·75	9·00
		$\Sigma A = 24$			$\Sigma AD = 96$		$\Sigma I_{AB} = 757{\cdot}00$

FIG. 81.—TABULAR METHOD FOR MOMENT OF INERTIA.

the rivet holes being subsequently regarded as negative areas and entered as such in the table. In most practical cases the effect of rivet hole allowance on the N.A. position may be neglected (see plate girder section, later).

Compound Girders

It is usual to allow for rivet holes in both flanges, in dealing with compound girder sections. The number of holes allowed will depend upon the style of riveting. The properties of the B.S.B.s in the following examples are obtained from the tables on pages 104 to 107.

EXAMPLES

(1) *Find the maximum moment of inertia, and the maximum section modulus, for the plated beam section given in Fig.* 82.

I_{xx} (gross section) :

Flange Plates.—These may be dealt with in several ways. Section books often provide tables giving the I values for

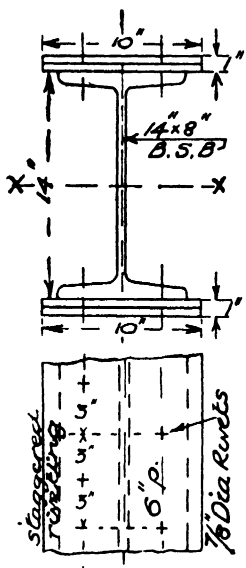


FIG. 82.—PLATED JOIST SECTION.

plates at stated distances apart. The principle of parallel axes may be employed, or the plates may be computed by regarding them as the difference between an outer and an inner rectangle.

(*a*) *Difference method*

$$I_{XX} = \left[\frac{10 \times 16^3}{12} - \frac{10 \times 14^3}{12}\right] \text{ins.}^4 = 1126{\cdot}66 \text{ ins.}^4$$

(*b*) *Parallel axis method*

$$I_{XX} = 2\left[\frac{10 \times 1^3}{12} + (10 \times 1) \times 7{\cdot}5^2\right] \text{ins.}^4 = 1126{\cdot}66 \text{ ins.}^4$$

B.S.B.

$I_{XX} = 705{\cdot}58$ ins.^{4} (from tables).

Total I_{XX} (gross) $= [1126{\cdot}66 + 705{\cdot}58]$ ins.^{4}
$= 1832{\cdot}24$ ins.^{4}

Rivet Hole Allowance

I_{XX} for a rivet hole $= I_{CG} + AD^2$.

I_{CG} is extremely small, and may be neglected. The rivet hole allowance is therefore AD^2 per rivet. A $\frac{7}{8}''$ diameter rivet will require a $\frac{15}{16}''$ diameter hole. The flange thickness of a $14'' \times 8'' \times 70$ lb. B.S.B. is $\cdot 92''$ (see section tables), so that the area of one rivet hole $= (1{\cdot}92'' \times \frac{15}{16}'')$ and its c.g. distance from axis XX $= 7{\cdot}04''$.

One rivet hole must be allowed in each flange in this case.

I_{XX} for rivet holes $= 2AD^2$
$= [2 \times 1{\cdot}92 \times \frac{15}{16} \times 7{\cdot}04^2]$ ins.^{4}
$= 178{\cdot}4$ ins.^{4}

Net I_{XX} for girder section $= [1832{\cdot}24 - 178{\cdot}4]$ ins.^{4}
$= 1653{\cdot}84$ ins.^{4}

$$Z_{XX} = \frac{I_{XX}}{y} = \frac{1653{\cdot}84}{8} \text{ ins.}^3 = 206{\cdot}7 \text{ ins.}^3$$

(2) *Calculate the safe total U.D. load, for an effective span of 20′, for a compound girder of section given in Fig. 83. Working stress = 8 tons/ins.²*

I_{XX} (gross section):

	Ins.^{4}
Plates	
$2\left[\frac{18 \times 1^3}{12} + (18 \times 1) \times 6{\cdot}5^2\right] =$	1524
B.S. Beams	
$2 \times 487{\cdot}77 =$	975·54
Total =	2499·54

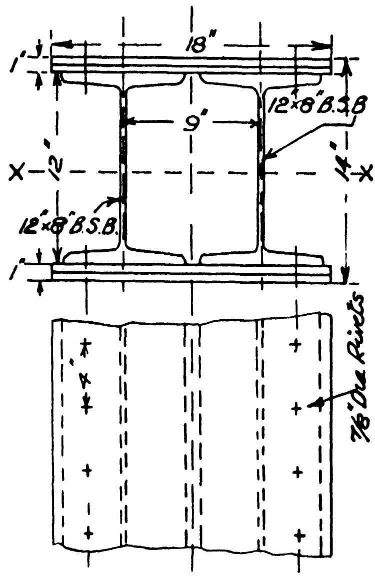


FIG. 83.—COMPOUND GIRDER.

Rivet allowance

$$4 \times \tfrac{15}{16} \times 1{\cdot}904 \times 6{\cdot}05^2 \qquad = \quad 261{\cdot}00$$

$$I_{XX} \text{ (net section)} \qquad = \quad 2238{\cdot}54$$

$$Z \text{ maximum} = \frac{2238{\cdot}54}{7} = 320 \text{ ins.}^3$$

$$M = fZ.$$

$$\frac{W \times 20 \times 12}{8} = 8 \times 320.$$

$$W = 85{\cdot}3 \text{ tons.}$$

Plate Girder Sections

In designing plate girder sections, it is common practice to allow for rivet holes in the tension flange, but not in the compression flange. The displacement of the neutral axis from the mid-depth position by this—or by the fact that the flange section is not the same for each flange—is relatively small and may be neglected in practice.

EXAMPLE. *Determine I_{XX} for the plate girder section given*

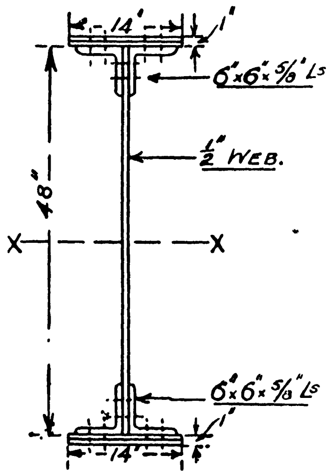


FIG. 84.—PLATE GIRDER SECTION.

in Fig. 84, *allowing* $\frac{13}{16}''$ *diameter rivet holes in the tension flange only.*

I_{XX} (gross section) :

Plates

$$I_{XX} = 2\left[\frac{14 \times 1^3}{12} + (14 \times 1) \times 24{\cdot}5^2\right] \text{ins.}^4 = 16809{\cdot}3 \text{ ins.}^4$$

Angles

From angle section tables, it is found that a $6'' \times 6'' \times \frac{5}{8}''$ angle has an area of $7{\cdot}11$ ins.^{2}, and that the position of the c.g. of the section is $1{\cdot}71''$ from the back of the angle. In the given example, therefore, the c.g. of each angle is $(24-1{\cdot}71)''=22{\cdot}29''$ from axis XX. I_{CG} for one angle is given as $23{\cdot}73$ ins.^{4} (gross).

I_{XX} for the four angles

$$= 4\ [23{\cdot}73 + (7{\cdot}11 \times 22{\cdot}29^2)] \text{ ins.}^4$$
$$= 14224{\cdot}9 \text{ ins.}^4$$

Web

$$I_{XX} = \frac{bd^3}{12} = \frac{\frac{1}{2} \times 48^3}{12} \text{ ins.}^4 = 4608 \text{ ins.}^4$$

Total I_{XX} (gross section) $= 35642{\cdot}2$ ins.^{4}

Rivet Hole Allowance.—Allowance must be made for two $\frac{13}{16}''$ diameter holes (through flange plates and angles) and one $\frac{13}{16}''$ diameter hole (through angles and web plate). For the latter

the hole farther from axis XX is taken. The rivet hole positions in standard sections are themselves standardised (see Appendix II), and in the case of a 6″ × 6″ × $\frac{5}{8}$″ angle, the first hole centre is 2·25″ from the back of the angle. In the example, this will mean a distance of 21·75″ from axis XX.

I_{XX} for flange rivet holes

$= 2 \times \frac{13}{16}'' \times 1\frac{5}{8}'' \times 24{\cdot}19^2 = 1545$ ins.^{4}

I_{XX} for web rivet hole

$= \frac{13}{16}'' \times 1\frac{3}{4}'' \times 21{\cdot}75^2 \quad = 672{\cdot}6$ ins.^{4}

Total I_{XX} for rivet holes $= 2217{\cdot}6$ ins.^{4}

Net I_{XX} for girder section

$= [35642{\cdot}2 - 2217{\cdot}6]$ ins.^{4}

$= 33424$ ins.^{4}.

EXERCISES 7

(Consult pages 104 to 107 for section properties of B.S.B.s.)

(1) A steel beam has the following dimensions: Overall depth = 12″, flange width = 6″, flange and web thicknesses, 1″ and $\frac{1}{2}$″ respectively. Calculate I_{XX}, (*a*) using the principle of parallel axis, (*b*) by any standard formula. Calculate also I_{YY} by any method.

(2) Two rectangular sections, each 6″ deep × 1″ wide, are placed side by side. Calculate the distance between their central vertical axes if $I_{XX} = I_{YY}$, for the two sections combined.

(3) Check the value of I_{XX} for the T-section, given in Fig. 78, by means of the tabular method.

(4) Find I_{XX} for the portion of trough section given in Fig. 85.

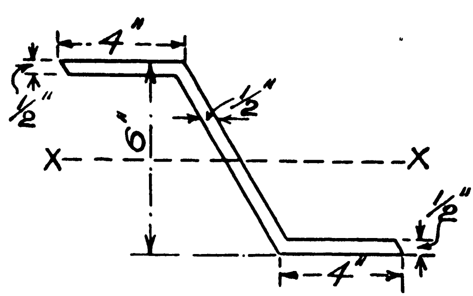


FIG. 85.

(5) A $12'' \times 10''$ compound girder is composed of one $10'' \times 8'' \times 55$ lb. B.S.B., with plates on each flange to form $10'' \times 1''$. Calculate I (maximum) and Z (maximum), allowing one $\frac{15}{16}''$ diameter rivet hole in each flange.

(6) A $14'' \times 14''$ compound girder is built up of two $13'' \times 5'' \times 35$ lb. B.S.B.s with one $14'' \times \frac{1}{2}''$ flange plate, top and bottom. The rivets used are $\frac{3}{4}''$ (nominal) diameter. Calculate the safe total uniformly distributed load for this girder, for an effective span of 20′, working stress = 8 tons per sq. in. (Allow two rivet holes in each flange.)

(7) Find the value of I (maximum) and Z (maximum) for the girder section shown in Fig. 86. The value of I_{XX} for one

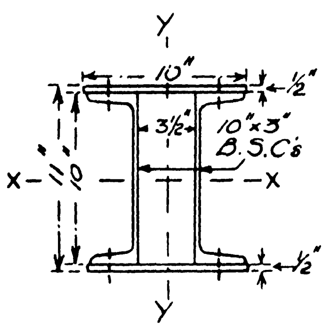


FIG. 86.

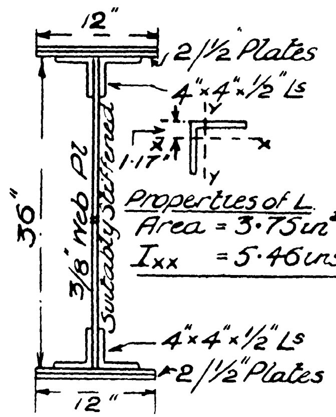


FIG. 87.

$10'' \times 3'' \times 19{\cdot}28$ lb. B.S.C. = $82{\cdot}66$ ins.^{4}. Allow two $\frac{3}{4}''$ rivets in each flange. Flange thickness of B.S.C. = $\cdot 45''$.

(8) A plate girder, of the section given in Fig. 87, has an effective span of 40′. Allowing two $\frac{13}{16}''$ diameter holes in each flange, calculate the total safe uniformly distributed load for the girder (maximum stress allowable in each flange = 8 tons/in.^{2}).

CHAPTER VIII

DEFLECTION OF BEAMS. THEORY AND PRACTICE

Introduction.—Not only have beams to be designed to support the applied loads without unduly stressing the fibres of the beam material, but *they must be made stiff enough to prevent excessive deflection.* The result of a large deflection in a beam carrying a plastered ceiling will be apparent, but the importance of ' stiffness ' in beams lies deeper than this. The steel frame is composed of a large number of separate units which are, however, often connected together in a manner which prevents them acting entirely independently. A beam which is connected to a stanchion by a rigid form of connection tends to transmit to the stanchion the end slope it would have if it were freely supported. A good deal of research work is being carried on with respect to the interdependence of members in a composite frame and the bending moment transmitted through stiff connections. Also such questions as the alternate loading of floors in a steel framed building are being investigated experimentally.

Permissible Deflection Values.—L.C.C. By-law No. 84 states: *The span of any filler floor beam encased in concrete shall not exceed 32 times the depth, measured from the bottom flange of the floor beam to the top surface of the concrete. The span of any other beam shall not exceed 24 times its depth, unless the calculated deflection of the beam is less than one three-hundred-and-twenty-fifth part of the span.*

The deflection allowance in any given case is governed by the particular application of the beam. For beams carrying brick walls—if the span exceeds 12′—the maximum deflection should not exceed $\frac{1}{40}''$ per foot of span. Timber beams of large span are especially liable to excessive deflection if designed for strength alone. Usually the deflection consideration governs the design in such cases. Beams supporting plastered ceilings should not deflect more than $\frac{1}{30}''$ per foot of span.

As already referred to in a previous chapter, the suitability of a B.S.B. section, from the deflection point of view, is indicated in tables of safe loads by the insertion of a zig-zag black line (see page 104). When beams are carrying their *full loads* corresponding to a working stress of 8 tons/in.^{2}, the adoption of spans to the right of this line would result in excessive deflection. The tabular values to the right of the line, in the tables on pages 104 to 107, do not correspond to a working stress of 8 tons/in.^{2}, but to a prescribed deflection allowance.

The connection between the deflection allowance and the position of the zig-zag line is discussed a little later.

Deflection Coefficients.—An inspection of the tables of safe loads will show that *deflection coefficients* are given for the various standard sections. For beams simply supported at their ends, and carrying U.D. loads of such value as to produce a maximum fibre stress of 8 tons/in.^{2}, the maximum deflection is obtained by *multiplying the square of the span in feet by the appropriate deflection coefficient.*

The complete deflection problem is to determine the deflection anywhere in a beam, and not merely the maximum value. The various methods employed in such calculations will now be considered.

Circular Bending

In the theory of bending relationship $\frac{E}{R} = \frac{M}{I}$, R = the radius of curvature of the beam (i.e. of its neutral layer). For the beam to deflect to the arc of a circle, R must be constant, i.e. $\frac{M}{EI}$ must be constant. Assuming E constant, this means that $\frac{M}{I}$ must remain constant throughout the beam span.

(i) **If I is constant,** i.e. if the beam has a constant section, **M must be constant.** This is not the usual case, as the bending moment generally varies from point to point in the span. Beams of constant section normally therefore do not bend in a circular manner. Fig. 88 illustrates a special case of a beam undergoing circular deflection.

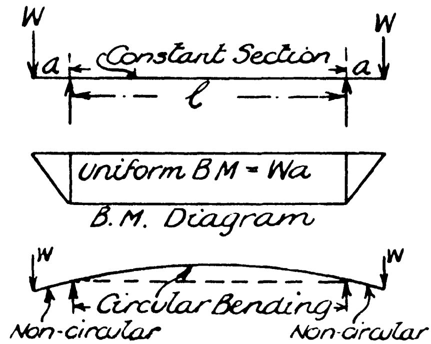


FIG. 88.—CIRCULAR BENDING.

(ii) **If M varies, I must vary in proportion** so as to make the ratio $\frac{M}{I}$ constant. I does vary as M in the economical design of a plate girder, flange plates being dispensed with, as the bending moment falls away. Plate girders, so designed, may be regarded as having circular deflection.

By the geometry of the circle (see Fig. 89)

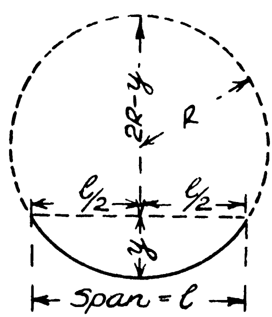


FIG. 89.

$$y(2R - y) = \frac{l}{2} \times \frac{l}{2}$$

$$2Ry - y^2 = \frac{l^2}{4}$$

y^2 is extremely small compared with the other quantities, and may be neglected.

$$\therefore\ 2Ry = \frac{l^2}{4}.$$

$$y = \frac{l^2}{8R}.$$

$$\text{But } \frac{1}{R} = \frac{M}{EI}.\quad \therefore\ y = \frac{Ml^2}{8EI}.$$

Maximum deflection in circular bending $= \dfrac{\mathbf{M}l^2}{\mathbf{8EI}}.$

(See page 285 for use of formula in a plate girder design.)

EXAMPLE. *A plate girder, fulfilling the requirements for circular deflection, has an effective span of* 30′, *and carries a U.D. load of* 60 *tons. The value of I for the girder section at the centre being* 8000 *ins.*^{4}, *and taking E as* 13000 *tons/in.*^{2}, *calculate the maximum deflection.*

$$y_{maximum} = \frac{Ml^2}{8EI}.$$

$$M = \frac{Wl}{8} = \frac{60 \times 30 \times 12}{8} \text{ tons ins.} = 2700 \text{ tons ins.}$$

$$\therefore\ y_{maximum} = \frac{2700 \times 30 \times 12 \times 30 \times 12}{8 \times 13000 \times 8000} = \cdot 42''.$$

E is sometimes taken as 12000 tons/in.^{2} in beam deflection problems, especially in cases where the parts of a beam are connected by riveting.

General Case of Deflection

It will be useful to consider the question of the *slope* of a beam at the same time as that of its deflection. The 'slope' is expressed in radians, the angle being measured to the horizontal.

Mohr's Theorem for the Deflection and Slope of a Cantilever. In Fig. 90 C and D are assumed to be so close together that the radius of curvature is R throughout the length δx of the cantilever.

The circular measure of the angle between the two radii shown $= \dfrac{\text{arc}}{\text{radius}} = \dfrac{\delta x}{R}.\quad \therefore\ \theta = \dfrac{\delta x}{R}.$

The tangents to the bent beam, at points C and D respec-

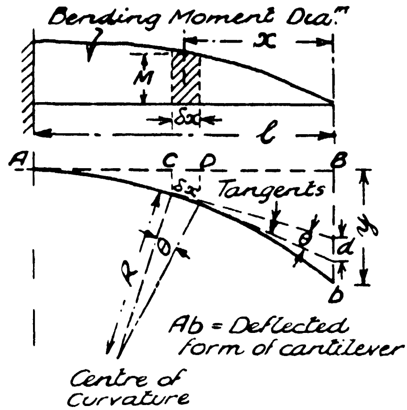


FIG. 90.—DEFLECTION OF A CANTILEVER.

tively, will also contain the angle θ, and its circular measure may be expressed very closely as $\frac{d}{x}$.

$$\therefore\ \theta = \frac{\delta x}{R} = \frac{d}{x} \qquad \therefore\ d = \frac{x\delta x}{R}.$$

$$\text{But } \frac{1}{R} = \frac{M}{EI} \qquad \therefore\ d = \frac{xM\delta x}{EI}.$$

The total deflection is the sum of all such little intercepts as d.

$$\therefore\ y_{maximum} = \Sigma d = \frac{\Sigma xM\delta x}{EI}.$$

$M\delta x$ represents the area of the little element of B.M. diagram standing on δx as base, and x is its distance from the end B of the cantilever.

$\therefore\ \Sigma xM\delta x = A\bar{x}$, where A is the total area of the B.M. diagram and $\bar{x}$ is its centre of gravity distance from B.

$$\therefore\ y_{maximum} = \frac{A\bar{x}}{EI}.$$

The diagrams in Fig. 91 show how the result can apply to any point in the cantilever.

The total slope of the cantilever at B (Fig. 90) will be

$$\Sigma\theta = \Sigma\frac{\delta x}{R} = \Sigma\frac{M\delta x}{EI}$$

$$= \frac{A}{EI}.$$

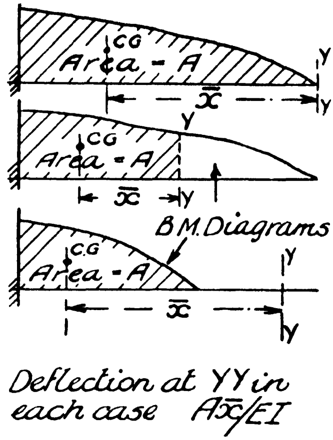


FIG. 91.

The theorem will now be applied to a few standard cases of beams.

(i) Cantilever with Concentrated Load W at the Free End

In Fig. 92, $A = \frac{1}{2}$ base $\times$ height

$$= \tfrac{1}{2}l \times Wl = \frac{Wl^2}{2}.$$

$$\bar{x} = \tfrac{2}{3}l.$$

$$y_{maximum} = \frac{A\bar{x}}{EI} = \frac{\frac{Wl^2}{2} \times \frac{2}{3}l}{EI}$$

$$= \frac{1}{3}\frac{Wl^3}{EI}.$$

$$\text{Slope at B} = \frac{A}{EI} = \frac{1}{2}\frac{Wl^2}{EI}.$$

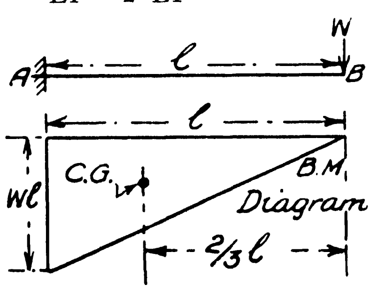


FIG. 92.—CANTILEVER WITH SINGLE END LOAD.

(ii) Cantilever with Uniformly Distributed Load of Total Value W

In Fig. 93, A = Area of parabola

$$= \tfrac{1}{3} \text{ base} \times \text{height (see Appendix III)}$$

$$= \tfrac{1}{3}l \times \frac{Wl}{2} = \frac{Wl^2}{6}.$$

$$\bar{x} = \tfrac{3}{4}l.$$

$$y_{maximum} = \frac{A\bar{x}}{EI} = \frac{\frac{Wl^2}{6} \times \frac{3}{4}l}{EI}$$

$$= \frac{1}{8}\frac{Wl^3}{EI}.$$

$$\text{Slope at B} = \frac{A}{EI} = \frac{1}{6}\frac{Wl^2}{EI}.$$

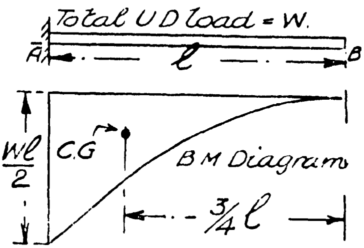


FIG. 93.—CANTILEVER WITH U.D. LOAD.

EXAMPLE. *A cantilever projecting* 5′ *from its support* (*Fig.* 94) *carries a load of* 2 *tons at its free end. The moment*

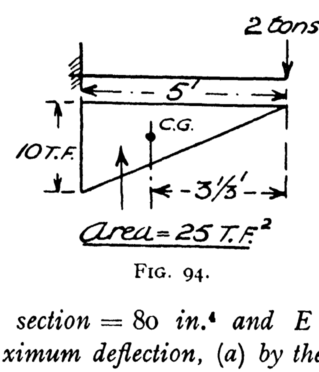


FIG. 94.

of inertia of the section = 80 *in.*⁴ *and E* = 13000 *tons/in.*². *Calculate the maximum deflection,* (*a*) *by the standard formula,* (*b*) *by Mohr's theorem.*

$$(a)\ y_{maximum} = \frac{1}{3}\frac{Wl^3}{EI} = \frac{\frac{1}{3} \times 2 \times (5 \times 12)^3}{13000 \times 80}$$

$$= \frac{144000}{1040000} = \cdot 138''.$$

(*b*) B.M. maximum = 2 tons × 5 ft. = 10 tons ft.

$$A = \tfrac{1}{2}\text{ base} \times \text{height} = (\tfrac{1}{2} \times 5 \times 10)\text{ tons ft.}^2$$

$$= 25\text{ tons ft.}^2$$

$$\bar{x} = \tfrac{2}{3} \times 5' = 3\tfrac{1}{3}'.$$

$$y_{maximum} = \frac{A\bar{x}}{EI} = \frac{25 \times 3\frac{1}{3} \times 12 \times 12 \times 12}{13000 \times 80}\text{ ins.}$$

(The three 12's in the numerator are required to reduce the 'feet cubed' to 'inch' dimensions.)

$$y_{maximum} = \cdot 138''\text{ as before.}$$

Simply supported beams may—by regarding them as pairs of inverted cantilevers—be made adaptable to Mohr's theorem, in the form given. A more direct mode of solution may, however, be employed in such cases.

Secondary B.M. Method for Deflection

It can be shown that *deflection* bears the same type of relationship to *bending moment* as *bending moment* does to *loading*. If therefore we treat the B.M. diagram for a beam as its load system, and recalculate the B.M. value for a given beam section on this basis, the value obtained will be a measure of the deflection at the section. The product EI has to be introduced to obtain the exact value.

The principle is illustrated in Fig. 95. The deflection at C is given by

$$y_C = \frac{M'}{EI}$$

where M′ is the secondary B.M. at C.

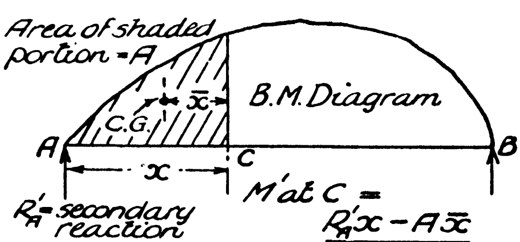


FIG. 95.—DEFLECTION BY SECONDARY BENDING MOMENT.

(iii) Simply Supported Beam with Single Concentrated Central Load W

In Fig. 96, R_A' = secondary reaction = $\frac{1}{2}$ total area of B.M. diagram.

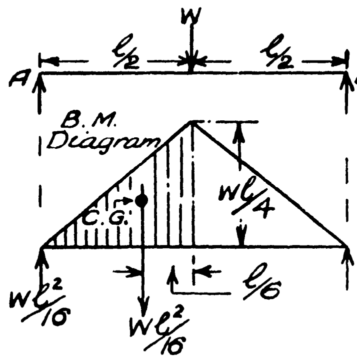


FIG. 96.—SIMPLY SUPPORTED BEAM WITH SINGLE CONCENTRATED CENTRAL LOAD.

$$= \left(\frac{1}{2} \times l \times \frac{Wl}{4}\right) \div 2$$

$$= \frac{Wl^2}{16}.$$

$$M' \text{ (at centre of span)} = \left[\frac{Wl^2}{16} \times \frac{l}{2}\right] - \left[\frac{Wl^2}{16} \times \left(\frac{1}{3} \times \frac{l}{2}\right)\right]$$

$$= \frac{Wl^3}{32} - \frac{Wl^3}{96} = \frac{Wl^3}{48}.$$

$$y_{maximum} = \frac{M'}{EI} = \frac{1}{48}\frac{Wl^3}{EI}.$$

(iv) Simply Supported Beam with U.D. Load of Total Value W

In Fig. 97, $R_A' = \frac{1}{2}$ ($\frac{2}{3}$ × base × height)

$$= \tfrac{1}{2}\left(\frac{2}{3} \times l \times \frac{Wl}{8}\right) = \frac{Wl^2}{24}.$$

$$M' \text{ (at centre of span)} = \left[\frac{Wl^2}{24} \times \frac{l}{2}\right] - \left[\frac{Wl^2}{24} \times \left(\frac{3}{8} \times \frac{l}{2}\right)\right]$$

$$= \frac{Wl^3}{48} - \frac{3Wl^3}{384} = \frac{5}{384} Wl^3.$$

$$y_{maximum} = \frac{M'}{EI} = \frac{5}{384}\frac{Wl^3}{EI}.$$

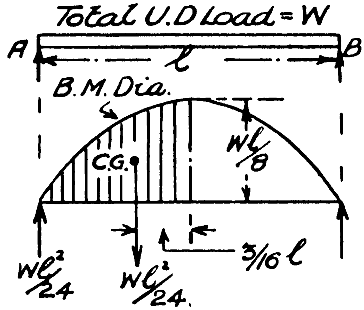


FIG. 97.—SIMPLY SUPPORTED BEAM WITH U.D. LOAD.

The reader should refer to Appendix III for the properties of a parabola.

EXAMPLES

(1) *Calculate the maximum deflection of a* 9″ × 4″ × 21 lb. *B.S.B. when it is carrying* 8 *tons U.D. load, for an effective span of* 12′ (*Fig.* 98).

$I_{max.}$ for section = 81·13 ins.^{4}, E = 13000 tons/in.^{2}

$$y_{max.} = \frac{5}{384}\frac{Wl^3}{EI} = \frac{5}{384} \times \frac{8 \times (12 \times 12)^3}{13000 \times 81 \cdot 13} \text{ ins.}$$
$$= \cdot 295''.$$

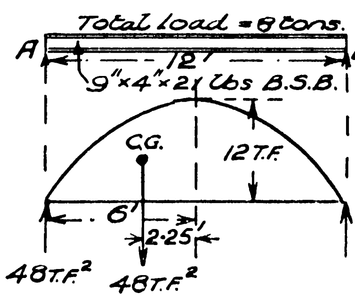


FIG. 98.

The tables give ·002051 as the deflection coefficient for this case (page 107, last column).

$$y_{max.} = \cdot 002051 \times (\text{span in feet})^2$$
$$= \cdot 002051 \times 12^2 \text{ ins.}$$
$$= \cdot 295'' \text{ as before.}$$

The result may also be checked by using the secondary B.M. method as follows :

$$\frac{Wl}{8} = \frac{8 \times 12}{8} \text{ tons ft.} = 12 \text{ tons ft.}$$

$$\begin{aligned} \text{Area of parabola} &= \tfrac{2}{3} \text{ base} \times \text{height} \\ &= (\tfrac{2}{3} \times 12 \times 12) \text{ tons ft.}^2 \\ &= 96 \text{ tons ft.}^2 \end{aligned}$$

$$\begin{aligned} \text{Secondary reaction at A} = R_A' &= \frac{96}{2} \text{ tons ft.}^2 \\ &= 48 \text{ tons ft.}^2 \end{aligned}$$

Taking moments about the centre of the beam for the secondary bending moment, we get :

$$M' = [(48 \times 6) - (48 \times 2{\cdot}25)] \text{ tons ft.}^3$$

Care must be taken with the dimensions of the results in these calculations, in order that correct reduction from ' feet ' to ' inch ' units may be effected, when desired.

$$\begin{aligned} M' \text{ at centre} &= 180 \text{ tons ft.}^3 \\ &= (180 \times 12 \times 12 \times 12) \text{ tons ins.}^3 \\ y_{maximum} &= \frac{M'}{EI} = \frac{180 \times 12 \times 12 \times 12}{13000 \times 81{\cdot}13} \text{ ins.} \\ &= {\cdot}295''. \end{aligned}$$

(2) *A* $7'' \times 4'' \times 16$ *lb. B.S.B. carries a single concentrated load of* 3 *tons, as shown in Fig.* 99. *I max. for this section* $= 39{\cdot}51$ *ins.*^{4}, *and* $E = 13000$ *tons/in.*^{2}. *Calculate the deflection under the load, and also determine the position and value of the maximum deflection.*

$$\text{B.M.}_C = \frac{3 \times 4 \times 8}{12} \text{ tons ft.} = 8 \text{ tons ft.}$$

In the case of a single load, as in this example, it will be found that the maximum deflection will always occur at a point in the larger portion of the span. It will be convenient therefore to calculate R_B'.

Area of triangle ADC $= \frac{1}{2} \times 4 \times 8$ tons ft.^{2} $= 16$ tons ft.^{2}
Area of triangle DCB $= \frac{1}{2} \times 8 \times 8$ tons ft.^{2} $= 32$ tons ft.^{2}

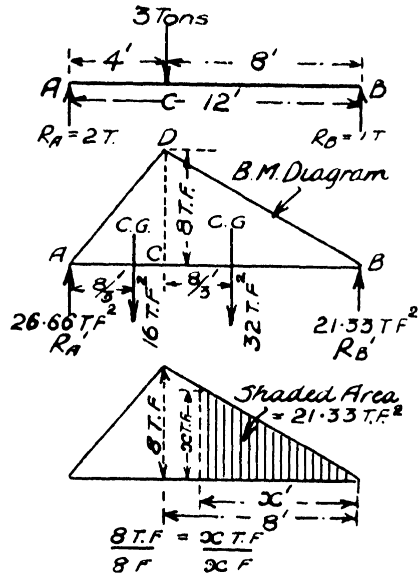


FIG. 99.

Moments about A

$$R_B' \times 12 = \left[32 \times \left(4 + \frac{8}{3}\right)\right] + \left[16 \times \left(\frac{2}{3} \times 4\right)\right]$$

$$12\ R_B' = \left(32 \times \frac{20}{3}\right) + \left(16 \times \frac{8}{3}\right)$$

$$R_B' = 21{\cdot}33 \text{ tons ft.}^2.$$

$$M' \text{ at } C = (21{\cdot}33 \times 8) - \left(32 \times \frac{8}{3}\right) \text{ tons ft.}^3$$

$$= 85{\cdot}31 \text{ tons ft.}^3$$

$$y_C = \frac{M_C'}{EI} = \frac{85{\cdot}31 \times 12 \times 12 \times 12}{13000 \times 39{\cdot}51} \text{ ins.} = {\cdot}287''.$$

To find the position of maximum deflection, we make use of a rule (explained in Chapter IX) which states that the maximum B.M. in a beam occurs at the point where the shear force is zero. As deflection $= \frac{M'}{EI}$, maximum deflection corresponds to maximum secondary B.M. value.

To find the position of maximum deflection a point will have

to be found at such distance from B, that the area of the B.M. diagram up to this point = R_B'.

Let x' = the distance.

B.M. at this section = x tons ft. (see Fig. 99). This is because at 8 ft. the B.M. = 8 tons ft., i.e. a numerical ratio of 1 : 1.

$$\therefore \frac{x \times x}{2} = 21{\cdot}33, \text{ i.e. } x^2 = 42{\cdot}66.$$

$$\therefore x = 6{\cdot}54'.$$

$$M' \text{ at this section} = \left[(21{\cdot}33 \times 6{\cdot}54) - \left(21{\cdot}33 \times \frac{6{\cdot}54}{3}\right)\right]$$

$$= 93 \text{ tons ft.}^3$$

$$y_{maximum} = \frac{M'}{EI} = \frac{93 \times 1728}{13000 \times 39{\cdot}51} \text{ ins.} = {\cdot}31''.$$

Graphical Method for Deflection

The reader will recall that if a link polygon be drawn for the load system on a beam, it forms, with the closing line, the B.M. diagram for the beam. By treating the B.M. diagram (as in the previous examples) as the load diagram, and drawing a link polygon for this new 'load system,' we will have a diagram which, to a certain scale, will give deflection values for all points on the beam (see Fig. 100). The graphical method is by far the

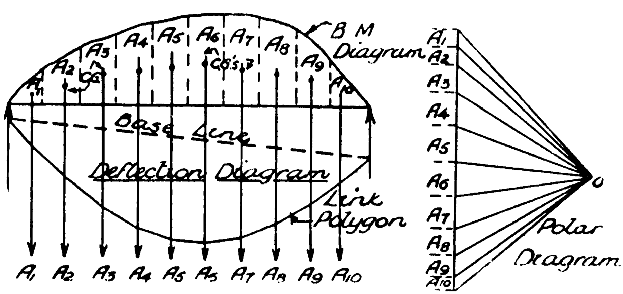


FIG. 100.—GRAPHICAL METHOD FOR DEFLECTION.

easiest method of dealing with deflections, if the loading is at all complicated. It has the advantage of exhibiting the deflection for all points in the beam span. Great care must be exercised in arriving at the correct scale for reading off the deflections from a diagram thus obtained.

EXAMPLE. *Check the value of the deflection at the load point, and also the value of maximum deflection, for the example given in Fig.* 99.

The B.M. diagram is divided into a convenient number of strips, strips 1′ wide being taken in the given example (Fig.

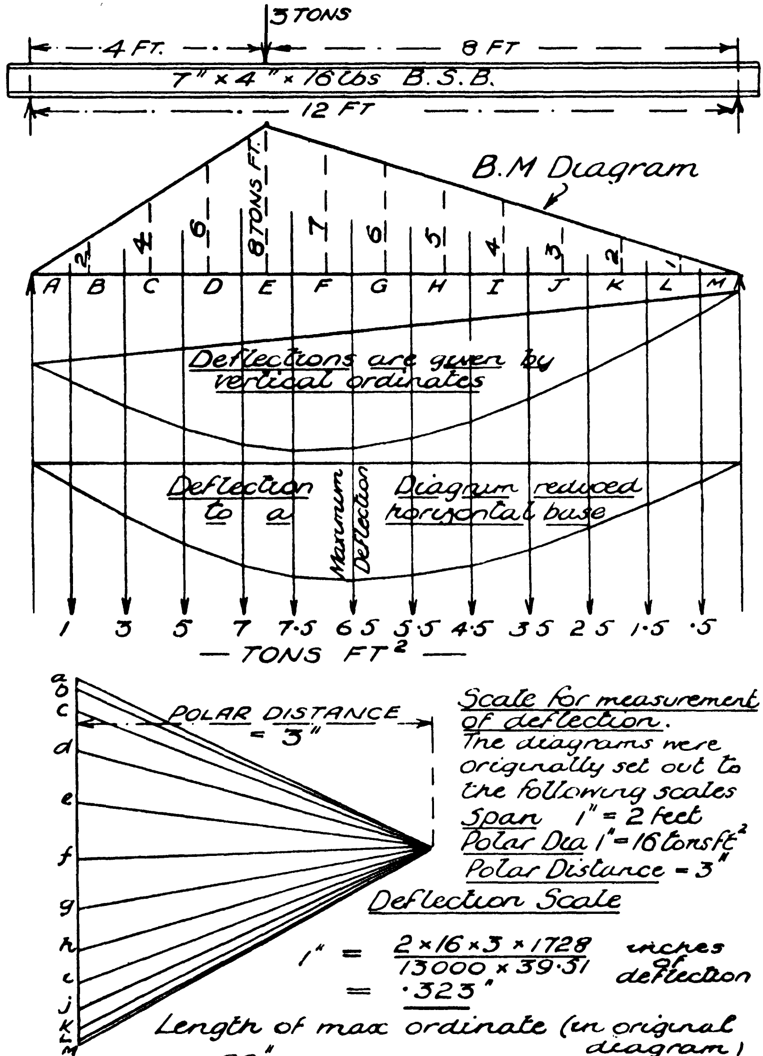


FIG. 101.—EXAMPLE OF GRAPHICAL METHOD.

101). The areas of these strips are computed, and set down in the polar diagram to scale—just as loads are treated in the usual construction. The polar distance is made a round figure, e.g. 3″, to yield a convenient deflection scale. The latter is obtained by multiplying together the span scale, 'load' scale and the polar distance, and then dividing the result by EI. Thus the diagram in Fig. 101, as originally set out, had a scale for deflections:

$$1'' = \frac{2 \times 16 \times 3 \times 1728}{13000 \times 39{\cdot}51} \text{ ins. of deflection}$$
$$= {\cdot}323''.$$

Length of maximum ordinate was ·98″, therefore the maximum deflection = (·98 × ·323) ins.

$$= {\cdot}316''.$$
$$\text{Ordinate at load point} = {\cdot}89'' \text{ (actual inches)}.$$
$$\therefore \text{Deflection at load point} = ({\cdot}89 \times {\cdot}323) \text{ ins.}$$
$$= {\cdot}287''.$$

With average care, the graphical method gives very accurate results.

Relationship between Span and Deflection

Taking the case of uniformly distributed loading (*the loading for which the limiting zig-zag black line is given in the tables*) we have:

$$y_{maximum} = \frac{5}{384} \frac{Wl^3}{EI}$$
$$= \frac{5}{48} \times \frac{Wl}{8} \times \frac{l^2}{EI}$$
$$= \frac{5}{48} \times \frac{M}{EI} \times l^2 \left[M = \frac{Wl}{8}\right]$$
$$\text{But } \frac{M}{I} = \frac{f}{y}$$
$$\therefore y_{maximum} = \frac{5}{48} \times \frac{f}{y} \times \frac{l^2}{E}.$$

Assuming a maximum stress of 8 tons/in.², putting $y = \frac{D}{2}$ (½ depth of beam) and inserting 13000 tons/in.² for E, we get—

for the usual case of beam sections symmetrical about the neutral axis—

$$y_{maximum} = \frac{5}{48} \times \frac{16}{D} \times \frac{l^2}{13000}.$$

$$\frac{y_{maximum}}{l} = \frac{l}{7800D}.$$

Taking the L.C.C. By-law value for maximum deflection, i.e. $\frac{1}{325}$th of the span,

$$\frac{1}{325} = \frac{l}{7800D}$$

$$\text{or} \qquad l = \frac{7800D}{325} = 24D.$$

This means that the span must not exceed 24 *times the depth of the beam.* For example, a beam 9″ deep must not (if fully loaded) be used for a span greater than (24 × 9)″ = 18′, which limit is indicated in the tables. The figure '24,' given in maximum deflection regulations, will now be understood.

Deflection Coefficients

$$y_{maximum} = \frac{l^2}{7800D} \text{ (from above).}$$

If L = span in feet and D = depth in inches,

$$y_{maximum} \text{ (inches)} = \frac{L^2 \times 144}{7800D}$$

$$= \frac{6}{325D} \times L^2 = \text{deflection coefficient} \times L^2.$$

Taking, for example, a 10″ × 8″, a 10″ × 6″, a 10″ × 5″ or a 10″ × 4½″, B.S.B., the deflection coefficient

$$= \left(\frac{6}{325 \times 10}\right) = \cdot001846.$$

This figure will be found in the tables on page 107 for joists with 10″ depth.

Simple Rule.—Instead of completely evaluating the deflection coefficient we may express the maximum deflection in terms of both D and L:

$$y_{maximum} = \frac{6}{325} \times \frac{L^2}{D}$$

$$\text{or } y_{maximum} = \frac{L^2}{54D} = \frac{\text{(Span in feet) squared}}{54 \times \text{depth in inches}}.$$

Mathematical Treatment of Deflection

Readers not familiar with the Calculus should omit the remainder of this chapter.

It is only intended to exemplify this method of solution in a few simple cases. A full treatment of this part of the subject will be found in the many excellent books on the theory of structures.

Convention of Signs.—In Fig. 102 x is positive to the right and y is positive upwards. The slope $\frac{dy}{dx}$ is positive, y increasing as x increases. Slopes upwards, as we proceed to

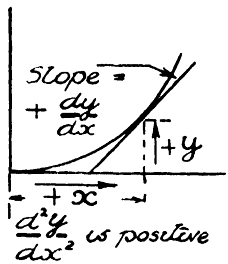


FIG. 102.

the right, will therefore be positive. The curvature shown is such that $\frac{dy}{dx}$ increases positively as x increases, therefore $\frac{d^2y}{dx^2}$ will be positive in this case. Also this type of bending will have been brought about by bending moments which are positive, according to the convention adopted. We must therefore associate positive B.M.s with $+\frac{d^2y}{dx^2}$. The application of the signs is illustrated in the selected examples given below.

Curvature is given in the Calculus by the expression $\frac{d^2y}{dx^2}$, and in elasticity by the expression $\frac{M}{EI}$. Equating these values, we get the relationship which forms the basis of the integrations leading to deflection values.

$$\frac{d^2y}{dx^2} = \frac{M}{EI}.$$

Case (i). Cantilever with Single End Load

In Fig. 103, B.M. at $x = -\text{W}(l - x)$.

$$-\frac{d^2y}{dx^2} = \frac{\text{W}(l - x)}{\text{EI}}.$$

$$-\text{EI}\frac{d^2y}{dx^2} = \text{W}(l - x).$$

Integrating, $-\text{EI}\frac{dy}{dx} = \text{W}lx - \frac{\text{W}x^2}{2} + \text{C}_1$ (a possible constant).

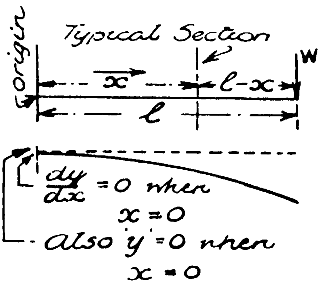


FIG 103.

To find the constant (if any), known values must be inserted. $\frac{dy}{dx}$ (i.e. the slope) $= 0$, when $x = 0$; therefore $\text{C}_1 = 0$.

$$\therefore\ -\text{EI}\frac{dy}{dx} = \text{W}lx - \frac{\text{W}x^2}{2}.$$

Integrating again,

$$-\text{EI}y = \frac{\text{W}lx^2}{2} - \frac{\text{W}x^3}{6} + \text{C}_2 \text{ (another possible constant).}$$

$y = 0$, when $x = 0$. $\therefore$ C_2, in this case, also $= 0$.

$$\therefore\ -\text{EI}y = \frac{\text{W}lx^2}{2} - \frac{\text{W}x^3}{6}.$$

This expression will give the deflection at any point in the cantilever, by inserting the proper value of x.

For maximum deflection put $x = l$.

$$-\text{EI}y = \frac{\text{W}l}{2} \times l^2 - \frac{\text{W}}{6} \times l^3$$

$$= \frac{\text{W}l^3}{3}.$$

$$y_{maximum} = -\frac{1}{3}\frac{\text{W}l^3}{\text{EI}}.$$

The minus sign indicates that the deflection is downwards.

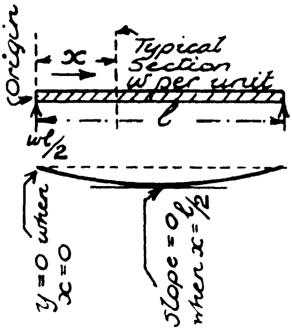


FIG. 104.

Case (ii). Simply Supported Beam with U.D. Load

In Fig. 104, B.M. at $x = \frac{wlx}{2} - \frac{wx^2}{2}$ (positive).

$$EI\frac{d^2y}{dx^2} = \frac{wlx}{2} - \frac{wx^2}{2}.$$

$$\therefore EI\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} + C_1.$$

$$\frac{dy}{dx} = 0 \text{ at mid-span, when } x = \frac{l}{2}.$$

$$\therefore EI \times 0 = \frac{wl}{4} \times \left(\frac{l}{2}\right)^2 - \frac{w}{6}\left(\frac{l}{2}\right)^3 + C_1.$$

$$\therefore C_1 = -\frac{wl^3}{24}.$$

$$\therefore EI\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^3}{24}.$$

Integrating again,

$$EIy = \frac{wlx^3}{12} - \frac{wx^4}{24} - \frac{wl^3x}{24} + C_2.$$

$$y = 0, \text{ when } x = 0. \quad \therefore C_2 = 0.$$

$$\therefore EIy = \frac{wlx^3}{12} - \frac{wx^4}{24} - \frac{wl^3x}{24}.$$

For maximum deflection, put $x = \frac{l}{2}$.

$$\text{EI}y = \left(\frac{wl}{12} \times \frac{l^3}{8}\right) - \left(\frac{w}{24} \times \frac{l^4}{16}\right) - \left(\frac{wl^3}{24} \times \frac{l}{2}\right)$$

$$= wl^4\left(\frac{1}{96} - \frac{1}{384} - \frac{1}{48}\right)$$

$$= -\frac{5}{384}wl^4.$$

$$y_{maximum} = -\frac{5}{384}\frac{\text{W}l^3}{\text{EI}}.$$

The remaining standard cases already considered by the *area-moment* method may be taken, by the reader, as exercises in the mathematical method.

Beams with Several Load Systems

Resolve the loading into simple systems, and deal with each system separately, by graphical or analytical methods. The net deflection at any given point will be the algebraic sum of the component deflections.

Exercises 8

(All beams are assumed to be simply supported at the ends. E is to be taken as 13,000 tons/in.^{2} in the case of steel beams.)

(1) A 12″ × 8″ × 65 lb. B.S.B. has I maximum = 487·77 ins.^{4}. Calculate the maximum deflection for a U.D. load of 27 tons, the span being 16′. Check the result by the tabular deflection coefficient for this beam section (·001538).

(2) A 9″ × 4″ × 21 lb. B.S.B. projects 4′ horizontally from its support, and carries at its end a concentrated load of 3 tons. Taking I maximum = 81 ins.^{4}, calculate:

(*a*) the maximum stress in the steel,

(*b*) the maximum deflection in the cantilever.

(3) Draw the B.M. diagram for the cantilever of question 2, and check the value of the maximum deflection by applying Mohr's theorem. Find also the maximum slope.

(4) In an experiment to determine Young's modulus for a given beam material, by a deflection experiment, it was found that a single central load of 450 lb. produced a deflection of

$\cdot 06''$. The span was $40''$, and I for the beam section 10 ins.^{4}. Find E from these results.

(5) What would be the maximum permissible span for a $14'' \times 6'' \times 46$ lb. B.S.B. if fully loaded and the L.C.C. regs. maximum allowance applied to its deflection? What maximum stress is assumed in this computation? Show that the maximum U.D. load, corresponding to the span obtained, is 12 tons. ($Z_{maximum}$ for the section $= 63\cdot 22$ ins.^{3}.)

(6) A steel beam of $20'$ span carries a single central load of 6 tons. $I_{maximum}$ for the beam section $= 300$ ins.^{4}

Determine the maximum deflection of the beam (*a*) by the standard formula, (*b*) by means of the secondary bending moment method.

(7) Obtain the maximum deflection for the beam of question 6, by graphical construction.

(Divide span into $2'$ bays. The areas of B.M. diagram will be, in order from the left, up to the mid-point of the beam: 6, 18, 30, 42 and 54 tons ft.^{2} respectively.)

(8) Find the maximum deflection for the compound girder given in Fig. 83, assuming an effective span of $20'$ and a total U.D. load (including the self-weight of the girder) of 85 tons.

(9) Treating the half-span of a simply supported beam as an inverted cantilever and using the expression $\frac{A}{EI}$, show that the maximum slope (i.e. the slope at the ends) of a simply supported beam is (*a*) $\frac{Wl^2}{16EI}$ for a single central load W, and (*b*) $\frac{Wl^2}{24EI}$ for a U.D. load W.

CHAPTER IX

SHEAR AND ITS APPLICATIONS

Relationship between Shear Force and Bending Moment in Beams

In this chapter will be explained some of the important applications of shear in structural calculations.

Fig. 105 shows a simply supported beam, carrying a load system of a general type. The B.M. at section 1 is assumed

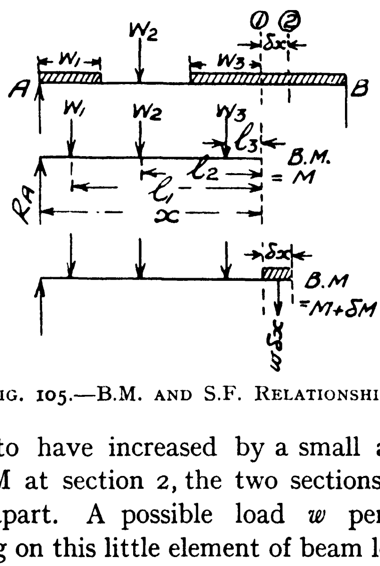


Fig. 105.—B.M. and S.F. Relationship.

to be M, and to have increased by a small amount δM to a value $M + \delta M$ at section 2, the two sections being a small distance δx apart. A possible load w per unit run is shown as acting on this little element of beam length.

$$
\begin{aligned}
M &= \text{Reaction moment} - \text{load moments} \\
&= R_A x - W_1 l_1 - W_2 l_2 - W_3 l_3. \\
M + \delta M &= R_A(x + \delta x) - W_1(l_1 + \delta x) - W_2(l_2 + \delta x) \\
&\quad - W_3(l_3 + \delta x) - \tfrac{1}{2}w\delta x^2.
\end{aligned}
$$

By subtraction and dividing by δx

$$\frac{\delta M}{\delta x} = R_A - W_1 - W_2 - W_3 - \tfrac{1}{2}w\delta x.$$

If δx be taken extremely small, $\frac{1}{2}w\delta x$ vanishes, so that, using the Calculus notation, we have:—

$\frac{dM}{dx} = R_A - W_1 - W_2 - W_3 = S$, where S represents the shear force at section 1.

$$\therefore S = \frac{dM}{dx}.$$

Integrating both sides, and changing over,

$$M = \int S dx.$$

In simple language these two important results may be expressed as follows:

(1) **The value of the shear force at any section of a beam is given by the slope of the B.M. diagram at that section.**

(2) **The difference in bending moment values, for any two given sections of a beam, equals the area of the shear force diagram between these two sections.**

The case of a simple beam has been taken to investigate the foregoing relationships, but a similar analysis, in the cases of the beam types referred to later in the book, will show that the laws enunciated are true for all types of beams.

Illustrative Example. As an overhanging beam involves both positive and negative bending moments, this type has been chosen to exemplify the relationships between B.M. and S.F. in beams.

In Fig. 106 the B.M. and S.F. diagrams have been constructed in the usual way.

Portion CA. From C to A, the slope of the B.M. diagram is uniform, negative, and equals $\frac{4 \text{ cwts. ft.}}{2 \text{ ft.}} = 2$ cwts.

Therefore, by law (1), the shear force in this portion is constant, negative, and equals 2 cwts.

Portion AD. Slope of B.M. diagram $= +\left(\frac{12 \cdot 8 + 4}{6}\right)$cwts. $= +\ 2 \cdot 8$ cwts., which equals the S.F. over this portion of the beam. The reader can now trace the remaining slopes, and show that they give the true S.F. values. We will now reverse the process, and find B.M. values from the S.F. diagram.

Portion CA. Area of S.F. diagram = − (2 cwts. × 2 ft.) = − 4 cwts. ft. By law (2), this must be the difference in B.M. values between the sections at C and A. But B.M. at C = 0. ∴ B.M. at A = − 4 cwts. ft.

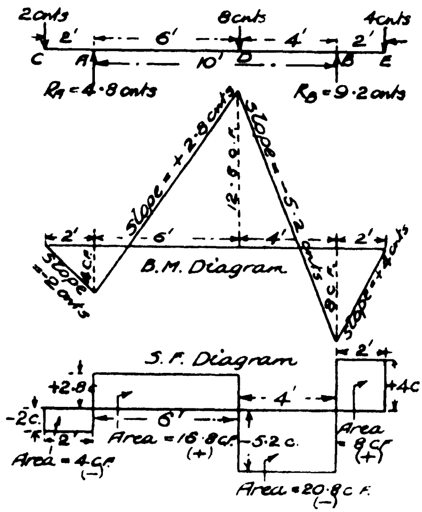


FIG. 106.

Portion CD. Total net area of S.F. diagram = [(− 2 × 2) + (2·8 × 6)] = + 12·8 cwts. ft., which gives the B.M. at D. The B.M. at B may similarly be found, and it can be verified easily that the net area of the S.F. diagram from C to E equals zero, giving B.M. at E = zero.

Applications of S.F.–B.M. Relationships

The importance of the laws referred to does not lie in the mere deduction of B.M. from S.F., or vice versa. One important application gives a method of determining the **position of maximum B.M. in a beam.**

The slope of the B.M. diagram (shown in Fig. 107) is zero at the point where the B.M. reaches its maximum value. If the diagram were made up of straight lines, the slope would suddenly change from a positive to a negative value, at the section corresponding to maximum B.M. But the slope of the B.M.

diagram is given by the S.F. We have therefore the following very important rules:

(1) **The B.M. will be a maximum at the beam section at which the S.F. diagram crosses its base, i.e. passes through a zero value.**

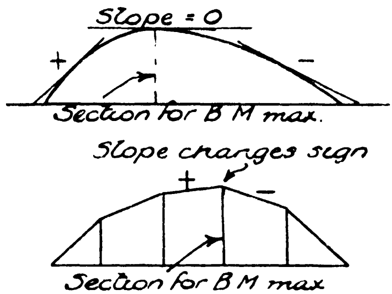


FIG. 107.

(2) **If we proceed across the beam from the left end, just sufficiently far enough to take up load of equal value to the left end support reaction, the point arrived at will be that of maximum B.M.**

EXAMPLE. *Find the position, and value, of maximum B.M. for the example given in Fig.* 108.

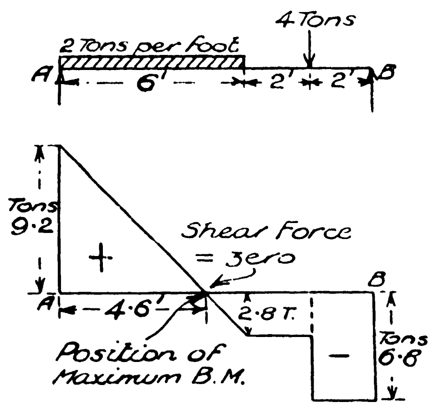


FIG. 108.

$$R_A \times 10 = [(2 \times 6) \times 7] + [4 \times 2]$$
$$10R_A = 84 + 8 = 92$$
$$R_A = 9{\cdot}2 \text{ tons.}$$

To make up 9·2 tons, we must proceed 4·6 ′ across the beam from A. B.M. maximum is therefore at 4·6 ′ from A.

$$\text{B.M. maximum} = (9{\cdot}2 \times 4{\cdot}6) - \left(9{\cdot}2 \times \frac{4{\cdot}6}{2}\right) \text{ tons ft.}$$
$$= 21{\cdot}16 \text{ tons ft.}$$

If, on arriving at a concentrated load by this method, it is found that its inclusion makes the load total too great, and that its exclusion gives too small a value, the maximum B.M. will actually occur at the load point. Fig. 106 shows an example of a S.F. diagram cutting its base line three times, each position corresponding to a *local* B.M. maximum value.

Rules for Constructing S.F. Diagrams

The following rules will be found helpful in constructing S.F. diagrams for simply supported beams :

(i) Where there is no load on the beam, the diagram will be horizontal.

(ii) A vertical load will cause a corresponding vertical jump in the diagram—vertically upwards for a reaction, and vertically downwards for a concentrated load.

(iii) The diagram will slope uniformly for U.D. load. Throughout the diagram, the slope will remain constant for constant load per unit run. If the rate of load per unit run increase, the slope will correspondingly increase. The slope is always downwards towards the right.

These rules will be found to apply in the cases of ‘ fixed ’ beams and ‘ continuous ’ beams respectively (see Chapter X).

EXAMPLE. *Draw the S.F. diagram for the case given in Fig.* 109.

$$R_A \times 20 = (20 \times 19) + (6 \times 12) + (24 \times 7).$$
$$20\,R_A = 380 + 72 + 168$$
$$= 620.$$
$$R_A = 31 \text{ cwts.}$$
$$R_B = 19 \text{ cwts.}$$

From the left end of the beam up to the point A, the slope of the S.F. diagram will be uniform, and the total drop will be (2 cwts. per foot × 4 feet) = 8 cwts. The diagram will then jump vertically upwards a distance equivalent to 31 cwts.,

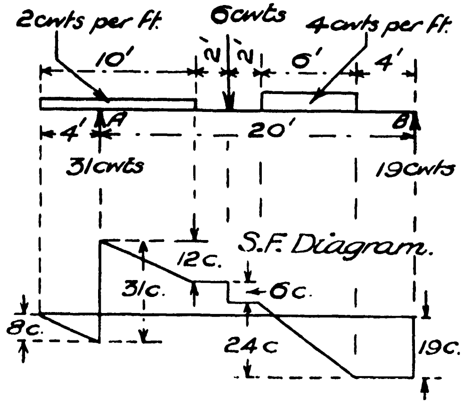


FIG. 109.

owing to the vertical reaction at A. The remainder of the diagram may be similarly followed through.

Complementary Shear Stress

ABCD (Fig. 110) is a very small square block of metal, forming part of the web of a beam, the sides AB and CD being vertical. Assuming the block to be situated near the left end of the beam, the sides AB and CD will be subjected to the type of shear stress indicated in the figure.

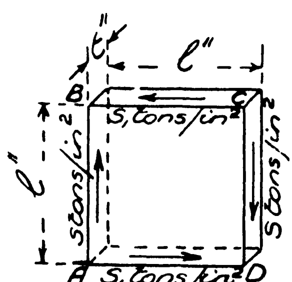


FIG. 110.

The total load carried by each face $= s$ (tons/in.^{2}) $\times$ $(l \times t)$ sq. ins. $= slt$ tons. These two forces constitute a couple tending to rotate the block in a clockwise manner with a moment of magnitude slt tons $\times$ l ins. $= sl^2t$ tons ins. It is clear that an equal and opposite couple must be acting on

the block, to maintain equilibrium. The forces in this balancing couple are brought about by the stress induced in the fibres of metal along the horizontal faces BC and AD of the block. If s_1 tons/in.^{2} be this stress value, the corresponding couple will have a moment

$$s_1lt \text{ tons} \times l \text{ ins.} = s_1l^2t \text{ tons/ins.}$$
$$\therefore\ s_1l^2t = sl^2t \text{ for equilibrium,}$$
$$\text{i.e. } s_1 = s.$$

A vertical shear stress of a certain value at a given point in the ~~web is~~, therefore, accompanied by a horizontal shear stress of equal ~~intensity, at the point~~.

Further investigation shows that other complementary stresses are involved. Fig. 111(a) indicates how the shear

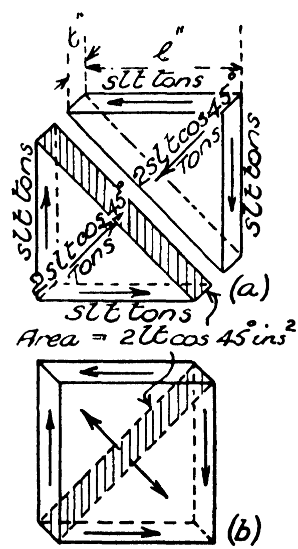


FIG. 111.—COMPLEMENTARY STRESSES.

stresses, already referred to, result in a *compressive stress* in the material Resolving the shear loads at right angles to the internal diagonal plane of the block, and dividing by the area of the plane, to obtain the intensity of stress, we get

$$\text{Compressive stress} = \frac{\text{Load}}{\text{Area}} = \frac{2slt \cos 45^\circ \text{ tons}}{2lt \cos 45^\circ \text{ sq. ins.}}$$
$$= s \text{ tons/in.}^2.$$

Similarly, the *tensile stress* induced = s tons/in.² (Fig. 111(b)). The original vertical shear stress is thus accompanied by both compressive and tensile stresses of equal intensity to its own—across planes at 45° to the horizontal. The theory would, of course, hold for the case of any stress units and may be extended to include any elastic material subjected to shear stress, not necessarily forming part of a beam web.

Variation of Shear Stress in a Beam Web

The result obtained by dividing the shear load (as obtained from the shear force diagram) by the area of the web, gives, for any given beam section, the average shear ~~stress in the web~~. The shear stress is not in fact a constant value for all points in the web depth, and the nature of the variation will now be discussed.

In Fig. 112, AB and CD are two vertical sections of a beam, assumed to be very close together. The bending mo-

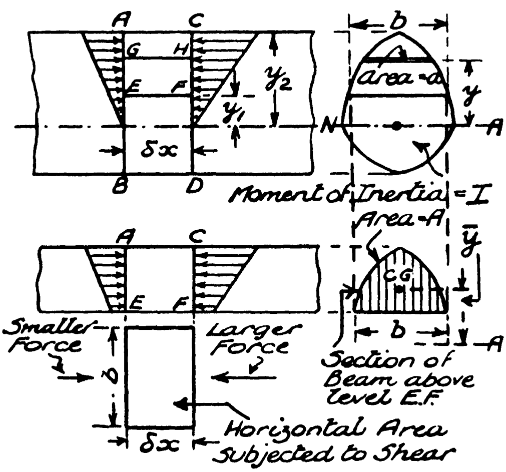


FIG. 112.—HORIZONTAL SHEAR STRESS.

ment at AB is M and that at CD is M + δM. EF is a horizontal layer of material lying between the two vertical sections, its distance from the neutral layer of the beam being y_1. Consider the forces acting on the portion of beam from EF to the top (shown in the lower diagrams of Fig. 112).

Owing to the difference of B.M. at sections AB and CD, the end faces of this little piece of beam will be subjected to different stress intensities—and therefore to different resultant thrusts. The difference of these two end thrusts represents a force tending to slide the portion of beam over its base at EF. We can obtain the corresponding horizontal shear stress by dividing this force by the horizontal area of the base.

GH represents a typical layer of beam situated between level EF and the top of the beam. At the level of GH in section AB the compressive stress will be $\frac{My}{I}$ $\left(\text{obtained from the standard formula } M = \frac{fI}{y}\right)$. The load on the corresponding elemental strip of cross-section of area a—at AB—will therefore be $\frac{Mya}{I}$.

The compressive stress at the level GH for section CD will be $\frac{(M + \delta M)y}{I}$, and the load on the corresponding cross-sectional strip $= \frac{(M + \delta M)ya}{I}$. For this one little strip the difference of the end loads will therefore be

$$\frac{(M + \delta M)ya}{I} - \frac{Mya}{I} = \frac{\delta Mya}{I}.$$

To obtain the total difference of thrusts referred to, we must add up all these little differences—from level EF to the top of the beam, i.e. from $y = y_1$ to $y = y_2$.

$$\text{Net resultant thrust} = \Sigma \frac{\delta May}{I}$$

$$= \frac{\delta M}{I} \times \Sigma ay \text{ (between the stated levels).}$$

But $\Sigma ay = A\bar{y}$ where $A =$ the total area of beam section above EF and $\bar{y}$ is its c.g. distance from the *neutral axis* of the section.

$$\therefore \text{Net resultant thrust} = \frac{\delta M}{I} A\bar{y}.$$

$$\text{Shear stress at level EF} = \frac{\text{Shear load}}{\text{Area}} = \frac{\delta M}{\delta x \times b} \times \frac{A\bar{y}}{I}.$$

As we take δx smaller and smaller, the value of $\frac{\delta M}{\delta x}$ becomes $\frac{dM}{dx}$, i.e. S, the shear force at the section of beam under consideration.

If s = the intensity of horizontal (or vertical) shear stress at the level EF,

$$s = \frac{S}{bI} A\bar{y}.$$

Application to a Rectangular Beam Section.—For the shear stress at the section EF (Fig. 113) we have

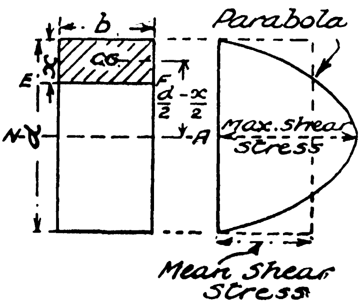


FIG. 113.—SHEAR STRESS IN RECTANGULAR BEAM.

$$\begin{aligned} b \text{ (in formula)} &= b \\ I \quad ,, \quad &= \frac{bd^3}{12} \\ A \quad ,, \quad &= bx \\ \bar{y} \quad ,, \quad &= \left(\frac{d}{2} - \frac{x}{2}\right) \end{aligned}$$

$$s = \frac{S}{bI} A\bar{y}$$

$$\therefore\ s = \frac{S}{b \times \frac{bd^3}{12}} \times bx \times \left(\frac{d}{2} - \frac{x}{2}\right)$$

$$= \frac{12S}{bd^3} \times \frac{x(d-x)}{2} = \frac{6S}{bd^3} \times x(d-x).$$

If we plotted a diagram showing the shear variation for different values of x, the graph would be parabolic.

When $x = 0$, $s = 0$.

$$,, \quad x = \frac{d}{2}, \; s = \frac{6S}{bd^3} \times \frac{d}{2} \times \frac{d}{2} = \frac{3S}{2bd}.$$

But the average shear stress would be $\frac{\text{Load}}{\text{Area}} = \frac{S}{bd}$, so that our result shows that **the maximum shear stress, in the case of a rectangular beam section, is $1\frac{1}{2}$ times the mean value.**

EXAMPLE. *A rectangular steel beam (Fig. 114) is used to carry a total U.D. load of 6 tons, the section being 6" deep × 3"*

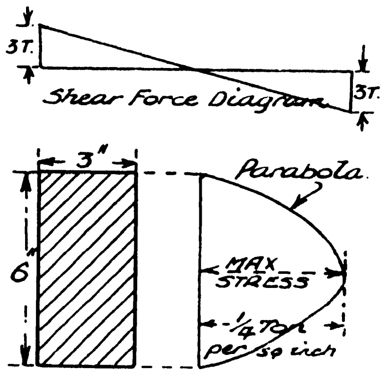


FIG. 114.

wide, and the span 12'. Draw a diagram showing the variation of shear (horizontal and vertical) stress down the beam section, and write down the maximum shear stress in the steel.

$$\text{Maximum shear load} = \frac{W}{2} = 3 \text{ tons.}$$

$$\text{Maximum shear stress} = \frac{3}{2} \times \text{mean stress.}$$

$$\text{Mean stress} = \frac{\text{Load}}{\text{Area}} = \frac{3}{3 \times 6} \text{ tons/in.}^2 = \frac{1}{6} \text{ tons/in.}^2$$

$$\therefore \text{ Maximum shear stress} = \frac{3}{2} \times \frac{1}{6} \text{ tons/in.}^2 = \frac{1}{4} \text{ tons/in.}^2$$

The maximum tensile and compressive bending stresses in this example will be found to be 6 tons/in.^{2}. The steel is only required to develop a maximum shear stress value of $\frac{1}{4}$ tons/in.^{2}, whereas 5 tons/in.^{2} is permissible in the usual case. As we have seen, the B.S.B. form of beam section is more economical from

the point of view of flexural stress, and the heavier loads carried will cause the maximum shear stress in the thin web to approach nearer its safe value, so that it requires to be checked in design.

Distribution of Shear Stress in a B.S.B. Type of Section

EXAMPLE. *Illustrate the shear stress distribution at the given*

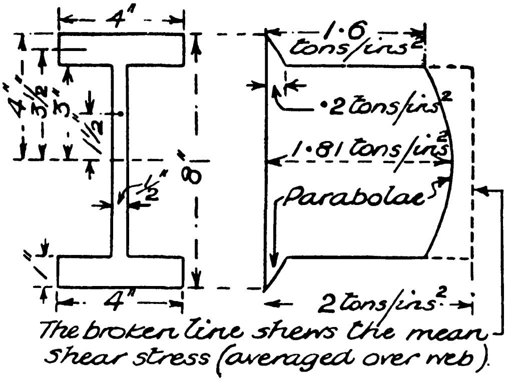


FIG. 115.—SHEAR STRESS DISTRIBUTION.

beam section (Fig. 115), *for a shear load of* 6 *tons. Compare the maximum shear stress with the mean value, as usually computed.*

$$\text{I for section} = \frac{\mathrm{BD}^3}{12} - \frac{bd^3}{12}$$

$$= \frac{4 \times 8^3}{12} - \frac{3{\cdot}5 \times 6^3}{12} = 107{\cdot}66 \text{ ins.}^4$$

Shear stress values :

(i) *At level of flange and web junction.*

(*a*) Just inside flange.

$$s = \frac{\mathrm{S}}{b\mathrm{I}} \mathrm{A}\bar{y}.$$

$\mathrm{S} = 6$ tons ; $b = 4''$; $\mathrm{I} = 107{\cdot}66$ ins.^{4} ; $\mathrm{A} = 4$ ins.^{2} ; $\bar{y} = 3\tfrac{1}{2}''$.

$$s = \left(\frac{6}{4 \times 107{\cdot}66} \times 4 \times 3{\cdot}5\right) \text{tons/in.}^2 = {\cdot}2 \text{ tons/in.}^2$$

(*b*) Just inside web.

$$s = \left(\frac{6}{{\cdot}5 \times 107{\cdot}66} \times 4 \times 3{\cdot}5\right) \text{tons/in.}^2 = 1{\cdot}6 \text{ tons/in.}^2$$

(ii) *At neutral axis of beam.*

$$b = \tfrac{1}{2}''.$$

$$A\bar{y} = (4 \times 3\tfrac{1}{2}) + [(3 \times \tfrac{1}{2}) \times 1\tfrac{1}{2}] = 16{\cdot}25 \text{ ins.}^3$$

$$s = \frac{S}{bI} A\bar{y}$$

$$= \left(\frac{6}{\tfrac{1}{2} \times 107{\cdot}66} \times 16{\cdot}25\right) \text{tons/in.}^2$$

$$= 1{\cdot}81 \text{ tons/in.}^2$$

Maximum shear stress $= 1{\cdot}81$ tons/in.²

$$\text{Mean shear stress} = \frac{\text{Shear Load}}{\text{Area of web}} = \frac{6 \text{ tons}}{6'' \times \tfrac{1}{2}''}$$

$= 2$ tons/in.² or $= 1{\cdot}5$ tons/in.² if the web is taken as 8″ deep.

The maximum and mean values approximately agree. In most cases of standard sections, it will be sufficiently accurate to compute the shear stress by averaging over the web sectional area. It will be remembered that the shear stress, *thus calculated,* may be 5 tons/in.², provided the tendency to side buckling of the web is safeguarded. The full beam depth is usually taken, in practice, in calculating web area for shear calculations.

Shear Strain

The method of measurement of shear strain has been referred to in Chapter I. The proportional law of elasticity applies to shear stresses and shear strains. The modulus of elasticity—corresponding to Young's modulus for tension and compression—is termed the ' Shear Modulus.' It is sometimes referred to as the ' Modulus of Rigidity.' In the case of mild steel the value of the shear modulus is about 6000 tons/in.². Shear strains in beams are very small, and may be neglected in calculations involving deflection.

Application of Theory of Shear to Built-up Beams

Stiffening of Plate Girder Webs.—Fig. 116 shows a portion of a plate girder. The shear force in this case is assumed to be negative, so that the compressive stresses in the web act at 45°, across the plane of compression indicated. If the web stiffeners shown are placed close enough together to both cut a plane of

compression, i.e. if they are not farther apart than the depth *d* in diagram, the lengths of inclined strips of web which have to act as virtual columns will be lessened. As will be seen later, in Chapter XI, the length of a column is a vital factor in its strength, so that the employment of stiffeners greatly

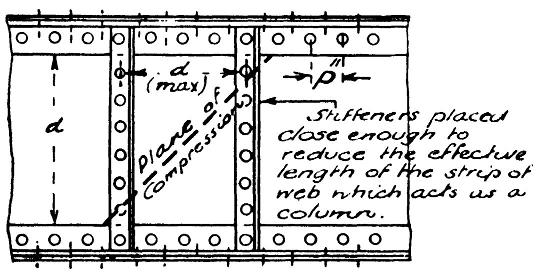


FIG. 116.—BUCKLING IN PLATE GIRDER WEB.

increases the resistance of the web to ' buckling ' or failure as a column. Various formulæ, of the column type, are in use, relating the spacing of stiffeners to web thickness. Practically, the spacing is a combination of theoretical principles and constructional requirements (see Chapter XV).

Riveting in Compound and Plate Girders.—The detail of the riveting of the flange angles to the web, and to the flange plates, is concerned with the horizontal shear which accompanies the vertical shear. The shear load per foot length of girder flange has to be resisted by the rivet strength provided per foot of length.

Consider the rivets connecting the flange angles to the web in Fig. 116. Expressing the equality of horizontal and vertical shear stress for the faces of a rectangular block of web, one foot deep, one foot long and *t* feet thick, we have (assuming S_V and S_H to be the shear loads) :

$$\frac{S_V}{1 \times t} = \frac{S_H}{1 \times t} \text{ or } S_V = S_H,$$

i.e. the *horizontal shear load per foot length of girder = the vertical shear load per foot of depth.* This method of investigation is necessarily approximate, as variation of web stress has been neglected, but a closer analysis, based on the stress variation

theory, leads to the same result for the position in which the rivet line, or lines, are fixed.*

Let V tons = value of one rivet = maximum shear load per pitch length.

Let p ins. = pitch of rivets (single riveting as in figure).

Let D ins. = depth of web (sometimes taken as depth between rivet lines).

Let S tons = shear force at the portion of the girder where the riveting is being considered.

Applying the above result we get

$$\frac{V}{p} = \frac{S}{D} \text{ or } p = \frac{VD}{S}.$$

In double riveting, as in $5'' \times 5''$ or $6'' \times 6''$ angles, V = value of two rivets, and 'p' represents the straight line pitch.

EXAMPLE. *A plate girder of depth (web) 3′ carries a total U.D. load of 80 tons. The web is $\frac{3}{8}''$ thick and $\frac{3}{4}''$ diameter rivets are used. Find the maximum permissible rivet pitch at the girder ends.*

The value of one rivet in this case $= dtf_b = (\frac{3}{4} \times \frac{3}{8} \times 12)$ tons = 3·37 tons.

Method 1. S.F. maximum $= \frac{W}{2} = \frac{80 \text{ tons}}{2} = 40$ tons.

$\therefore$ Shear load per foot of depth $= \frac{40}{3}$ tons.

$\therefore$ Shear load per foot length of girder $= \frac{40}{3}$ tons.

Number of rivets required per foot $= \frac{40}{3} \div 3{\cdot}37 = 4$, i.e. the maximum pitch = 3″.

Method 2. $p = \frac{VD}{S} = \frac{3{\cdot}37 \times 36''}{40} = 3''$.

The rivet pitch may be changed to 4″, if desirable, at the section of the beam where the shear force has fallen to a value $S = \frac{VD}{4 \text{ ins.}}$, i.e. $\frac{3{\cdot}37 \times 36}{4}$ tons = 30 tons.

This position may be fixed by means of the S.F. diagram, or by calculation.

* See *Structural Engineering*, by Husband and Harby.

The rivet pitch in the flange plates will normally correspond with that in the 'angle to web' connection.

Shear Reinforcement in R.C. Beams.—Fig. 117 illustrates how the induced tensile and compressive forces are resisted in

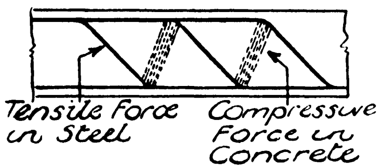


FIG. 117.—SHEAR REINFORCEMENT IN R.C. BEAM.

the case of a reinforced concrete beam. The weakness of concrete in tension necessitates steel reinforcement. As the B.M. falls away, some of the main reinforcing bars may be dispensed with, as far as their employment in providing moment of resistance is concerned. These are turned up to provide the necessary resistance in tension. In the diagram the inclined steel cuts the tension planes (as in Fig. 111(*b*)) at right angles, thus taking up the stress brought about by the positive vertical shear force in the beam.

EXERCISES 9

(1) Draw the shear force diagram for the beam given in Fig. 118. Show that the area (in cwts. ft. units) above the base line equals that below. Why is this ?

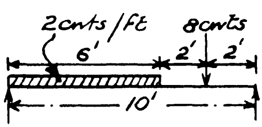


FIG. 118.

(2) The maximum B.M. for the beam given in Fig. 119 occurs at 9′ from the left end. Verify this by drawing a shear force diagram, and calculate the value of B.M. maximum.

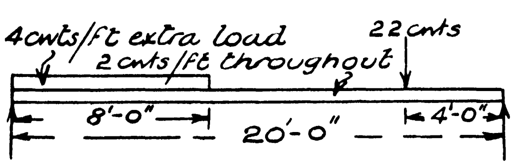


FIG. 119.

(3) Deduce the load system which will result in the B.M. diagram given in Fig. 120. (Construct the S.F. diagram by the

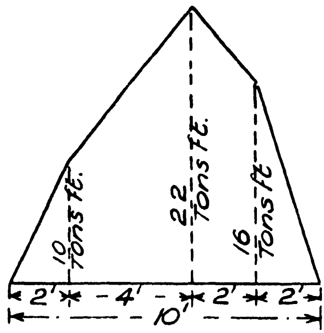


FIG. 120.

slope of B.M. diagram method and determine the loads from the vertical jumps in this diagram.)

(4) Obtain (without drawing a S.F. diagram) the position, and value, of the maximum B.M. for a beam of 12′ span, which carries two U.D. load systems, viz. 2 tons per foot for the whole span and 4 tons per foot in addition for the first 3′ of span, measured from the left end.

(5) A rectangular beam section, 2″ wide × 6″ deep, is subjected to a vertical shear load of 24 tons. Calculate the intensity of shear stress at a level 2″ below the upper surface. Obtain the value of maximum shear stress, and construct a diagram showing the variation of shear stress down the section.

(6) A steel joist section has the following dimensions: flange width = 8″, flange thickness = 1″, overall depth = 16″, and web thickness = $\frac{1}{2}$″. Construct a diagram showing the shear stress distribution across the section, for a vertical shear load of 21 tons.

(7) A plate girder carries a total U.D. load of 240 tons. Taking the particulars given, determine a suitable rivet pitch for the girder near the supports.

Depth of web = 48″.
Thickness of web = $\frac{1}{2}$″.
Rivet diameter = $\frac{7}{8}$″ (f_b = 12 tons/in.^{2}).

The flange angles are 6″ × 6″ × $\frac{5}{8}$″, requiring two lines of rivets.

CHAPTER X

FIXED AND CONTINUOUS BEAMS

Fixed Beams

THE bending and shear effects produced in a beam by a given load system depend upon the way in which the ends of the beam are held in position. In Fig. 121(*a*), the ends are subjected to no end restraint, and the beam is said to be *simply supported.*

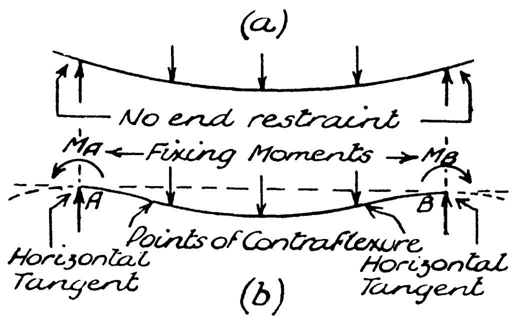


FIG. 121.

In Fig. 121(*b*), the ends are constrained to bend *until the tangent to the beam at each support is horizontal.* Such constraint represents *perfect fixture,* and the beam is termed a *fixed beam.* It is important to note that a beam is only partially fixed if it does not fulfil the qualification of zero slope at the supports. Most practical beams belong to the partially fixed class and, for design purposes, are treated as simply supported. A beam rigidly fixed at its ends by top, web and base cleats (or by welding) to a stanchion transmits a moment to the stanchion which the latter must be capable of resisting. The nature of the bending moments set up in a fixed beam will now be considered.

Relationship between Fixed and Overhanging Beams.—The negative support moments necessary to bring about the con-

dition of fixed ends may be produced by overhanging the ends of the beam, as shown in Fig. 122. In (*a*) we have the ordinary B.M. diagram, as for free ends. In (*b*) is shown the B.M.

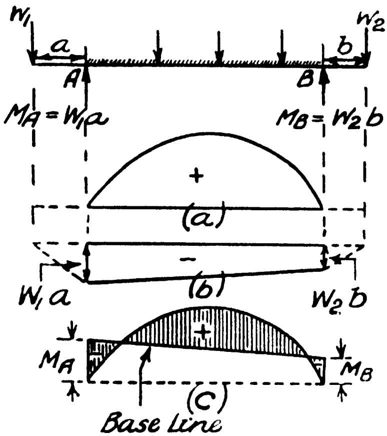


FIG. 122.—B.M. DIAGRAM FOR FIXED BEAM.

diagram (negative) for loads W_1 and W_2—the loads introduced to create the required 'end-fixing' moments. The positive and negative diagrams are superimposed in (*c*), the final net B.M. diagram being shown hatched. It should be noted that, in all such diagrams with a sloping base line, the *B.M. values are obtained by scaling vertically*—and not at right angles to the base. **The B.M. diagram for a fixed beam is thus the summation of two diagrams : (i) the ordinary B.M. diagram as for ends free (a positive diagram), and (ii) a negative B.M. diagram in the form of a trapezium.** The problem in any given case is, therefore, to obtain the dimensions of the *negative fixing trapezium* in order to superimpose it on the *free-end* B.M. diagram which is first drawn by the usual methods.

Properties of the Fixing Trapezium.—The negative fixing trapezium referred to above has two important relationships with the 'free-end' B.M. diagram. These are determined by an application of Mohr's theorem for the deflection of a cantilever. It is assumed in Fig. 123 that the ends A and B are at the same level, and that the beam is of constant section. We may regard the beam as being a cantilever loaded with a positive

and with a negative B.M. diagram, in such a way that, with respect to the support end A, the deflection at the end B is zero.

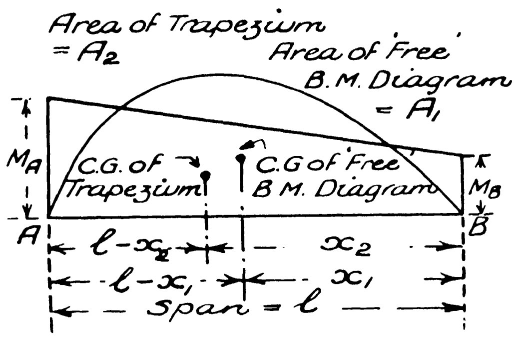


FIG. 123.

Similarly, the support end may be assumed to be at B, and the deflection at A taken as zero. Using the symbols given in Fig. 123, and expressing Mohr's theorem for the case of the support at A, we get :

$$\frac{A_1 x_1}{EI} - \frac{A_2 x_2}{EI} = 0.$$

$$\therefore \; A_1 x_1 = A_2 x_2.$$

For the support at B, and zero deflection at A, the theorem gives :

$$\frac{A_1(l - x_1)}{EI} - \frac{A_2(l - x_2)}{EI} = 0.$$

$$\therefore \; A_1(l - x_1) = A_2(l - x_2),$$

$$\text{i.e.} \quad A_1 l - A_1 x_1 = A_2 l - A_2 x_2.$$

$$\text{But as } A_1 x_1 = A_2 x_2, \quad A_1 l = A_2 l,$$

$$\text{i.e. } A_1 = A_2$$

$$\text{and } x_1 = x_2.$$

The following relationships are thus established :

(i) **The area of the fixing trapezium is equal to that of the 'free' B.M. diagram.**

(ii) **The centres of gravity of the two diagrams lie in the same vertical line, i.e. are equidistant from a given end of the beam.**

Fixed Beam with Central Concentrated Load

The fixing trapezium is in this case a rectangle, as its centre of gravity has to be at mid-span. It must clearly also have half the height of the triangular free B.M. diagram in order to be of equal area. The net diagram for this case is, therefore, as shown hatched in Fig. 124. The maximum B.M. is halved by

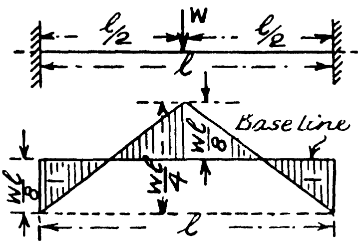


FIG. 124.—FIXED BEAM WITH SINGLE CONCENTRATED CENTRAL LOAD

fixing the ends of the beam, but the supports are required to resist a B.M. of $\frac{Wl}{8}$, which is equal to that at the centre of the fixed beam.

Fixed Beam with Uniformly Distributed Load

The fixing trapezium will again be a rectangle (Fig. 125). The area of the parabolic free B.M. diagram $= \frac{2}{3}$ base $\times$ height, so that the height of the rectangle for equal area must

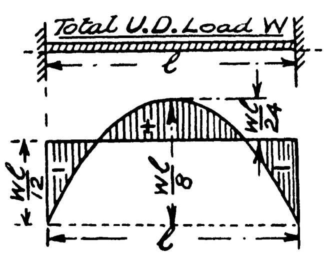


FIG. 125.—FIXED BEAM WITH U.D. LOAD.

be $\frac{2}{3} \times$ height of parabola, $= \frac{2}{3} \times \frac{Wl}{8} = \frac{Wl}{12}$. In this case it will be seen that the maximum B.M. is not at the centre of the beam, but at the supports. This illustrates the im-

portance of the consideration of the bending moments transmitted by such beams to members with which they are connected.

Fixed Beam with a Symmetrical Load System

In all such cases the fixing trapezium will clearly be a rectangle, as both c.g.s will be at mid-span. The height of the rectangle is found by equating its area to that of the free B.M. diagram.

EXAMPLE. *Construct the B.M. diagram for the fixed beam given in Fig.* 126.

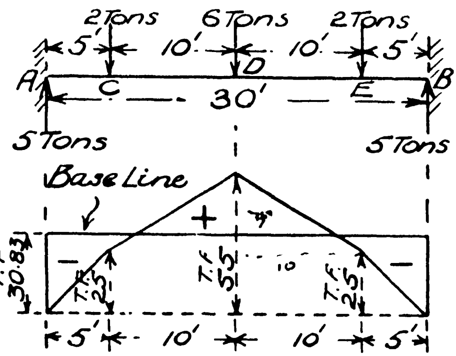


Fig. 126

Free B.M. diagram :

$$R_A = R_B = \frac{10 \text{ tons}}{2} = 5 \text{ tons}.$$

$$BM_C = (5 \times 5) \text{ tons ft.} = 25 \text{ tons ft.}$$

$$BM_D = [(5 \times 15) - (2 \times 10)] \text{ tons ft.} = 55 \text{ tons ft.}$$

$$\text{Area of diagram} = 2\left(\frac{5 \times 25}{2}\right) + 2\left(\frac{25 + 55}{2} \times 10\right) \text{ tons ft.}^2$$

$$= (125 + 800) \text{ tons ft.}^2$$

$$= 925 \text{ tons ft.}^2.$$

Fixing trapezium :

The height of the rectangle will be 925 tons ft.^{2} $\div$ 30 ft.

$$= 30{\cdot}83 \text{ tons ft.}$$

The top of the rectangle forms the base line of the 'fixed beam' diagram.

Shear Force Diagrams for Fixed Beams

In Chapter IX it was explained that the shear force at any section of a beam is given by the slope of the B.M. diagram at the given section. The slope at any section will remain unaltered, if the base of the B.M. diagram remain horizontal. In all cases of symmetrical loading, therefore, the shear force values will be identical for both 'free' and 'fixed ended' beams, and the shear force diagrams will be the same for both cases.

If M_A, the fixing moment at A, is not equal to M_B, the fixing moment at B, the base line for the fixed beam B.M. diagram will be inclined, so that all slopes will be altered—but by the same amount, i.e. by the increase in slope. The *positive* increase in slope $= \dfrac{M_A - M_B}{l}$, so that it will be necessary to *lower* the base line of the original 'free' S.F. diagram by this amount, for the fixed end condition. If the value is negative, i.e. if M_B is greater than M_A, then the base line must be *raised*.

Fixed Beam with a Single Non-central Concentrated Load

This case lends itself to a simple solution derived from the properties of a trapezium.

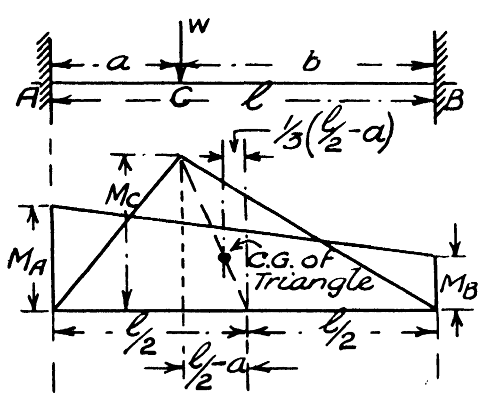


FIG. 127.

In Fig. 127 M_C = the free B.M. at C

$$= \frac{Wab}{l}.$$

Equating the areas of the triangle (representing the 'free' diagram) and the 'fixing trapezium'

$$\frac{M_C \times l}{2} = \frac{M_A + M_B}{2} \times l,$$

$$\text{i.e. } M_C = M_A + M_B.$$

$$\text{Distance of c.g. of triangle from B} = \frac{l}{2} + \frac{1}{3}\left(\frac{l}{2} - a\right)$$

$$= \frac{l}{2} + \frac{l}{6} - \frac{a}{3} = \frac{2l - a}{3}.$$

Distance of c.g. of trapezium from B (using the standard expression for the c.g. position in a trapezium)

$$= \frac{2M_A + M_B}{3(M_A + M_B)} \times l.$$

Equating these distances,

$$\frac{2M_A + M_B}{3(M_A + M_B)} \times l = \frac{2l - a}{3}.$$

Let $M_A = x\ M_B$.

$$\therefore \frac{(2x\ M_B + M_B)l}{3(xM_B + M_B)} = \frac{2l - a}{3}.$$

$$\therefore \frac{(2x + 1)l}{3(x + 1)} = \frac{2l - a}{3}, \text{ i.e. } 6xl + 3l = 3(x + 1)(2l - a).$$

$$6xl + 3l = 6lx - 3ax + 6l - 3a.$$

$$3ax + 3a = 3l = 3a + 3b.$$

$$\therefore ax = b, \text{ i.e. } x = \frac{b}{a}.$$

$$\therefore \frac{M_A}{M_B} = \frac{b}{a}.$$

$$\therefore \frac{M_A}{M_A + M_B} = \frac{b}{a + b} = \frac{b}{l}.$$

$$\text{But } M_A + M_B = M_C.$$

$$\therefore M_A = \frac{b}{l} \times M_C.$$

$$\text{and } M_B = \frac{a}{l} \times M_C.$$

M_A and M_B having been obtained, the base line for the fixed beam can be constructed. The final diagrams are as

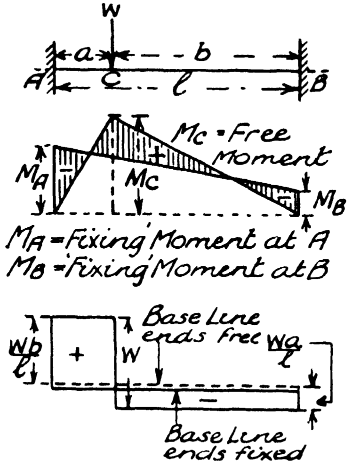


FIG. 128.—FIXED BEAM WITH SINGLE NON-CENTRAL LOAD.

shown in Fig. 128. The base line of the S.F. diagram is *lowered*, the broken line representing the base for ' ends free.'

EXAMPLE. *Construct the B.M. and S.F. diagrams for the fixed beam given in Fig.* 129.

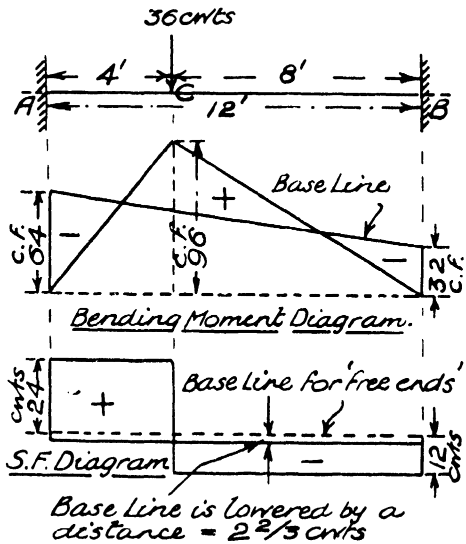


FIG. 129.

$$M_C \text{ (free B.M. at C)} = \frac{36 \times 8 \times 4}{12} \text{ cwts. ft.} = 96 \text{ cwts. ft.}$$

$$M_A = \frac{b}{l} \times M_C = \left(\frac{8}{12} \times 96\right) \text{ cwts. ft.} = 64 \text{ cwts. ft.}$$

$$M_B = \frac{a}{l} \times M_C = \left(\frac{4}{12} \times 96\right) \text{ cwts. ft.} = 32 \text{ cwts. ft.}$$

The diagram is constructed as shown.

$$R_A \text{ (for free ends)} = \frac{36 \times 8}{12} \text{ cwts.} = 24 \text{ cwts.}$$

$$R_B \text{ (for free ends)} = \frac{36 \times 4}{12} \text{ cwts.} = 12 \text{ cwts.}$$

The base line for the S.F. diagram (free ends) is shown by the broken line. For fixed end condition this must be *lowered* by $\frac{M_A - M_B}{l}$

$$= \frac{64 - 32}{12} \text{ cwts.} = \frac{32}{12} \text{ cwts.} = 2\tfrac{2}{3} \text{ cwts.}$$

Fixed Beam with Several Concentrated Loads

The B.M. diagram for free ends is first drawn. The fixing moment at the left end, corresponding to each load taken

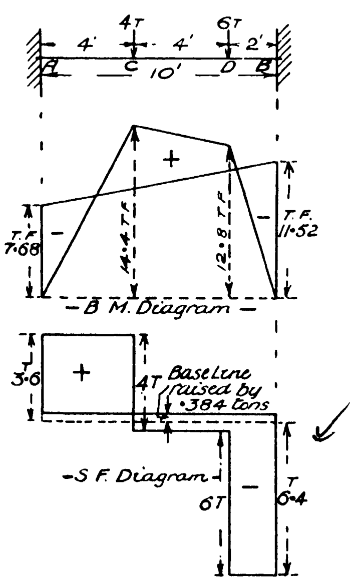


FIG. 130.

separately, is computed and the total fixing moment obtained by addition. Similarly the total fixing moment at the right end is determined.

EXAMPLE. *Fig.* 130 *shows a fixed beam carrying two concentrated loads. Construct the B.M. and S.F. diagrams for the beam.*

Free B.M. diagram:

$$R_A \times 10 = (6 \times 2) + (4 \times 6) = 36.$$
$$R_A = 3{\cdot}6 \text{ tons}, \ R_B = 6{\cdot}4 \text{ tons}.$$
$$M_C = (R_A \times 4) \text{ tons ft.} = 14{\cdot}4 \text{ tons ft.}$$
$$M_D = (R_B \times 2) \text{ tons ft.} = 12{\cdot}8 \text{ tons ft.}$$

Note: M_C and M_D are 'free' moments throughout.

$$M_C \text{ (due to 4 tons load alone)} = \frac{4 \times 6 \times 4}{10} \text{ tons ft.}$$
$$= 9{\cdot}6 \text{ tons ft.}$$
$$\therefore M_A \text{ (due to 4 tons load alone)} = \frac{9{\cdot}6 \times 6}{10} \text{ tons ft.}$$
$$= 5{\cdot}76 \text{ tons ft.}$$
$$\therefore M_B \text{ (due to 4 tons load alone)} = \frac{9{\cdot}6 \times 4}{10} \text{ tons ft.}$$
$$= 3{\cdot}84 \text{ tons ft.}$$
$$M_D \text{ (due to 6 tons load alone)} = \frac{6 \times 8 \times 2}{10} \text{ tons ft.}$$
$$= 9{\cdot}6 \text{ tons ft.}$$
$$\therefore M_A \text{ (due to 6 tons load alone)} = \frac{9{\cdot}6 \times 2}{10} \text{ tons ft.}$$
$$= 1{\cdot}92 \text{ tons ft.}$$
$$\therefore M_B \text{ (due to 6 tons load alone)} = \frac{9{\cdot}6 \times 8}{10} \text{ tons ft.}$$
$$= 7{\cdot}68 \text{ tons ft.}$$

Total $M_A = (5{\cdot}76 + 1{\cdot}92)$ tons ft. $= 7{\cdot}68$ tons ft.

Total $M_B = (3{\cdot}84 + 7{\cdot}68)$ tons ft. $= 11{\cdot}52$ tons ft.

The diagram is constructed as indicated.

M_B is greater than M_A, therefore the base line of the free end S.F. diagram must be *raised*.

$$\frac{M_A - M_B}{l} = \frac{7{\cdot}68 - 11{\cdot}52}{10} \text{ tons}$$
$$= -{\cdot}384 \text{ tons.}$$

Deflection of Fixed Beams

(a) Fixed Beam with Single Concentrated Central Load

Fig. 131 illustrates a convenient method of solution. The beam is regarded as being a double inverted cantilever.* The cantilever shown to the left of the central section is loaded with

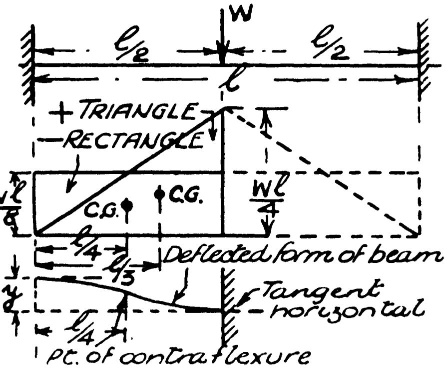


FIG. 131.

two B.M. diagrams, and by applying Mohr's theorem for the maximum upward deflection y we get:

$$y = \frac{A\bar{x}}{EI} = \frac{1}{EI}\left[\left(\frac{Wl}{4} \times \frac{l}{4} \times \frac{l}{3}\right) - \left(\frac{Wl}{8} \times \frac{l}{2} \times \frac{l}{4}\right)\right]$$
$$= \frac{1}{EI}\left(\frac{Wl^3}{48} - \frac{Wl^3}{64}\right)$$
$$= \frac{1}{192}\,\frac{Wl^3}{EI}.$$

(b) Fixed Beam with Uniformly Distributed Load

Adopting the same method as in the last case, the expression for y (Fig. 132) $= \dfrac{A\bar{x}}{EI}$

$$= \frac{1}{EI}\left[\left(\frac{2}{3} \times \frac{l}{2} \times \frac{Wl}{8} \times \frac{5}{16}l\right) - \left(\frac{Wl}{12} \times \frac{l}{2} \times \frac{l}{4}\right)\right]$$
$$= \frac{1}{EI}\left(\frac{5Wl^3}{384} - \frac{Wl^3}{96}\right)$$
$$= \frac{1}{384}\,\frac{Wl^3}{EI}.$$

* See *Experimental Building Science*, Vol. II, Manson and Drury.

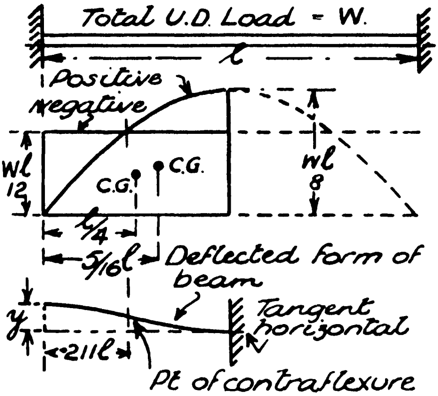


FIG. 132.

For other cases of loading, the graphical method given later may be employed.

Points of Contraflexure

These are the points in a beam at which the character of the bending changes from positive to negative, or vice versa. In the case of a single central load (Fig. 131) the points are clearly situated at one quarter of the span, respectively, from each support. In Fig. 132, in which the load is uniformly distributed, the points can be fixed by equating the general expression for the 'free B.M.' at a given point in the span to $\frac{Wl}{12}$, the end fixing moment. If w be the load per unit run, the 'free' B.M. at x from the left end $= \frac{wlx}{2} - \frac{wx^2}{2}$.

$$\therefore \frac{wlx}{2} - \frac{wx^2}{2} = \frac{wl^2}{12}$$

$$6x^2 - 6lx + l^2 = 0$$

$$x = \frac{6l \pm \sqrt{36l^2 - 24l^2}}{12} = \frac{l}{2} \pm \sqrt{\frac{l^2}{12}}$$

$$= \cdot 5l \pm \cdot 289l$$

$$= \cdot 211l \text{ or } \cdot 789l,$$

i.e. the points of contraflexure are $\cdot 211l$ from each support.

Continuous Beams

A continuous beam is one which covers more than one span, so that it has at least three supports. Continuous beams are not so common in steelwork as in reinforced concrete. The calculation of B.M. values for a beam of this type is a reversal of the normal procedure in B.M. calculations. In the case of continuous beams the B.M. values are determined first, and the support reactions deduced therefrom. Evaluations of bending moments are effected by means of a theorem known as the '**theorem of three moments.**' A proof of this theorem will be found in books on the theory of structures.

Theorem of Three Moments

In Fig. 133 we have a continuous beam for which the *free* B.M. diagrams are shown for two adjacent spans. These diagrams are drawn as if AB and BC were separate beams,

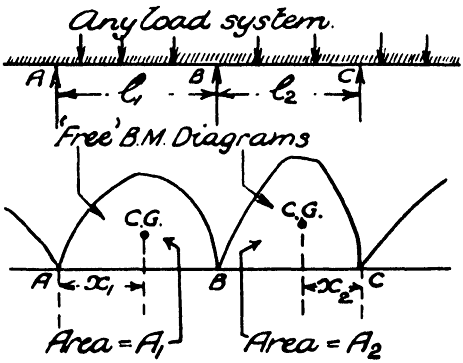


FIG. 133.—CONTINUOUS BEAM.

having freely supported ends. Owing to continuity there will be bending moments at the supports A, B and C—similar in character to the fixing moments in fixed beams. The theorem connects up the values of these three 'moments' with quantities derived from the free B.M. diagrams.

If A_1 be the area of the diagram on span AB, and x_1 its c.g. distance from end A, and if A_2 be the area for span BC, and x_2 the c.g. distance from end C, the theorem is expressed as follows :

$$M_A\, l_1 + 2\, M_B\, (l_1 + l_2) + M_C\, l_2 = 6\left(\frac{A_1 x_1}{l_1} + \frac{A_2 x_2}{l_2}\right)$$

M_A, M_B and M_C represent the *numerical* values of the support bending moments at A, B and C respectively.

The theorem assumes uniform beam section throughout, and is true only provided the supports at A, B and C are at the same level.

Expression of the Theorem for U.D. Load.—If for each span in the series of spans the loading be uniformly distributed, the theorem may be expressed in a simpler form.

In Fig. 134, $A_1 = \frac{2}{3}$ base $\times$ height $= \frac{2}{3} \times l_1 \times \frac{w_1 l_1^2}{8} = \frac{w_1 l_1^3}{12}$

and $x_1 = \frac{l_1}{2}$. Similar values will be obtained for the second span.

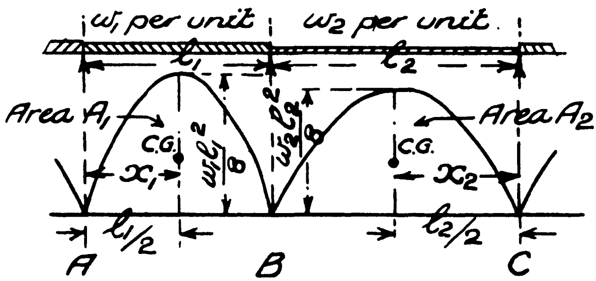


FIG. 134.—UNIFORMLY DISTRIBUTED LOAD.

Inserting these special values in the general expression of the theorem, we get :

$$M_A l_1 + 2M_B\,(l_1 + l_2) + M_C l_2 = 6\left(\frac{\frac{w_1 l_1^3}{12} \times \frac{l_1}{2}}{l_1} + \frac{\frac{w_2 l_2^3}{12} \times \frac{l_2}{2}}{l_2}\right)$$

$$= \tfrac{1}{4}(w_1 l_1^3 + w_2 l_2^3).$$

EXAMPLE (1). *Draw the B.M. and S.F. diagrams for the continuous beam given in Fig.* 135. *The ends A and C are freely supported.*

Free B.M. diagrams :

Span AB. B.M. maximum $= \frac{Wl}{8} = \frac{20 \times 10}{8}$ cwts. ft.

$= 25$ cwts. ft.

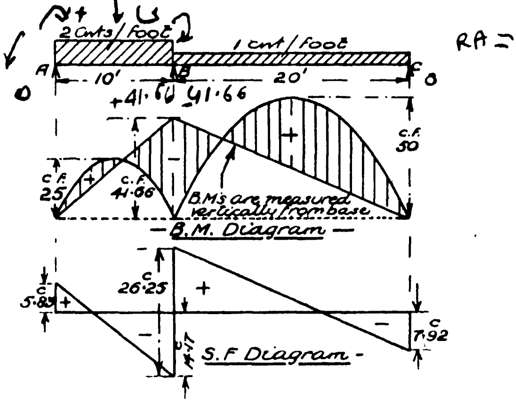


FIG. 135.

Span BC. B.M. maximum $= \dfrac{Wl}{8} = \dfrac{20 \times 20}{8}$ cwts. ft.

$= 50$ cwts. ft.

The free B.M. diagrams are parabolæ, as shown.

Expressing the theorem of three moments :

$$M_A l_1 + 2M_B (l_1 + l_2) + M_C l_2 = \tfrac{1}{4}(w_1 l_1^3 + w_2 l_2^3).$$

$M_A = M_C = 0$, as the extreme ends are 'free.'

$$\therefore 2M_B (10 + 20) = \tfrac{1}{4}(2 \times 10^3 + 1 \times 20^3).$$

$$60\, M_B = 2500.$$

$$\therefore M_B = 41{\cdot}66 \text{ cwts. ft.}$$

The negative support B.M. at B is therefore 41·66 cwts. ft. This is set up to scale at B, and the B.M. diagram completed by drawing in the base line. It is not necessary to reduce the diagram to a horizontal base, provided B.M. values are measured vertically from the jointed, inclined base shown.

Support Reactions

These are obtained by the ordinary methods of moments, *but care must be taken with the sign of a support moment.*

Taking moments about B :

$$(R_A \times 10) - (20 \times 5) = -\,41{\cdot}66 \text{ (note the sign).}$$

$$10\, R_A = 100 - 41{\cdot}66 = 58{\cdot}34.$$

$$R_A = 5{\cdot}834 \text{ cwts.}$$

Moments about B (to obtain R_C) :

$$(R_C \times 20) - (20 \times 10) = -41{\cdot}66 .$$

$$20\ R_C = 200 - 41{\cdot}66 = 158{\cdot}34.$$

$$R_C = 7{\cdot}917 \text{ cwts.}$$

Moments about C :

$$(R_A \times 30) + (R_B \times 20) - (20 \times 25) - (20 \times 10) = 0.$$

$$20\ R_B = 500 + 200 - 175{\cdot}02 = 524{\cdot}98.$$

$$R_B = 26{\cdot}249 \text{ cwts.}$$

Check : $R_A + R_B + R_C = (5{\cdot}834 + 26{\cdot}249 + 7{\cdot}917)$ cwts.

$= 40$ cwts. (load on beam).

The S.F. diagram is constructed by the rules given in Chapter IX, page 152.

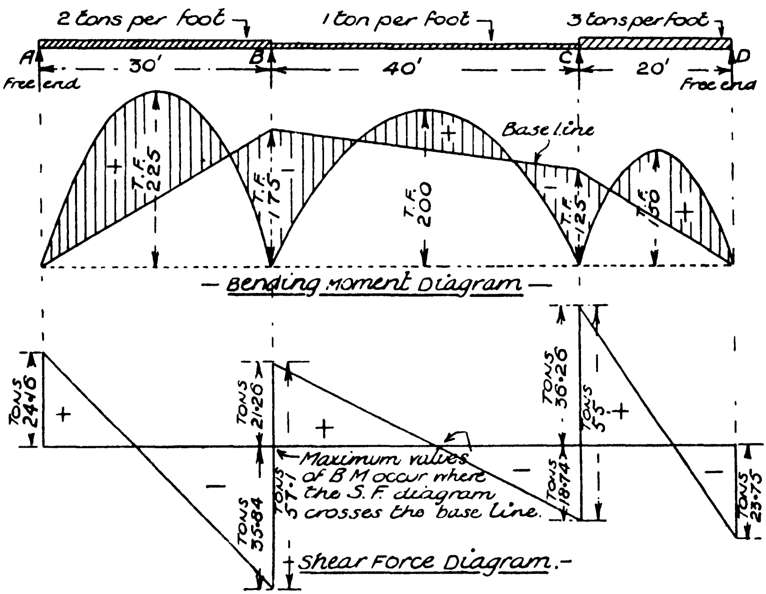


FIG. 136.

EXAMPLE (2). *Fig. 136 shows a beam continuous over three spans, the extreme ends being freely supported. Construct the B.M. and S.F. diagrams for the beam.*

Free B.M. maximum values :

Span AB : B.M. max. $= \dfrac{Wl}{8} = \dfrac{60 \times 30}{8} = 225$ tons ft.

Span BC: B.M. max. $= \frac{Wl}{8} = \frac{40 \times 40}{8} = 200$ tons ft.

Span CD: B.M. max. $= \frac{Wl}{8} = \frac{60 \times 20}{8} = 150$ tons ft.

Expressing the ' theorem of three moments ' for the first two spans, viz. AB and BC, we have:

$$M_A l_1 + 2M_B (l_1 + l_2) + M_C l_2 = \tfrac{1}{4}(w_1 l_1^3 + w_2 l_2^3).$$

But $M_A = 0$.

$$\therefore\ 2M_B (30 + 40) + (M_C \times 40) = \tfrac{1}{4}(2 \times 30^3 + 1 \times 40^3).$$

$$140\ M_B + 40\ M_C = 29500.$$

$$\therefore\ 7\ M_B + 2\ M_C = 1475\ (1).$$

In order to find M_B and M_C we require another simultaneous equation. This is obtained by considering spans BC and CD.

$$M_B l_2 + 2\ M_C (l_2 + l_3) + M_D l_3 = \tfrac{1}{4}(w_2 l_2^3 + w_3 l_3^3).$$

But $M_D = 0$.

$$\therefore\ 40\ M_B + 2\ M_C (40 + 20) = \tfrac{1}{4}(1 \times 40^3 + 3 \times 20^3).$$

$$40\ M_B + 120\ M_C = 22000.$$

$$M_B + 3\ M_C = 550\ (2).$$

Combining these equations:

$$7\ M_B + 2\ M_C = 1475\ (1).$$

$$M_B + 3\ M_C = 550\ (2).$$

Multiplying (2) by 7, and subtracting (1),

$$19\ M_C = 2375.$$

$$M_C = 125 \text{ tons ft.}$$

$$\text{and } M_B = 175 \text{ tons ft.}$$

Support reactions:

$$(R_A \times 30) - (60 \times 15) = -175.$$

$$R_A = 24{\cdot}16 \text{ tons.}$$

$$(R_B \times 40) + (24{\cdot}16 \times 70) - (60 \times 55) - (40 \times 20) = -125.$$

$$R_B = 57{\cdot}1 \text{ tons.}$$

$$(R_D \times 20) - (60 \times 10) = -125.$$

$$R_D = 23{\cdot}75 \text{ tons.}$$

$$(R_C \times 40) + (23{\cdot}75 \times 60) - (60 \times 50) - (40 \times 20) = -175.$$

$$R_C = 55 \text{ tons.}$$

The sum of the reactions is 160 tons, which checks the numerical working.

EXAMPLE (3). *AC (Fig. 137) is a continuous beam, freely*

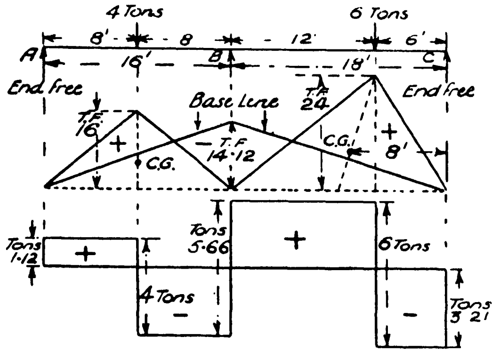


Fig. 137.

supported at A and C. Construct the B.M. and S.F. diagrams for the beam, which carries the two given concentrated loads.

Area of free B.M. diagram for AB $= A_1 = (\frac{1}{2} \times 16 \times 16)$ tons ft.^{2}

$= 128$ tons ft.^{2}.

Area of free B.M. diagram for BC $= (\frac{1}{2} \times 18 \times 24)$ tons ft.^{2}

$\therefore A_2 = 216$ tons ft.^{2}.

$x_1 = 8'$ (measured to A).

$x_2 = 8'$ (measured to C).

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = 6\left(\frac{A_1 x_1}{l_1} + \frac{A_2 x_2}{l_2}\right)$$

$M_A = M_C = 0.$

$$\therefore 2M_B(16 + 18) = 6\left(\frac{128 \times 8}{16} + \frac{216 \times 8}{18}\right)$$

$$68M_B = 6 \times 160.$$

$$M_B = 14{\cdot}12 \text{ tons ft.}$$

Reactions at supports:

$$(R_A \times 16) - (4 \times 8) = -14{\cdot}12.$$

$$16\,R_A = 17{\cdot}88.$$

$$R_A = 1{\cdot}12 \text{ tons.}$$

$$(R_B \times 18) + (R_A \times 34) - (4 \times 26) - (6 \times 6) = 0.$$

$$18\,R_B = 36 + 104 - (34 \times 1{\cdot}12).$$

$$18\,R_B = 101{\cdot}92.$$

$$R_B = 5{\cdot}66 \text{ tons.}$$

$$(R_C \times 18) - (6 \times 12) = -14{\cdot}12.$$

$$18\,R_C = 72 - 14{\cdot}12 = 57{\cdot}88.$$

$$R_C = 3{\cdot}21 \text{ tons.}$$

The diagrams are completed as shown in Fig. 137.

Graphical Method for Deflection

The method shown in Chapter VIII for obtaining the deflection of a simply supported beam graphically, may be extended to fixed and continuous beams. Fig. 138 illustrates the method applied to one span of a continuous beam. Negative areas are drawn

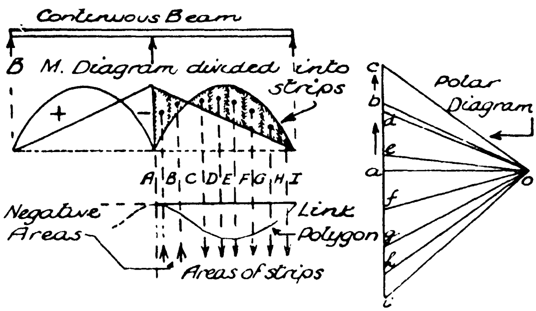


FIG. 138.

upwards in the polar diagram, otherwise the procedure is as for simple beams. As the slope in space A = zero, in the example chosen, the pole *o* is conveniently chosen on a level with point *a* in the polar diagram. This is done to obtain a horizontal base for the deflection diagram.

Characteristic Points

Bending Moment and Shear Force diagrams for fixed and continuous beams are sometimes constructed by the aid of points termed *characteristic points.* The method is particularly useful when the conditions are rather complicated for the use of the 'theorem of three moments.' The theory is due to Professor Claxton Fidler. Readers interested in the application of the method may with advantage consult 'Selected Engineering Paper No. 46' of the Institution of Civil Engineers. This paper, entitled 'Characteristic Points,' is written by Dr. E. H. Salmon, M.Inst.C.E., and in it the author shows how the method may be developed and extended. It will be only possible here to refer briefly to the use of the theory in a few of the examples already considered.

Each free B.M. diagram for a span in a continuous beam (or for a given fixed beam span) will have two characteristic

points. To obtain these points the span is divided into three equal parts, and, at the third points in the span, ordinates are erected of a certain height. The tops of these ordinates are the characteristic points required.

For a given span AB, the ordinate nearer A must have such a height x as to satisfy the following equation:

$x \times \text{span}^2 =$ twice the moment of the free B.M. diagram about B.

Note that for the ordinate nearer A, moment is taken about B, and vice versa.

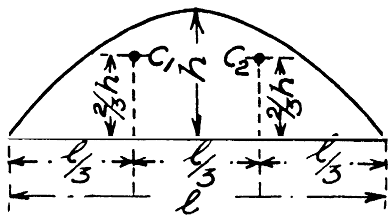


FIG. 139.—CHARACTERISTIC POINTS.

Applying the rule to a parabolic B.M. diagram (Fig. 139) of height h we have:

$$x \times l^2 = 2 \times \left(\frac{2}{3} l \times h\right) \times \left(\frac{l}{2}\right)$$

$$x = \frac{2}{3} h.$$

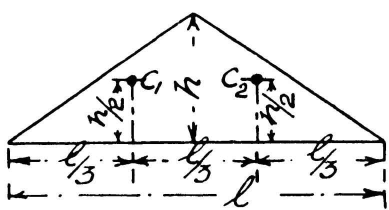


FIG. 140.—CHARACTERISTIC POINTS.

The points are shown as C_1 and C_2 in Fig. 139. For the case of Fig. 140 (representing the B.M. diagram for a single central load):

$$x \times l^2 = 2 \times \left(\frac{1}{2} \times l \times h\right) \times \frac{l}{2}.$$

$$x = \frac{h}{2}.$$

Having constructed the free B.M. diagrams in the usual manner, the base line is drawn in—using the characteristic points—with the aid of the following rules :

Fixed Beams.—Simply draw the base line through the two characteristic points. This clearly gives the required B.M. diagram in the cases given in Figs. 139 and 140, and should be tested for the other fixed beam diagrams already taken.

Continuous Beams.—The method is a little more complicated, but will be set out as briefly as possible.

Take the example of Fig. 141 (previously solved in Fig. 136). The point C_1 (being adjacent to a free end) is not required.

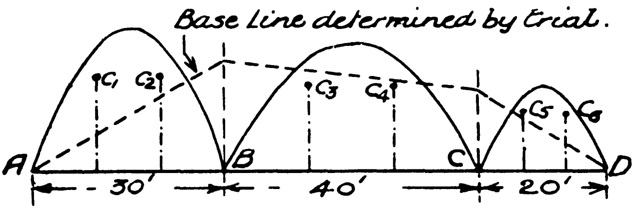


FIG 141.

The base line must obviously pass above C_2, through C_2 or below C_2. If above C_2, it must be below C_3 ; if below C_2, it must be above C_3, i.e. it must alternate, above and below, for either side of a support. For two points, such as C_3 and C_4 *in the same span*, there is no such required relationship. If the base line passes through C_2, it must pass through C_3 (but not necessarily through C_4). There is one further relationship, governing the base line position with respect to C_2 and C_3 (or C_4 and C_5). *The respective vertical distances between the base line and a pair of such points must be inversely as the spans in which they are situated.* Thus the distance below C_2 (in the example of Fig. 141) is to that above C_3, in the same ratio, as span BC is to span AB, i.e. $\frac{40}{30}$. The method is not so involved as it appears in a description, and after a few trial lines the correct base line can usually be fitted in. The reader is advised to draw out the given example to a fairly big scale, and to check the base line position by the support moments already computed for this case.

For the example given in Fig. 142 (see also Fig. 137) the

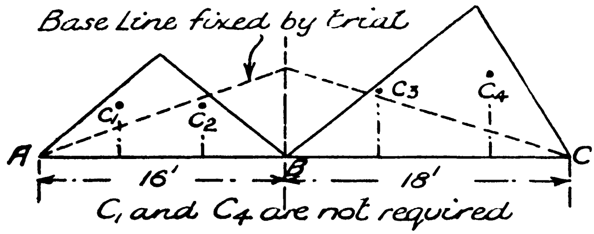


FIG. 142.

characteristic point is just below the base line for span AB and slightly less above for span BC, the ratio of distances being 18 : 16. Its position in AB will be $\frac{16}{3}$ ft. ($= 5\frac{1}{3}$ ft.) towards A from B, and the height above base AB $= \frac{16\text{T.F.}}{2} = 8$ T.F. (to scale). In BC it will be $\frac{18}{3}$ ft. ($=$ 6 ft.) from B, and x above base where

$$x \times 18^2 = 2 \times \frac{18 \times 24}{2} \times 8 \left[= \begin{matrix}\text{twice moment} \\ \text{of area about C}\end{matrix}\right]$$

i.e. $x = 10\frac{2}{3}$ T.F. (to scale). [T.F. = tons ft.]

The characteristic points for an unsymmetrical B.M. diagram—as that for span BC in the last example—will not, of course, both be at the same height above the base line of the free B.M. diagram. To find the height of C_4, if it were actually required, the moment of the B.M. diagram area would be taken about B.

EXERCISES 10

(1) A steel beam of 10′ span carries a total U.D. load of 13 tons. Taking a working stress of 8 tons/in.^{2}, calculate the necessary section modulus for the beam, assuming (*a*) ends simply supported, (*b*) ends fixed. Draw the B.M. and S.F. diagrams for case (*b*).

(2) An 8″ × 4″ × 18 lb. B.S.B. (Z = 13·91 ins.^{3}) is securely held by its end connections to pillars, so that the ends may be regarded as being completely fixed. The span is 8′ and the loads carried are 6·9 tons (central) plus 3·45 tons (uniformly distributed). Find the B.M. transmitted to each pillar, and deduce the maximum stress in the steel of the beam.

(3) A beam, fixed at both ends, has a span of 21′. A concentrated load of 9 tons is carried at 7′ from the left end. Calculate the *fixing moment* at each support, and draw the B.M. and S.F. diagrams for the beam.

(4) Draw the B.M. diagram for the example given in Exercise (3) by means of *characteristic points*.

(5) A continuous girder of 30′ total length, and of constant section, carries a uniform load of 2 tons per foot run. It rests freely on three supports, at the same level, one at each end and one 12′ from the left end. Calculate the B.M. at the intermediate support and the reaction at each support. Draw the B.M. and S.F. diagrams for the beam.

(6) A beam, fixed at one end, is propped at the other end so that both ends of the beam are at the same level. Assuming the beam to carry a total U.D. load W, obtain an expression for the value of the pressure on the prop. (Assume the beam to be half a continuous beam of two equal spans, with a load W on each span.)

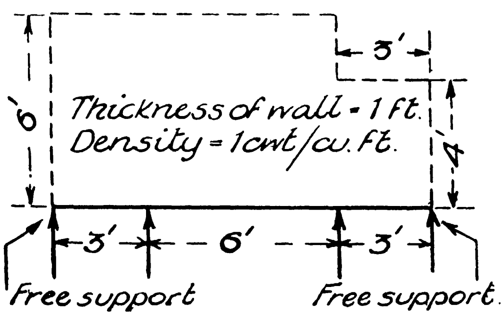


FIG. 143.

(7) Fig. 143 shows a continuous beam carrying a wall of uniform thickness. Calculate the support bending moments and reactions, and draw the B.M. and S.F. diagrams for the beam.

(8) A fixed beam of 12′ span carries two concentrated loads, 8 tons at 3′ from left end and 4 tons at 3′ from right end. Calculate the positions of the characteristic points and complete the B.M. diagram. Obtain the end fixing moments and draw the S.F. diagram for the beam.

CHAPTER XI

PRACTICAL DESIGN OF COMPRESSION MEMBERS

Introduction.—In the computation of the strength of a compression member, several factors have to be considered which do not influence the calculations in the corresponding example of a member in tension. In the latter case, the question of length does not usually arise. Further, the methods by which the ends of a tie are fixed are important only in so far as they may influence the axiality of the load. In a compression member, these considerations are of vital importance. The word *strut* will be used in a general sense throughout the chapter to include all members in compression, such as *columns*, *pillars* or *stanchions*.

Length of a Strut.—The terms *long* and *short* are of a special relative character, when used in connection with 'struts.' A *long strut* may be actually shorter than a *short strut*. The terms have reference to the relationship between the actual length of the member and its cross-sectional dimensions. Thus a concrete cube, 6″ high, would be a *short* strut, but a needle 2″ long would be a *long* strut.

Long struts are liable to failure by side-bending or *buckling*, as well as by direct crushing, and the 'longer' the member is, the greater is the importance of the buckling tendency.

Classification of Struts

(*a*) *Short struts*—in which failure is due to the direct crushing of the material, without the complication of buckling. The design of such members depends simply upon the permissible working stress in compression for the material. The London Building Act permits a maximum strut stress of 6·5 tons/in.^{2}. The maximum value given in the By-laws is 7·2 tons/in.^{2}.

(*b*) *Medium struts*—in which failure is a combination of crushing (direct stress) and buckling. This group includes the majority of practical struts, and is discussed in detail later.

(c) *Long struts*—in which the direct stress plays an unimportant part in comparison with that due to buckling.

The actual numerical limiting values to be given to each of these groups cannot be assigned, until we have considered the exact way in which the cross-sectional dimensions enter into the question of *length*.

Radius of Gyration

In Chapter VI two properties of section were considered. These had important implications in the design of beams. The property of section, now to be considered, is of equal importance in the design of 'struts.' It involves the 'moment of inertia' of the strut cross-section, and also its 'sectional area.' Experiment has shown that the association of these two quantities is in the form of the square root of their 'ratio.'

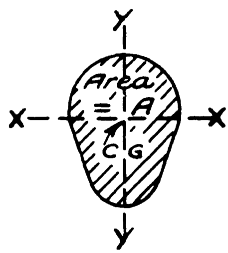


FIG. 144.

If I_{XX} = moment of inertia of the section given in Fig. 144 about the axis XX, and A = its sectional area, the radius of gyration of the section with respect to the axis XX is given by the expression $\sqrt{\frac{I_{XX}}{A}}$.

Various symbols are employed to denote this property. The L.C.C. By-laws adopt the letter *r*, B.S.S. 449–1937 uses *k*, while text-books frequently refer to the radius of gyration as *g*.

$$r_{XX} = \sqrt{\frac{I_{XX}}{A}}.$$

Similarly, $r_{YY} = \sqrt{\frac{I_{YY}}{A}}.$

It is possible to constrain a strut so that its tendency to bend must be about a particular axis of its section. In the usual case

no such restraint is present, and the radius of gyration has to be evaluated for the principal axis of section for which it has the lesser value. This is termed *least r* (or *least k*, etc.).

Calculation of Radius of Gyration Values

The following examples illustrate the nature of the calculations involved in the derivation of this property. Rivet holes are not allowed for in obtaining r values, the gross section being always taken.

EXAMPLES

(1) *Find r_{XX} for a rectangle b (wide) $\times$ d (deep) (Fig. 145).*

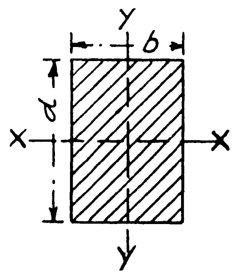


FIG. 145.

$$I_{XX} = \frac{bd^3}{12}. \qquad A = b \times d.$$

$$r_{XX} = \sqrt{\frac{I_{XX}}{A}} = \sqrt{\frac{\frac{bd^3}{12}}{b \times d}} = \frac{d}{\sqrt{12}}.$$

Similarly, $r_{YY} = \dfrac{b}{\sqrt{12}}$.

(2) *Obtain an expression for r_{XX} (i.e. 'r' about a diameter) for a solid circular section of diameter D (Fig. 146).*

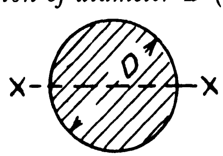


FIG. 146.

$$r_{XX} = \sqrt{\frac{I_{XX}}{A}} = \sqrt{\frac{\frac{\pi D^4}{64}}{\frac{\pi D^2}{4}}} = \sqrt{\frac{D^2}{16}} = \frac{D}{4}.$$

(3) *Calculate the value of least r for the joist column section given in Fig.* 147.

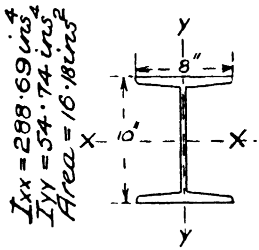


FIG. 147.

I_{XX} (from tables) $= 288{\cdot}69$ ins.^{4}.
I_{YY} ,, ,, $= 54{\cdot}74$ ins.^{4}.
A ,, ,, $= 16{\cdot}18$ ins.^{2}.

$$r_{XX} = \sqrt{\frac{I_{XX}}{A}} = \sqrt{\frac{288{\cdot}69}{16{\cdot}18}} = 4{\cdot}22 \text{ ins.}$$

$$r_{YY} = \sqrt{\frac{I_{YY}}{A}} = \sqrt{\frac{54{\cdot}74}{16{\cdot}18}} = 1{\cdot}84 \text{ ins.}$$

Least r is therefore 1·84″. It is clear that the **least r value must be associated with the least I value.**

The values above may be checked by means of the tables given on page 227, in which the least r values are emphasised by being printed in bolder type.

(4) *Find the values of r_{XX} and r_{YY} for the compound column section of Fig.* 148.

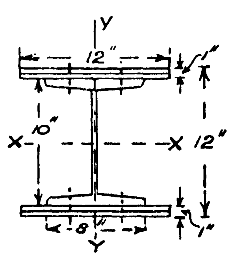


FIG. 148.

I_{XX} for B.S. section $= 288{\cdot}69$ ins.^{4}.
I_{XX} for plates $= 2\,[I_{CG} + AD^2]$

$$= 2\left[\frac{12 \times 1^3}{12} + (12 \times 1 \times 5{\cdot}5^2)\right] = 728 \text{ ins.}^4.$$

$$\text{Total } I_{XX} = (288 \cdot 69 + 728) \text{ ins.}^4 = 1016 \cdot 69 \text{ ins.}^4.$$

$$\text{Total area} = (16 \cdot 18 + 24) \text{ ins.}^2 = 40 \cdot 18 \text{ ins.}^2.$$

$$r_{XX} = \sqrt{\frac{I_{XX}}{A}} = \sqrt{\frac{1016 \cdot 69}{40 \cdot 18}} = 5 \cdot 03 \text{ ins.}$$

$$I_{YY} \text{ for B.S. section} = 54 \cdot 74 \text{ ins.}^4.$$

$$I_{YY} \text{ for plates} = 2\left(\frac{1 \times 12^3}{12}\right) = 288 \text{ ins.}^4.$$

$$\text{Total } I_{YY} = (288 + 54 \cdot 74) \text{ ins.}^4 = 342 \cdot 74 \text{ ins.}^4.$$

$$r_{YY} = \sqrt{\frac{I_{YY}}{A}} = \sqrt{\frac{342 \cdot 74}{40 \cdot 18}} = 2 \cdot 92 \text{ ins.}$$

(5) *Calculate the values of the greatest and the least radii of gyration respectively for the compound column section given in Fig.* 149.

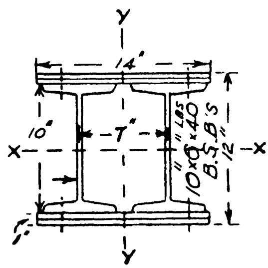


FIG. 149.

From section tables (page 107) the following properties of a 10" × 6" × 40 lb. B.S.B. are obtained.

$I_{XX} = 204 \cdot 80 \text{ ins.}^4$, $I_{YY} = 21 \cdot 76 \text{ ins.}^4$, $A = 11 \cdot 77 \text{ ins.}^2$.

$$\text{Total } I_{XX} \text{ for B.S.B.s} = 2 \times 204 \cdot 8 = 409 \cdot 6 \text{ ins.}^4.$$

$$\text{Total } I_{XX} \text{ for plates} = 2\left[\frac{14 \times 1^3}{12} + (14 \times 1 \times 5 \cdot 5^2)\right]$$
$$= 849 \cdot 34 \text{ ins.}^4.$$

$$\text{Total } I_{XX} \text{ for section} = (409 \cdot 6 + 849 \cdot 34) \text{ ins.}^4$$
$$= 1258 \cdot 94 \text{ ins.}^4.$$

$$\text{Total area of section} = [(2 \times 11 \cdot 77) + (2 \times 14 \times 1)] \text{ ins.}^2$$
$$= 51 \cdot 54 \text{ ins.}^2.$$

$$r_{XX} = \sqrt{\frac{I_{XX}}{A}} = \sqrt{\frac{1258 \cdot 94}{51 \cdot 54}} \text{ ins.} = 4 \cdot 94 \text{ ins.}$$

$$\text{Total } I_{YY} \text{ for B.S.B.s} = 2\,[I_{CG} + AD^2]$$
$$= 2\,[21{\cdot}76 + (11{\cdot}77 \times 3{\cdot}5^2)] \text{ ins.}^4$$
$$= 331{\cdot}88 \text{ ins.}^4.$$

$$\text{Total } I_{YY} \text{ for plates} = \frac{2 \times 1 \times 14^3}{12} \text{ ins.}^4$$
$$= 457{\cdot}33 \text{ ins.}^4.$$

$$\text{Total } I_{YY} \text{ for section} = (331{\cdot}88 + 457{\cdot}33) \text{ ins.}^4 = 789{\cdot}21 \text{ ins.}^4.$$

$$r_{YY} = \sqrt{\frac{I_{YY}}{A}} = \sqrt{\frac{789{\cdot}21}{51{\cdot}54}} = 3{\cdot}91 \text{ ins.}$$

End Fixture of Struts

It is important to distinguish between two terms which are used in connection with the end fixture of a strut.

(*a*) *Position fixed.*—This mode of fixture implies that the end is unable to move its position but also that the strut has freedom of bending, just as if the end were held by a pin joint (Fig. 150 (*a*)).

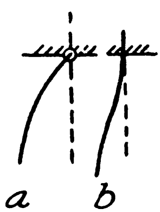


FIG. 150.

(*b*) Fig. 150 (*b*) illustrates a method of end fixture whereby the end of the strut is constrained to deflect in the manner of a fixed beam end. Such end fixture is termed *direction fixed.*

In the illustrations given later it will be seen that the form of the curve, into which a strut tends to deflect, depends upon the mode of end fixture. In each case there is a portion of the length of the strut which bends as if this part had pin-jointed ends. The length of this portion is known as the *equivalent* or *effective* length of the strut. The precise value to be taken for this length depends upon certain practical considerations (noted later). It will be convenient to explain here, in summary form, the incidence of *equivalent lengths* in the various methods used for strut calculations:

(*a*) In formulæ of the 'Euler' and 'Rankine' type (see later) the 'equivalent length' of the strut must be inserted.

(*b*) The London Building Act gives three separate formulæ, the particular formula to be used depending on the nature of the end fixture.

In these formulæ, the actual length of the strut must be used.

(*c*) B.S.S. 449–1937 and the L.C.C. By-laws revert to the use of *effective lengths*, and one table only of working stress values is given. B.S.S. 449–1937 includes working column stresses for high tensile steel.

Equivalent Length of a Strut

(1) *Hinged ends*, i.e. ends 'position fixed' only.

The strut bends freely from end to end (Fig. 151) and the 'equivalent length' is the actual length of the strut.

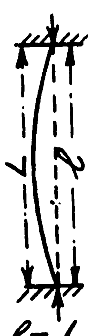


FIG. 151.

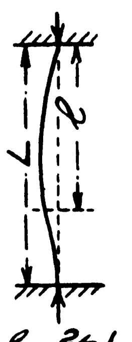


FIG. 152.

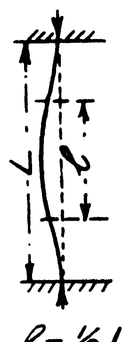


FIG. 153.

(2) *One end hinged, one end fixed*, i.e. 'position fixed' only at one end, 'position and direction fixed' at the other (Fig. 152). The value '$\frac{2}{3}$ × actual length' is commonly taken for the equivalent length in this case.

(3) *Both ends fixed*, i.e. 'position and direction fixed' at both ends. The equivalent length in this case is half the actual length—as indicated in Fig. 153.

(4) *One end fixed, one end free*, i.e. 'position and direction fixed' at one end, and no effective restraint at the other. This

is the extreme case, and the equivalent length, as shown in Fig. 154, is twice the actual length of the strut.

FIG. 154.

It is not easy to decide to which of these four sections any practical strut in a steel frame should be assigned, but commonly adopted standards will be found later in this chapter.

The Strength of Struts

The methods used for determining the strength of struts may be divided into three groups :

(i) Theoretical methods, based on certain ideal conditions—exemplified in Euler's theory.

(ii) Methods involving the use of formulæ having a theoretical background, but which are rendered *empirical* (or ' practical ') by the insertion of ' constants,' which are found by the actual testing of struts. Rankine's formula is an example of this group.

(iii) Methods based on lists of working stresses, which are laid down in regulations issued by local authorities, or in standard specifications representing the results of research.

Euler's Theory.—Euler neglected direct stress, and derived a formula on the assumption that the strut would fail by ' buckling.' He assumed that the loading was perfectly axial and the material homogeneous throughout, and that the length was such that the assumption as to buckling failure was permissible. It is of interest to test Euler's expression for the ultimate load for a strut by a series of approximations.

In Fig. 155, as a first approximation, assume that the strut bends to the arc of a circle.

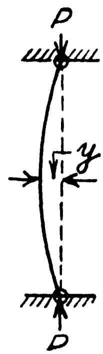


FIG. 155.

$$y = \frac{Ml^2}{8EI} \text{ (for circular deflection).}$$

But $M = P \times y$ in this case.

$$y = \frac{Pyl^2}{8EI}$$

$$\text{or } P = \frac{8}{l^2} \times EI.$$

As a nearer approximation, assume that the B.M. does not remain constant (as is assumed in circular deflection with a constant section of the member), but to vary, as in the case of a beam with U.D. load.

$$y = \frac{5}{384}\frac{Wl^3}{EI} = \frac{5}{48} \times \frac{Wl}{8} \times \frac{l^2}{EI}$$

$$= \frac{5}{48} \times M \times \frac{l^2}{EI} = \frac{5}{48} \times Py \times \frac{l^2}{EI}.$$

$$\therefore P = \frac{9{\cdot}6}{l^2} EI.$$

Euler in his theory obtained π^2 instead of 9·6, and defined P as the *least load required to produce instability*. He assumes that the strut remains perfectly straight until this critical load is reached, and that collapse takes place without any intermediate state of equilibrium.

$$P = \frac{\pi^2}{l^2} EI.$$

l = the equivalent length of the strut in ins.

E = Young's modulus (tons/in.^{2}).

I = least moment of inertia (ins.^{4}).

P = crippling load (axial) in tons.

A factor of safety of 4 is usually used in this theory for mild steel. The theory gives inadmissible results if the ratio of the length of the strut to its least radius of gyration is less than about 110 to 120.

EXAMPLE. *A 4″ × 4″ × ⅜″ B.S. angle is used as a strut in a truss. It may be assumed to have 'one end hinged' and 'one end fixed.' Its actual length is 8′ 6″. Calculate the safe axial thrust.*

Least I (from tables) = 2·76 ins.^{4}.

E = 13,000 *tons/in.*^{2}.

Factor of safety = 4.

Equivalent length of strut = ⅔ × 102″ = 68″.

$$P = \frac{\pi^2}{l^2} EI = \left(\frac{\pi^2}{68^2} \times 13{,}000 \times 2{\cdot}76\right) \text{tons}$$

$$= 77 \text{ tons.}$$

$$\text{Safe axial load} = \frac{P}{4} = 19 \text{ tons.}$$

Rankine's Formula.— Rankine gave the following formula for the crippling load of a strut, axially loaded:

$$P = \frac{Af_c}{1 + a\left(\frac{l}{k}\right)^2}.$$

A = sectional area of strut.

l = equivalent length of strut.

k = least radius of gyration.

f_c = a practical constant (associated with the yield point of the strut material) commonly taken as 21 tons/in.^{2} for mild steel.

a = a constant, usually taken as $\frac{1}{7500}$ for mild steel.

Taking the case of hinged ends (the case for which these formulæ are originally considered) it can be shown that for low values of $\left(\frac{l}{k}\right)$ the formula becomes $P = A \times f_c$ and for high values P = the corresponding Eulerian value, so that it agrees with accepted theory at the extreme values of the $\frac{l}{k}$ range.

EXAMPLE. *Using Rankine's formula, find the safe axial load for a* 12″ × 6″ × 44 *lb. joist column* 10′ 10″ *high—to be regarded as having ends fixed—using the usual constants for mild steel and adopting a factor of safety of* 4. *The least radius of gyration for the given standard section is* 1·30″, *and the sectional area* 13 *ins.*².

$$P = \frac{Af_c}{1 + a\left(\frac{l}{k}\right)^2}.$$

$$l = \text{equivalent length of column} = \tfrac{1}{2} \times 130'' = 65''.$$

$$a\left(\frac{l}{k}\right)^2 = \frac{1}{7500} \times \left(\frac{65}{1\cdot3}\right)^2 = \frac{2500}{7500} = \cdot33.$$

$$P = \frac{13 \times 21}{1 + \cdot33} \text{ tons} = 205\cdot2 \text{ tons}.$$

$$\text{Safe axial load} = \frac{205\cdot2}{4} \text{ tons} = 51\cdot3 \text{ tons}.$$

London Building Act Column Formulæ.—The London Building Act, 1930, Third Schedule, paragraph 20, contains the following table of working stresses, in tons per square inch of section, for mild steel pillars.* The length referred to in the first column is the actual length of the member.

MILD STEEL COLUMNS

Ratio of length to least radius of gyration.	Working stresses in tons per square inch of section.		
	Hinged ends.	One end hinged and one end fixed.	Both ends fixed.
20	4·0	5·0	6·0
40	3·5	4·5	5·5
60	3·0	4·0	5·0
80	2·5	3·5	4·5
100	2·0	3·0	4·0
120	1·0	2·5	3·5
140	0·0	2·0	3·0
160		1·0	2·5
180		0·0	1·5
200			0·5
210			0·0

FIG. 156.—WORKING STRESSES IN MILD STEEL COLUMNS.

* The word 'column' replaces 'pillar' in the more recent L.C.C. Regulations.

It will be observed that the working stress in the case of each mode of end fixture gradually falls to a certain value and then decreases at a uniform but quicker rate. This is illustrated in the graphs shown in Fig. 157, which give the relationships between working stress and $\frac{l}{r}$ values for each of the three cases.

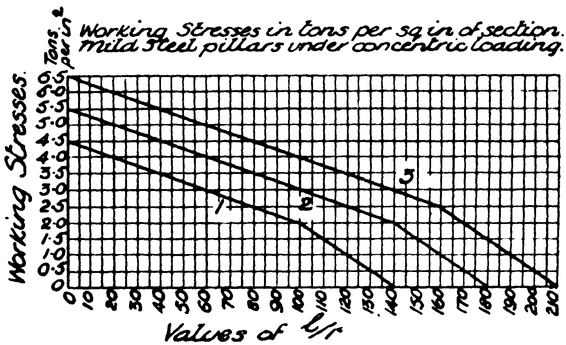


FIG. 157.—LONDON BUILDING ACT (1930) COLUMN STRESSES.

Graph 1 is for 'hinged ends.' The maximum working stress allowed is 4·5 tons/in.². The value decreases to 2 tons/in.² uniformly, the latter stress corresponding to a $\frac{l}{r}$ value of 100. This represents the economic limit for this end fixture, as the working stress rapidly falls for higher values.

Graph 2 is for 'one end hinged, one end fixed.' The maximum working stress is 5·5 tons/in.², and the economic limit for $\frac{l}{r} = 140$.

Graph 3 is for 'both ends fixed,' the maximum working stress in this case being 6·5 tons/in.², and the economic limit 160.

The formulæ corresponding to the three graphs, in their economic ranges, are simple in form, and may be easily remembered.

Formulæ for working stresses in tons per sq. inch of column section :

Fixed ends. $6{\cdot}5 - \frac{l}{40r}$ (maximum $\frac{l}{r} = 160$).

One end hinged. *One end fixed.* $\Big\}$ $5{\cdot}5 - \dfrac{l}{40r}$ (maximum $\dfrac{l}{r} = 140$).

Hinged ends. $4{\cdot}5 - \dfrac{l}{40r}$ (maximum $\dfrac{l}{r} = 100$).

EXAMPLES

(1) *Calculate the safe concentric load for a solid circular mild steel column, 5″ diameter and 10′ high, taking the end fixture as 'one end hinged, one end fixed.'*

Working stress $= 5{\cdot}5 - \dfrac{l}{40r}$.

$r =$ least radius of gyration $= \dfrac{D}{4} = \dfrac{5''}{4} = 1{\cdot}25''$.

$\dfrac{l}{r} = \dfrac{120''}{1{\cdot}25''} = 96$. This is less than 140, therefore the formula is admissible.

$$\text{Working stress} = \left(5{\cdot}5 - \frac{96}{40}\right) \text{tons/in.}^2$$
$$= (5{\cdot}5 - 2{\cdot}4) \text{ tons/in.}^2$$
$$= 3{\cdot}1 \text{ tons/in.}^2.$$

$$\text{Sectional area of column} = \frac{\pi D^2}{4} = \frac{\pi \times 5^2}{4} \text{in.}^2 = 19{\cdot}635 \text{ in.}^2.$$

$$\therefore \text{ Safe concentric load} = (3{\cdot}1 \times 19{\cdot}635) \text{ tons}$$
$$= 60{\cdot}87 \text{ tons.}$$

The working stress $3{\cdot}1$ tons/in.^{2} may be checked from graph No. 2 (for $\dfrac{l}{r} = 96$).

(2) *A column 15′ high, whose ends may be regarded as being fixed, has to support a concentric load of 82 tons. Select a suitable joist column section.*

Try a 12″ × 8″ × 65 lb. standard beam section.

From tables on page 225 least $r = 1{\cdot}85''$.

$$\therefore \frac{l}{r} = \frac{15 \times 12}{1{\cdot}85} = 97{\cdot}3.$$

From graph 3 $\left[\text{or by formula } 6{\cdot}5 - \dfrac{l}{40r}\right]$ the corresponding working stress $= 4{\cdot}06$ tons/in.^{2}.

The sectional area of this section $= 19{\cdot}1$ in.^{2}.

$$\therefore \text{ Safe concentric load} = (4{\cdot}06 \times 19{\cdot}1) \text{ tons}$$
$$= 77{\cdot}5 \text{ tons.}$$

This section is therefore not suitable.

Try a 14″ × 8″ × 70 lb. joist column.

Least $r = 1{\cdot}80''$.

$$\therefore \frac{l}{r} = \frac{180''}{1{\cdot}80''} = 100.$$

Working stress = 4 tons/in.².

∴ Safe concentric load = 4 tons/in.² × sectional area.

Sectional area = 20·6 in.².

∴ Safe concentric load = (4 × 20·6) tons
= 82·4 tons.

Hence this selection is a suitable one.

(3) *A $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ B.S. equal angle is to be used as a strut in a truss to carry an axial load of $7\frac{1}{4}$ tons, the end conditions to be considered as 'both hinged.' Calculate the maximum permissible length for the strut. Least radius of gyration for this section = ·68″. Sectional area = 3·25 in.².*

Let l'' = maximum length.

$$\text{Actual stress in angle section} = \frac{7{\cdot}25 \text{ tons}}{3{\cdot}25 \text{ in.}^2}$$
$$= 2{\cdot}23 \text{ tons/in.}^2.$$

$$\text{Working stress} = 4{\cdot}5 - \frac{l}{40r}.$$

$$\therefore 2{\cdot}23 = 4{\cdot}5 - \frac{l}{40 \times {\cdot}68}.$$

$$\frac{l}{27{\cdot}2} = 2{\cdot}27.$$

$$\therefore l = 61{\cdot}74''$$
$$= 5'\ 1\tfrac{1}{2}''.$$

(4) *Find the safe concentric load for the compound column section given in Fig. 149. The ends are fixed and the distance from floor level to floor level (for which this is the column section) = 20′*

As previously determined, least $r = 3{\cdot}91$ ins.

$$\therefore \frac{l}{r} = \frac{20 \times 12}{3{\cdot}91} = 61{\cdot}4.$$

$$\text{Working stress} = 6{\cdot}5 - \frac{l}{40r} = (6{\cdot}5 - 1{\cdot}535) \text{ tons/in.}^2$$
$$= 4{\cdot}965 \text{ tons/in.}^2.$$

Sectional area of column $= 51\cdot54$ ins.^{2}.

$\therefore$ Safe concentric load $= (51\cdot54 \times 4\cdot965)$ tons

$= 255$ tons.

L.C.C. By-laws Methods

The working loads in column shafts are contained in clause 85 of the regs. Clause 85(*a*) deals with the maximum permissible ratio of effective column length to the least radius of gyration. For columns and struts forming part of the main structure of a building, the maximum permissible value of the ratio is 150, while that for subsidiary members in compression is 200.

Clause 85(*b*) is as follows:

The working loads per square inch in the shafts of columns and other compression members of structural steel shall not exceed those specified in the following table, except as provided in by-laws 87 *and* 90.

Ratio of effective column length to least radius of gyration. $= \frac{l}{r}$	Working loads in tons per square inch of gross section. $= F_1$	Ratio of effective column length to least radius of gyration. $= \frac{l}{r}$	Working loads in tons per square inch of gross section. $= F_1$
20	7·2	130	2·6
30	6·9	140	2·3
40	6·6	150	2·0
50	6·3	160	1·8
60	5·9	170	1·6
70	5·4	180	1·5
80	4·9	190	1·3
90	4·3	200	1·2
100	3·8	—	—
110	3·3	—	—
120	2·9	—	—

Intermediate values shall be obtained by interpolation.

FIG. 158.—COLUMN STRESSES.

Clauses 87 and 90 will be explained later. B.S.S. No. 15–1936 removes the former division of material into 'A' and 'B' steel.

In order to use the working loads given in the table, it is necessary to determine the 'effective length' of a column.

The values to be taken are the subject of Clause 86 in the L.C.C. By-laws.

L.C.C. By-laws (1938), *Clause* 86: *The effective column length to be assumed in determining the working load per square inch in accordance with By-law* 85 *shall be as follows:*

	Type of Column.	Effective column length.
Columns of one storey	Properly restrained at both ends in position and direction.	0·75 of the actual column length.
	Properly restrained at both ends in position but not in direction.	Actual column length.
	Properly restrained at one end in position and direction and imperfectly restrained in both position and direction at the other end.	A value intermediate between the actual column length and twice that length, depending upon the efficiency of the imperfect restraint.
Columns continuing through two or more storeys.	Properly restrained at both ends in position and direction.	0·75 of the distance from floor level to floor level.
	Properly restrained at both ends in position and imperfectly restrained in direction at one or both ends.	A value intermediate between 0·75 and 1·00 of the distance from floor level to floor (or roof) level, depending upon the efficiency of the directional restraint.
	Properly restrained at one end in position and direction and imperfectly restrained in both position and direction at the other end.	A value intermediate between the distance from floor level to floor (or roof) level and twice that distance, depending upon the efficiency of the imperfect restraint.

NOTE.—The effective column length values given above are in respect of typical cases only, and embody the general principles which shall be employed in assessing, to the satisfaction of the district surveyor, the appropriate value for any particular column.

FIG. 159.—TABLE OF EFFECTIVE COLUMN LENGTHS.

Practical Interpretation of End Fixture.—We have still to decide what constitutes adequate restraint in 'position,' or in 'direction,' or in both 'position and direction.' The following will be found to be common practice.

Columns of one storey or top length of a continuous column.

Let L = height from floor level to floor level.

With 1-way connection, effective length = 1·25L.

With 2-way connections, effective length = 1·00L.

With 3- or 4-way connections, effective length = ·875L.

The same values are taken for the topmost length in a continuous column through several storeys, L being then the height from top floor level to roof.

Columns continuing through two or more storeys or bottom length of a continuous column.

L = height from floor level to floor level.

With 1-way connection, effective length = 1·00L.

With 2-way connections, effective length = ·875L.

With 3- or 4-way connections, effective length = ·75L.

EXAMPLES

(1) *A solid steel circular column* 4″ *diameter is* 8′ 9″ *high. Its ends are adequately restrained in position, but not in direction. Calculate the safe concentric load.*

As this is a column of one storey the effective length = the actual length (for the end fixture given).

$$\therefore l = 105''.$$

Radius of gyration $= r = \frac{D}{4} = \frac{4''}{4} = 1''$.

$\therefore \frac{l}{r} = \frac{105''}{1''} = 105$. To get F_1, the working load in tons/in.² of gross section, we have to interpolate between the values given for $\frac{l}{r} = 110$ and $\frac{l}{r} = 100$.

$$\frac{l}{r} = 110, \ F_1 = 3\cdot3 \text{ tons/in.}^2.$$

$$\frac{l}{r} = 100, \ F_1 = 3\cdot8 \text{ tons/in.}^2.$$

Difference = 10 Difference = ·5 tons/in.².

For difference = 5 difference = ·25 tons/in.².

$\therefore$ for $\frac{l}{r} = 105$, $F_1 = 3\cdot3 + \cdot25 = 3\cdot55$ tons/in.².

Area of column section $= \frac{\pi D^2}{4} = \pi \times \frac{4^2}{4}$ in.² $= 12\cdot56$ in.².

$\therefore$ Safe concentric load $= (3\cdot55 \times 12\cdot56)$ tons
$= 44\cdot58$ tons.

(2) *A 14″ × 8″ × 70 lb. joist column is used as a continuous column through two floors, the connections to the column being 2-way. Calculate the maximum load, regarded as concentric, which may be transmitted to the bottom length of the column, the distance between the floor levels being 10′.*

The effective column length for this case will be:

$\cdot875 \times 10' = 8\cdot75' = 105''$.

From page 225 the value of $r = 1\cdot80''$.

$$\therefore \frac{l}{r} = \frac{105}{1\cdot8} = 58\cdot33.$$

For $\frac{l}{r} = 50$ $\quad F_1 = 6\cdot3$ tons/in.².

For $\frac{l}{r} = 60$ $\quad F_1 = 5\cdot9$ tons/in.².

Difference for 10 $= \cdot4$ tons/in.².
Difference for $8\cdot33 = \cdot33$ tons/in.².

$\therefore$ for $\frac{l}{r} = 58\cdot33$, $\quad F_1 = (6\cdot3 - \cdot33)$ tons/in.²
$= 5\cdot97$ tons/in.².

The area of section for this joist column $= 20\cdot6$ in.².
$\therefore$ Safe concentric load for the bottom length of column
$= (5\cdot97 \times 20\cdot6)$ tons
$= 123$ tons.

This value may be approximately checked by the table on page 224, which gives 127 tons for 8 ft. *effective height.*

Eccentric Loads

In the case of a column continuing through several floors of a building, the connections of the main floor beams will have to be made to the flange or to the web of the column. The loads transmitted by these beams will therefore not coincide with the axis of the column. The effect of eccentricity of loading is very

marked, and even small eccentricities may considerably reduce the carrying capacity of a column.

Consider the eccentric load W in the case of the column shown in Fig. 160. Its effect on the column is really two-fold:

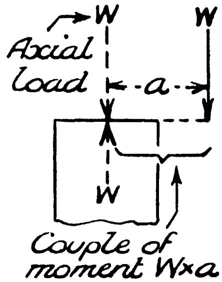


FIG. 160.—ECCENTRIC LOADING.

this may be demonstrated by introducing two vertical forces, as shown, each equal to W, at the centroid of the section. We have now an axial load W, producing direct compressive stress of a uniform value all over the section, together with a bending moment, of value W × arm of eccentricity, creating bending stresses. The resultant stress in any fibre in the section will be the algebraic sum of these stresses.

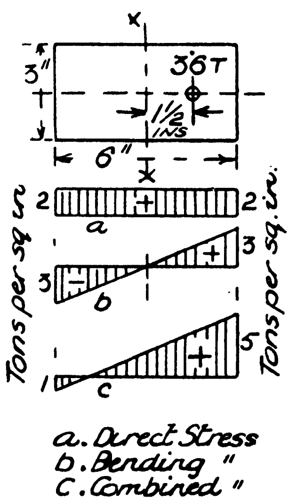


FIG. 161.

EXAMPLE. *A short steel column of rectangular section (indicated in Fig. 161) carries an eccentric load of 36 tons. Taking the dimensions given, calculate the maximum and minimum stresses produced and draw a diagram showing the stress distribution across the section.*

The eccentric load of 36 tons is equivalent in its effect to a concentric load of 36 tons, together with a bending moment of $(36 \times 1\frac{1}{2})$ tons ins.

$$\text{Direct stress} = \frac{\text{Load}}{\text{Area}} = \frac{36 \text{ tons}}{18 \text{ in.}^2} = 2 \text{ tons/in.}^2.$$

Section modulus of column section about axis XX

$$= \frac{bd^2}{6} = \frac{3 \times 6 \times 6}{6} \text{ ins.}^3 = 18 \text{ ins.}^3.$$

$$M = fZ \text{ or } f = \frac{M}{Z} = \frac{36 \times 1\frac{1}{2}}{18} \text{ tons/in.}^2 = 3 \text{ tons/in.}^2.$$

$\therefore$ Maximum compressive and tensile bending stresses

$= 3$ tons/in.^{2}.

Total maximum compressive stress $= (2 + 3)$

$= 5$ tons/in.^{2}.

Total minimum compressive stress $= (2 - 3)$

$= -1$ ton/in.^{2},

i.e. there is a resultant tensile stress of 1 ton/in.^{2}. As the column is short the maximum compressive stress is not excessive. We cannot decide, in a general case, whether such stress is too great for a practical column, until the length and end fixture are known. **The maximum stress thus computed must not exceed the appropriate 'column stress,' determined with reference to the axis of least radius of gyration—in this case YY.** A slight increase on the tabular column stress is permitted when part of the stress is due to eccentric loading.

L.C.C. By-law No. 87.—In the case of eccentric loading on a steel column, the bending moment about each principal axis shall be calculated and the resulting bending stresses added to the axial load per square inch. The working load per square inch may then be increased above that specified in By-law 85 up to a limit F_2, where

$$F_2 = f_c + 7{\cdot}5\left(1 - \frac{f_c}{F_1}\right)\left(1 - {\cdot}002\frac{l}{r}\right)$$

where F_1 denotes the working load per square inch specified in By-law 85, f_c the total load on the column in tons divided by the gross cross-sectional area of the column in square inches, and $\frac{l}{r}$ the ratio of effective column length to the least radius of gyration.

An example illustrating this clause is given later.

Equivalent Concentric Load.—In Fig. 162 let W_E = the actual eccentric load, and a the arm of eccentricity for the

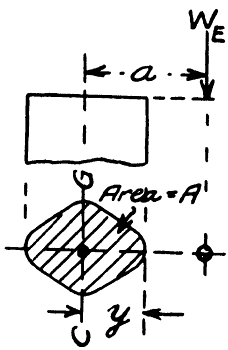


FIG. 162.

principal axis shown, Z = the section modulus for this axis, and y = distance of extreme fibre from the axis, measured towards the side on which the load is situated. If A = the sectional area of the column, the maximum compressive stress will be:

$$\text{Direct stress} + \text{bending stress}$$

$$= \frac{W_E}{A} + \frac{W_E \times a}{Z}.$$

Writing $\frac{I}{y}$ for Z,

$$\text{Maximum compressive stress} = \frac{W_E}{A} + \frac{W_E a y}{I}.$$

If r be the radius of gyration of the section for the given axis, $r = \sqrt{\frac{I}{A}}$ or $I = Ar^2$.

$$\therefore \text{Maximum compressive stress} = \frac{W_E}{A} + \frac{W_E a y}{Ar^2}$$

$$= \frac{W_E}{A}\left(1 + \frac{ay}{r^2}\right)$$

Let W_C = the concentric load which would produce, as a uniform stress all over the section, a stress intensity equal to the maximum actually created by the eccentric load.

$$\frac{W_C}{A} = \frac{W_E}{A}\left(1 + \frac{ay}{r^2}\right)$$

$$\therefore W_C = W_E\left(1 + \frac{ay}{r^2}\right)$$

Equivalent concentric load = Actual eccentric load ×

$$\left(1 + \frac{ay}{r^2}\right)$$

A more convenient form for the purpose of taking advantage of the allowances permitted in the L.C.C. By-laws (Nos. 87 and 88 (see later)) and the B.S.S. is:

Equivalent concentric load

= [Actual eccentric load + (eccentric load × $\frac{ay}{r^2}$)]

The portion $\frac{ay}{r^2}$ simply depends for its value on the properties of the column section and the 'arm of eccentricity.' It may therefore be evaluated for a given standard section in terms of the 'eccentric arm.' The symbols *ex* and *ey* are often used instead of *a* and they denote respectively *eccentricity with respect to axis XX* and *with respect to axis YY*. The phrase *with respect to* means in this connection *at right angles to*. The expression $\frac{ay}{r^2}$ is termed an *eccentricity coefficient*. Values of these coefficients will be found on page 225. The coefficient for the YY axis is tabulated in blacker type, as this corresponds to axis for least *r*.

EXAMPLE. *Fig.* 163 *shows a* 10″ × 8″ × 55 *lb. joist column carrying* (*a*) *a concentric load,* (*b*) *an eccentric load, with an eccentricity of* 6″ *with respect to the XX axis, and* (*c*) *an eccentric load having* 2″ *eccentricity with respect to the YY axis. Assuming the column to have an effective height of* 11′, *calculate the maximum value of W in each case.* (*No allowance is to be made in this example for the increased permissible stress referred to in L.C.C. By-law No.* 87.)

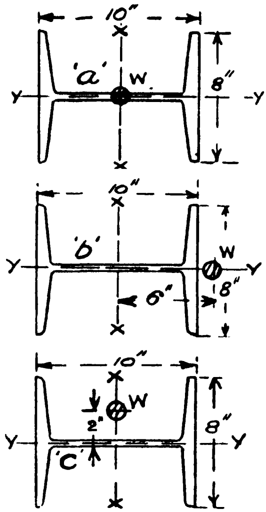


FIG. 163.

(a) Least r for this section $= 1{\cdot}84''$.

$$\frac{l}{r} = \frac{11 \times 12}{1{\cdot}84} = 71{\cdot}74.$$

For $\frac{l}{r} = 70$, $F_1 = 5{\cdot}4$ tons/in.^{2}.

For $\frac{l}{r} = 80$, $F_1 = 4{\cdot}9$ tons/in.^{2}.

For $\frac{l}{r} = 71{\cdot}74$, F_1 by interpolation $= (5{\cdot}4 - {\cdot}05 \times 1{\cdot}74)$ tons/in.^{2}

$= 5{\cdot}313$ tons/in.^{2}.

Area of section of column $= 16{\cdot}2$ in.^{2}.

$\therefore$ Safe concentric load $= (5{\cdot}313 \times 16{\cdot}2)$ tons.

$W = 86$ tons.

(b) (i) By first principles.

$$\text{Direct stress} = \frac{W}{16{\cdot}2} \text{ tons/in.}^2.$$

$$\text{Bending stress} = \frac{W \times 6''}{Z} = \frac{W \times 6}{57{\cdot}6} \text{ tons/in.}^2.$$

$$\therefore \frac{W}{16{\cdot}2} + \frac{6W}{57{\cdot}6} = 5{\cdot}313.$$

$$W = 32 \text{ tons.}$$

(ii) By tabular eccentricity coefficient. The value for the coefficient given on page 227 is $\cdot 28\ ex$.

$$\cdot 28ex = \cdot 28 \times 6'' = 1\cdot 68.$$

$$\therefore W + W \times 1\cdot 68 = 86 \text{ tons (from part } (a)).$$

$$W = \frac{86 \text{ tons}}{2\cdot 68} = 32 \text{ tons (as before).}$$

(c) (i) By first principles.

$$\text{Direct stress} = \frac{W}{16\cdot 2} \text{ tons/in.}^2.$$

$$\text{Bending stress} = \frac{W \times 2}{13\cdot 7} \text{ tons/in.}^2.$$

$$\therefore \frac{W}{16\cdot 2} + \frac{2W}{13\cdot 7} = 5\cdot 313.$$

$$W = 25\cdot 6 \text{ tons.}$$

(ii) By eccentricity coefficient.

$$\text{Coefficient} = 1\cdot 18 \times 2 = 2\cdot 36.$$

$$\therefore 1 + \text{eccentricity coefficient} = 1 + 2\cdot 36 = 3\cdot 36.$$

$$\therefore W = \frac{86}{3\cdot 36} = 25\cdot 6 \text{ tons.}$$

The reader will note that the value $5\cdot 313$ tons/in.^{2} must not be exceeded for the maximum stress whether the eccentricity is with respect to the XX axis or to the YY axis.

Continuing Columns with Eccentric Loads.—The problem of eccentric loading becomes a little more complicated when an allowance is claimed on the tabular column stress (as per Clause 87, L.C.C. By-laws). Further complication arises when the bending moment due to eccentricity is divided between the upper and lower lengths of a continuing column, as referred to in the regs., Clause 88. The latter clause reads thus :

In cases where a beam is connected to a continuing column, the bending moment in the column due to the eccentricity of the reaction from the beam may be regarded as divided between the column lengths above and below the level of the beam proportionately to their stiffnesses, account being taken of all bending moments or shearing forces at any joint.

'Stiffness' is obtained by dividing the appropriate moment of inertia of the column section by the equivalent length of the column.

$$\text{Stiffness} = \frac{I}{l}.$$

In practice, unless there is a big difference in sectional properties or length between the two column lengths concerned, it is common to take *half* the eccentric B.M. for the lower length.

L.C.C. By-law No. 89 is also important :

In a continuing column all bending moments due to eccentricities of loading at any one floor level may be considered as entirely dissipated at the levels of the floor beams immediately above and below, provided that the column at these latter levels is effectively restrained in the direction of the eccentricity.

Double Eccentricity.—When a load is eccentric with respect to both principal axes of a column section, the B.M. about each axis must be taken. The reduction in bending moment referred to in a previous paragraph may be applied to each of these B.M. values. If there are several eccentric loads, the sum of the loads is taken to compute the direct stress, and the resultant B.M. is calculated for each of the principal axes separately. The two compressive stresses corresponding to these B.M.s are then added to the direct stress.

Example Illustrating the Practical Stress Allowances

Fig. 164 *shows the proposed section of the middle length of a column continuous through three floors of a building. The upper*

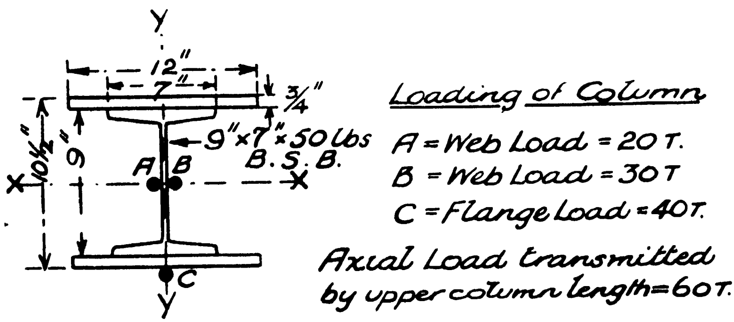


FIG. 164.—EXAMPLE OF CONTINUING COLUMN.

length of the column is 15′ *long and the middle length* 16′. *The loads indicated are transmitted by floor beams and by the column above. The upper length of the column has* 12″ × ½″ *plates in place of the* 12″ × ¾″ *plates shown. Test the suitability of the section.*

Note.—When a beam is connected by the usual form of connection to the face of the flange, or to the web of a column, the point of application of the load, for the purposes of calculation of eccentric bending moment, is sometimes assumed to be from $\frac{1}{2}''$ to $2''$ from the face. A distance of $2''$ will be taken in the present example.

$$ex = 5\tfrac{1}{4}'' + 2'' = 7{\cdot}25''.$$

ey, for both loads, $= 2'' + \frac{1}{2}$ web thickness

$$= 2'' + \frac{\cdot 40''}{2} = 2{\cdot}20''.$$

Equivalent lengths (for 3-way connections):

Upper length of column $= \cdot 875 \times 15 \times 12 = 157{\cdot}5''$.

Middle length of column $= \cdot 75 \times 16 \times 12 = 144''$.

Relative stiffnesses :

The I values will be obtained from the Z values given in table, page 229.

I_{XX} for upper column $= Z_{XX} \times \frac{1}{2}$ depth $= 95{\cdot}8 \times 5 = 479$ ins.^{4}.

I_{XX} for middle column $= 121{\cdot}3 \times 5{\cdot}25 = 637$ ins.^{4}.

Stiffness for upper column $= \dfrac{I}{l} = \dfrac{479}{157{\cdot}5} = 3{\cdot}04$.

Stiffness for middle column $= \dfrac{I}{l} = \dfrac{637}{144} = 4{\cdot}42$.

Sum of stiffnesses $= 7{\cdot}46$.

I_{YY} for upper column $= 30{\cdot}7 \times 6 = 184{\cdot}2$ ins.^{4}.

I_{YY} for middle column $= 42{\cdot}7 \times 6 = 256{\cdot}2$ ins.^{4}.

Stiffness for upper column $= \dfrac{I}{l} = \dfrac{184{\cdot}2}{157{\cdot}5} = 1{\cdot}17$.

Stiffness for middle column $= \dfrac{I}{l} = \dfrac{256{\cdot}2}{144} = 1{\cdot}78$.

Sum $= 2{\cdot}95$.

Loads carried by middle column length due to eccentricity alone:

XX axis: Eccentricity coefficient $= \cdot 27ex = \cdot 27 \times 7{\cdot}25 = 1{\cdot}96$.

$\therefore$ Load proportioned to middle length

$$= (40 \times 1{\cdot}96) \times \frac{4{\cdot}42}{7{\cdot}46} = 46{\cdot}5 \text{ tons.}$$

YY axis: Eccentricity coefficient $= \cdot 77\,ey = \cdot 77 \times 2{\cdot}2 = 1{\cdot}7$.

The unbalanced load causing B.M. is (30—20) tons = 10 tons at 2·2" eccentricity.

Equivalent concentric load proportioned to middle length

$$= (10 \times 1{\cdot}7) \times \frac{1{\cdot}78}{2{\cdot}95}$$
$$= 10{\cdot}3 \text{ tons.}$$

Total equivalent concentric load (due to eccentricity)

$$= (46{\cdot}5 + 10{\cdot}3) \text{ tons}$$
$$= 56{\cdot}8 \text{ tons.}$$

Total equivalent concentric load for middle length

$$= [(30 + 20 + 40 + 60) + (56{\cdot}8)] \text{ tons}$$
$$= 206{\cdot}8 \text{ tons.}$$

$$\text{Total corresponding stress} = \frac{206{\cdot}8 \text{ tons}}{\text{Sectional area}}$$
$$= \frac{206{\cdot}8}{32{\cdot}7} \text{ tons/in.}^2 = 6{\cdot}32 \text{ tons/in.}^2.$$

Permissible value of F_1:

$$\frac{l}{r} \text{ for this length of column} = \frac{144}{2{\cdot}8} = 51{\cdot}4.$$

By interpolation in the table of values for F_1, the value corresponding to 51·4 is 6·24 tons/in.². This is less than the value required, viz. 6·32 tons/in.², so we apply the formula of Clause 87 (L.C.C. By-laws):

$$F_2 = f_c + 7{\cdot}5\left(1 - \frac{f_c}{F_1}\right)\left(1 - {\cdot}002\frac{l}{r}\right)$$

$$f_c = \frac{\text{Total load}}{\text{Area of section}} = \frac{150}{32{\cdot}7} \text{ tons/in.}^2 = 4{\cdot}59 \text{ tons/in.}^2.$$

$$F_2 = 4{\cdot}59 + 7{\cdot}5\left(1 - \frac{4{\cdot}59}{6{\cdot}24}\right)\left(1 - {\cdot}002 \times 51{\cdot}4\right)$$
$$= (4{\cdot}59 + 1{\cdot}78) \text{ tons/in.}^2.$$
$$= 6{\cdot}37 \text{ tons/in.}^2.$$

As this revised stress exceeds 6·32 tons/in.² the section chosen for the middle length of the column is suitable.

Joints in Column Lengths

Clause 72, L.C.C. By-laws (1938), reads as follows:

All joints in steel columns shall occur as near as reasonably practicable to floor levels. Joints in columns where bending

actions produce tensile stresses shall be properly spliced to resist such bending actions in accordance with these by-laws. Column joints in which the resultant stress due to all loads and bending moments is wholly compressive shall be sufficiently spliced to retain the members accurately in place, provided that the length of each splice plate, on each side of the joint, shall be at least equal to the maximum breadth of the column flange, or at least 12 *inches, whichever is the greater.*

EXAMPLE. *A compressive load of* 55 *tons is transmitted through a column joint (Fig.* 165). *The upper length is composed*

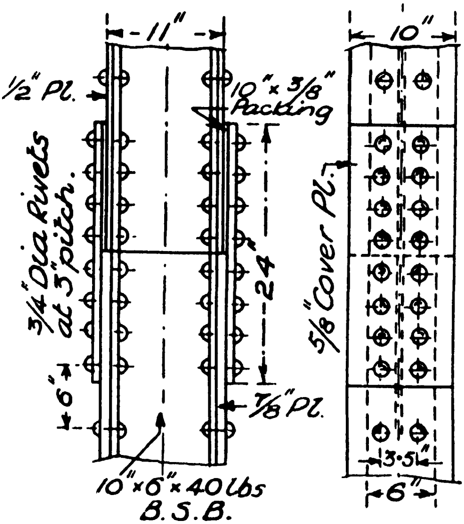


FIG. 165.—COLUMN JOINT.

of a 10" × 6" × 40 *lb. steel joist with a* 10" × ½" *plate on each flange. The lower length has the same standard section, but has a* 10" × ⅞" *plate on each flange. Design a suitable joint.*

The value of one ¾" diameter rivet in single shear at 5 tons/in.² = 2·21 tons. It is common to design the riveting in single shear to take 60% of the load transmitted. The ends of all column lengths have to be machined dead square to form a close butted joint.

60% of 55 tons = 33 tons.

Total number of rivets required $= \dfrac{33}{2 \cdot 21} = 15.$

A convenient number will be 16, giving 8 on each flange. The 6″ pitch usual for this column section will be lessened to 3″, to reduce length of cover. ⅜″ packing will be required, owing to the reduction in depth of column section.

Web angle cleats, with a division plate, are used when the joist section is not the same above and below the joint.

Web splice plates are employed when the joint is subjected to bending, and these, in conjunction with the flange covers, must be capable of resisting the applied moment.

Wind Forces on Stanchions

The following details from the L.C.C. By-laws (1938) have to be borne in mind when dealing with the subject of wind force.

1. *All buildings other than those indicated below shall be so designed as to resist safely a wind pressure in any horizontal direction of not less than 15 lb. per square foot upon the upper two-thirds of the vertical projection of the surface of such buildings, with an additional pressure of 10 lb. per square foot upon all projections above the general roof level.*

2. *If the height of a building is less than twice its width, wind pressure may be neglected, provided that the building is stiffened by floors, etc., so as to transmit the wind loads to the earth.*

3. *The working loads and stresses . . . for beams, columns and their connections as computed for all loads and forces other than wind pressure may be increased by 33⅓% in cases where such increase is solely due to stresses induced by wind pressure, provided that such increase shall not apply to the stresses given for grillage beams and filler joists.*

The reasons for the exemption of the buildings typified in Clause 2 are: (*a*) In a building which is wide in comparison with its height, the tendency to overturn due to the horizontal wind force is very small, and (*b*) the additional stresses induced by wind pressure are easily covered by the increase in working stresses allowed in Clause 3.

In high, narrow buildings, however, there is a considerable tendency to overturn, and we will consider the effect of wind pressure on a building of this type.

General Assumptions.—In the treatment of wind pressure given, the following assumptions will be made :

(1) The effect of the wind is to bend the stanchions and beams as shown in Fig. 166, giving points of contraflexure, or points where no B.M. occurs in the stanchions, at A and B, which

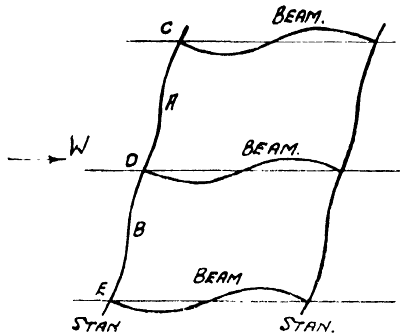


FIG. 166.

are in the centres of the floor heights, the maximum B.M. in the stanchion occurring at the floor levels.

(2) The wind force on any floor height, or panel, say CD, is assumed to act at the floor levels C and D, giving forces at these points which are equal, and, therefore, each equal to half the wind load on the floor height under consideration.

(3) As the points of contraflexure occur at the centre of the floor height, these positions are assumed to give the positions of maximum shear force, or reaction to the wind force, and the total load above any floor level, say C, is assumed to be resisted by a horizontal force at the point of contraflexure below that floor level—in this case at A. The bending moments at C and D due to this force at A are given by force at A multiplied by the distance AC or AD (which are each equal to half the floor height).

(4) If there are a number of stanchions in the width of the building, each of these stanchions is assumed to take its share of the total wind force on them, provided that they are all equally capable of resisting this force, i.e. that they all present the same axis to the windward side. If, out of several stanchions, one or more present the 'weak' axis to the wind-

ward side, they may be neglected, and the wind force divided equally between those stanchions which present their 'strong' axis. This assumption only holds good if the beams connecting the stanchions are strong enough, both in themselves and in their connections to the stanchions, to transmit the moments that occur at the floor levels. In this connection it is essential to bear in mind that the moment to be transmitted by any beam is equal to the sum of the moments above and below the point of connection to the stanchion.

(*N.B.*—Strictly, the respective capability of the stanchions to resist the wind force will be in direct ratio with the I of the stanchion about the axis at right angles to the direction of the wind force, but for practical purposes the assumption made above may be used.)

(5) The effect of the wind pressure on the vertical loads in the stanchions is as follows.

The *stress* on the stanchions will vary in proportion to their distances from the centre of gravity of the stanchions as a whole, i.e. the farther a stanchion is from the centre of gravity, the more heavily will it be stressed. The vertical load in each stanchion is, of course, the area of the stanchion multiplied by the appropriate stress.

The vertical loads in the stanchions to the leeward of the centre of gravity will be increased, while the loads in the stanchions to windward will be reduced. These reductions are called *uplifts*. The total uplift must, of course, be equal to the total increase in downward load.

The downward load on the base of any leeward stanchion must be added to the base load when designing the foundation, and the dead load on any stanchion must be greater than the maximum uplift.

EXAMPLE. In the worked example, a building one bay wide only is considered (Fig. 167).

The figures, showing the horizontal forces at the floor levels, are the actual forces due to wind; the figures at the points of contraflexure are the horizontal resistances supplied by the stanchions at these points; the figures shown with no units at the points of connection of the beams to the stanchions are the

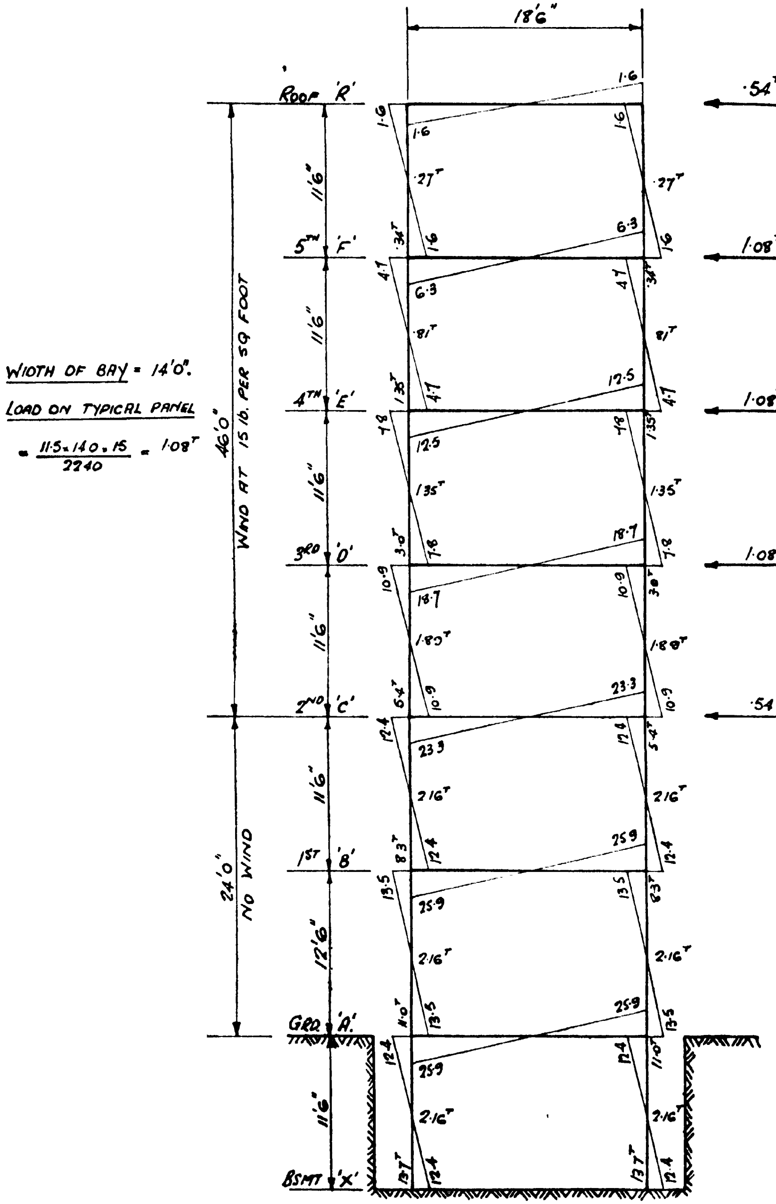
moments in foot tons in the various members of the structure, and the figures at these points shown thus—$8{\cdot}3^T$—give the downward loads, or the uplifts respectively at these points.

Maximum Stress in Stanchion.—The maximum stress in any length of the stanchion is determined by the sum of the direct stresses and the bending stresses. The direct stresses are those due to loading from the beams and also due to the increase in vertical load owing to wind pressure, while the bending stresses are those caused by eccentric loading from beams and by wind pressure. The bending stress due to wind pressure is obtained by dividing the bending moment in the particular length of stanchion by the appropriate section modulus.

The beams connecting the stanchions must be capable of resisting—in addition to the B.M. due to dead and superimposed loads—the B.M. due to wind loading, which, as previously explained, is the sum of the B.M.s in the lengths of stanchions respectively above and below the floor level under consideration.

(*N.B.*—The maximum B.M. due to dead and superimposed loads will occur generally near the centre of the beam, while the maximum B.M. due to wind occurs at the ends, and if these two quantities are simply added together a total B.M. will be obtained which is greatly in excess of the actual maximum B.M. The actual maximum B.M. should be obtained by superimposing the two B.M. diagrams one on the other.)

Connection of Beam to Stanchion.—The connection of any given beam to a stanchion must be capable of transmitting the total B.M. taken by the beam. In order to design such connections, the resistance moment of a connection is considered to be that of a couple, the forces in the couple being provided by the strength of the rivets or bolts in the top and bottom cleats, and the lever arm of the couple being taken as the depth of the beam. The web connections, or side cleats, do not contribute much to the resisting of the bending moment and may, especially in the case of the more shallow sections, be neglected. The number of rivets connecting the cleats to the stanchion should be equal to the number of rivets connecting the cleats to the beam.



WIND LOADS ON STANCHIONS.

FIG. 167.

B.M. in Stanchions.	*B.M. in Beams.*	*Vertical Loads in Stanchions.*
	1·6 ft.T.	
$\cdot 27 \times 5\cdot 75 = 1\cdot 6$ ft.T.		
	6·3 ft.T.	$\frac{1\cdot 08 \times 5\cdot 75}{18\cdot 5} = \cdot 34$ T.
$\cdot 81 \times 5\cdot 75 = 4\cdot 7$ ft.T.		
	12·5 ft.T.	$\frac{2\cdot 16 \times 11\cdot 5}{18\cdot 5} = 1\cdot 35$T.
$1\cdot 35 \times 5\cdot 75 = 7\cdot 8$ ft.T.		
	18·7 ft.T.	$\frac{3\cdot 24 \times 17\cdot 25}{18\cdot 5} = 3\cdot 0$T.
$1\cdot 89 \times 5\cdot 75 = 10\cdot 9$ ft.T.		
	23·3 ft.T.	$\frac{4\cdot 32 \times 23\cdot 0}{18\cdot 5} = 5\cdot 4$T.
$2\cdot 16 \times 5\cdot 75 = 12\cdot 4$ ft.T.		
	25·9 ft.T.	$\frac{4\cdot 32 \times 34\cdot 5}{18\cdot 5} = 8\cdot 3$T.
$2\cdot 16 \times 6\cdot 25 = 13\cdot 5$ ft.T.		
	25·9 ft.T.	$\frac{4\cdot 32 \times 47\cdot 0}{18\cdot 5} = 11\cdot 0$T.
$2\cdot 16 \times 5\cdot 75 = 12\cdot 4$ ft.T.		
		$\frac{4\cdot 32 \times 58\cdot 5}{18\cdot 5} = 13\cdot 7$T.

Exercises 11

(1) An angle iron strut in a truss has to carry an axial load of 12 tons. The length of the strut is 8′ and its ends are fixed. Using Rankine's formula and the usual constants (with a factor of safety of 4) show that a B.S.E.A., $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{5}{8}''$, would be suitable.

(Properties : Least radius of gyration = ·68″.
Area of section = 3·985 in.^{2}.

(2) Calculate the value of least r in each of the following cases :

(*a*) Rectangular section $3'' \times 9''$.
(*b*) Solid circular section 6″ diameter.
(*c*) $8'' \times 6'' \times 35$ lb. joist column. $Z_{YY} = 6{\cdot}51$ ins.^{3}.
Area = 10·3 ins.^{2}
(*d*) $10'' \times 10''$ compound column, composed of one steel joist $8'' \times 6'' \times 35$ lb. with plates on each flange to form $10'' \times 1''$.

(3) Using the formulæ of the London Building Act, calculate the safe concentric load for a $12'' \times 8'' \times 65$ lb. joist column for a height of 12′, the end fixture being 'one end hinged, one end fixed.' (Look up the necessary section properties.)

(4) For a $9'' \times 9''$ compound column (*K*302, page 228) the tables give a safe concentric load of 60·3 tons, for an effective height of 18′. Using the tabular working stresses of the L.C.C By-laws given on page 202, check this load.

(5) A column continuing through two storeys has for its bottom length a $9'' \times 10''$ compound column composed of an $8'' \times 6'' \times 35$ lb. B.S.B. with a $10'' \times \frac{1}{2}''$ plate on each flange. The connections to the column are two-way. Using the recommendations for effective length given on page 204, find the safe reduced concentric load for the bottom length of the column, the distance from floor level to floor level being 18′.

(Look up the necessary section properties and L.C.C. By-law working stresses.)

(6) A joist column, $18'' \times 6'' \times 55$ lb., carries a load of 24 tons at an eccentricity of 6″ with respect to the XX axis. Calculate the equivalent concentric load (*a*) by addition of direct and bending stresses, (*b*) by using the tabular eccentricity coefficient,

viz. $\cdot 17\ ex$. Draw a diagram showing the stress distribution across the section.

(Properties of 18″ × 6″ B.S.B. section: $Z_{XX} = 93 \cdot 5$ ins.^{3}. Sectional area $= 16 \cdot 2$ ins.^{2}.)

(7) Fig. 168 shows a 10″ × 6″ × 40 lb. joist column (bottom length) carrying an axial load of 12 tons, two balanced loads of

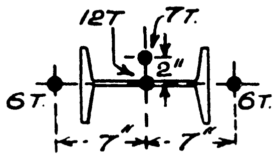


FIG. 168.

6 tons each and a load of 7 tons at 2″ eccentricity with respect to the YY axis. The column is continuous through two floors, the section remaining the same throughout, and the effective lengths for both floors being 144″. The least radius of gyration for this section is 1·36 ins. and the eccentricity coefficient for the YY axis is $1 \cdot 62\ ey$. Area of section $= 11 \cdot 80$ ins.^{2}. Give calculations to show that this section is suitable if the usual eccentricity allowances are taken advantage of.

(Use L.C.C. By-laws working stresses.)

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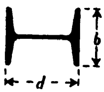

British Standard Sections (Revised 1932)

JOIST COLUMNS

Safe Loads

Reference Mark	Size $d \times b$ inches	SAFE CONCENTRIC LOADS IN TONS FOR EFFECTIVE HEIGHTS IN FEET													
		6	7	8	9	10	11	12	13	14	15	16	18	20	22
j140	24×7½	177	169	159	148	136	124	112	100	90·8	81·8	73·8	60·6		
j139	22×7	136	128	118	108	97·6	87·2	77·7	69·2	61·7	55·1	49·5			
j138	20×7½	168	160	151	142	131	120	109	98·9	89·3	80·6	71·9	60·0		
j137	20×6½	116	109	100	90·7	81·1	72·0	63·9	56·6	50·4	44·9	40·2			
j136	18×8	154	149	142	135	127	118	109	101	92·3	84·2	76·9	64·1	53·9	
j135	18×7	139	132	123	114	104	94·4	84·9	76·1	68·2	61·3	55·2	45·2		
j134	18×6	95·6	88·0	79·5	70·6	62·2	54·6	48·0	42·2	37·3	33·2				
j133	16×8	145	140	135	128	121	113	105	97·2	89·0	81·5	74·5	62·3	52·5	44·6
j132	16×6_H	108	99·5	90·1	80·3	70·8	62·2	54·7	48·1	42·6	37·9				
j131	16×6_L	87·8	81·2	73·8	66·0	58·4	51·4	45·3	39·9	35·3	31·4				
j130	15×6	78·8	72·7	66·0	58·9	52·0	45·8	40·2	35·5	31·4	27·9				
j129	15×5	64·6	56·4	48·3	41·1	35·0	30·0	25·9							
j128	14×8	136	132	127	121	114	107	100	92·8	85·5	78·4	71·8	60·2	50·9	43·3
j127	14×6_H	101	94·8	86·9	78·4	69·9	62·0	54·8	48·5	43·1	38·4	34·4			
j126	14×6_L	81·7	75·7	69·0	62·0	55·1	48·6	42·8	37·8	33·6	29·8				
j125	13×5	55·7	49·3	42·8	36·7	31·5	27·1	23·5							
j124	12×8	127	123	119	113	108	102	95·5	88·7	81·9	75·4	69·2	58·3	49·4	42·1
j123	12×6_H	97·5	91·2	84·1	76·4	68·6	61·1	54·2	48·1	42·9	38·3	34·3			
j122	12×6_L	79·1	73·8	67·7	61·2	54·6	48·5	42·9	38·0	33·8	30·1	27·0			
j121	12×5	50·4	44·4	38·3	32·8	28·1	24·1	20·8							

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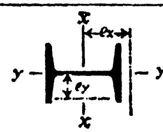
Dimensions and Properties



Size $d \times b$ inches	Weigh per foot in pounds	Area in square inches	Standard Thicknesses		Radii of Gyration		Moduli of Section		Eccentricity Coefficients	
			Web	Flange	Axis y—y	Axis x—x	Axis y—y	Axis x—x	Axis y—y	Axis x—x
24×7½	95	27·9	·57	1·011	1·50	9·52	16·7	211·1	1·68ey	0·13ex
22×7	75	22·1	·50	·834	1·36	8·72	11·7	152·4	1·88ey	0·14ex
20×7½	89	26·2	·60	1·010	1·55	7·99	16·7	167·3	1·57ey	0·16ex
20×6½	65	19·1	·45	·820	1·31	8·01	10·0	122·6	1·91ey	0·16ex
18×8	80	23·5	·50	·950	1·72	7·41	17·4	143·6	1·36ey	0·16ex
18×7	75	22·1	·55	·928	1·45	7·22	13·3	127·9	1·66ey	0·17ex
18×6	55	16·2	·42	·757	1·21	7·21	7·9	93·5	2·05ey	0·17ex
16×8	75	22·1	·48	·938	1·76	6·64	17·1	121·7	1·29ey	0·18ex
16×6	62	18·2	·55	·847	1·22	6·31	9·1	90·6	2·01ey	0·20ex
16×6	50	14·7	·40	·726	1·24	6·48	7·5	77·3	1·96ey	0·19ex
15×6	45	13·2	·38	·655	1·23	6·10	6·6	65·6	2·00ey	0·20ex
15×5	42	12·4	·42	·647	·98	5·89	4·7	57·1	2·62ey	0·22ex
14×8	70	20·6	·46	·920	1·80	5·85	16·7	100·8	1·24ey	0·20ex
14×6	57	16·8	·50	·873	1·29	5·64	9·3	76·2	1·80ey	0·22ex
14×6	46	13·6	·40	·698	1·26	5·71	7·2	63·2	1·90ey	0·22ex
13×5	35	10·3	·35	·604	1·03	5·25	4·3	43·6	2·38ey	0·24ex
12×8	65	19·1	·43	·904	1·85	5·05	16·3	81·3	1·17ey	0·24ex
12×6	54	15·9	·50	·883	1·33	4·86	9·4	62·6	1·69ey	0·25ex
12×6	44	13·0	·40	·717	1·30	4·94	7·4	52·8	1·76ey	0·25ex
12×5	32	9·5	·35	·550	1·01	4·84	3·9	36·8	2·44ey	0·26ex

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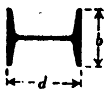

British Standard Sections (Revised 1932)

JOIST COLUMNS

Safe Loads

Reference Mark	Size d × b inches	SAFE CONCENTRIC LOADS IN TONS FOR EFFECTIVE HEIGHTS IN FEET													
		3	4	5	6	7	8	9	10	12	14	16	18	20	22
j120	10 × 8	116	113	110	107	104	100	96·2	91·3	80·4	68·9	58·2	49·0	41·4	35·3
j119	10 × 6	82·5	79·7	76·5	72·8	68·3	63·3	57·7	52·1	41·4	32·9	26·4			
j118	10 × 5	60·2	57·0	53·2	48·5	43·1	37·6	32·4	27·9	20·8					
j117	10 × 4½	49·1	45·9	42·0	37·2	32·0	27·2	23·0	19·4						
j116	9 × 7	104	102	99·2	95·9	92·1	87·7	82·8	77·5	65·8	54·7	45·1	37·4	31·3	
j115	9 × 4	40·2	36·8	32·4	27·5	22·8	18·7	15·5	13·0						
j114	8 × 6	72·3	69·9	67·2	64·0	60·2	55·9	51·1	46·2	37·0	29·5	23·7			
j113	8 × 5	56·7	54·1	50·8	46·8	42·3	37·4	32·6	28·3	21·4					
j112	8 × 4	34·4	31·3	27·5	23·2	19·2	15·8	13·1	10·9						
j111	7 × 4	31·0	28·5	25·3	21·6	18·0	14·9	12·4	10·4						
j110	6 × 5	50·5	48·1	45·2	41·7	37·6	33·3	29·0	25·2	19·0					
j109	6 × 4½	39·5	37·0	34·0	30·3	26·3	22·4	19·0	16·1	11·9					
j108	6 × 3	21·3	18·1	14·5	11·4	8·9	7·1								
j107	5 × 4½	40·0	38·0	35·4	32·4	28·9	25·2	21·8	18·8	14·0					
j106	5 × 3	20·0	17·3	14·1	11·2	8·9	7·1								
j105	4¾ × 1¾	7·5	4·9												
j104	4 × 3	18·0	15·6	12·7	10·1	8·0	6·4								
j103	4 × 1¾	5·6	3·6												
j102	3 × 3	15·7	13·8	11·5	9·2	7·3	5·9								
j101	3 × 1½	3·9	2·5												

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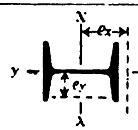

Size $d \times b$ inches	Weigh per foot in pounds	Area in square inches	Standard Thicknesses		Radii of Gyration		Moduli of Section		Eccentricity Coefficients	
			Web	Flange	Axis y—y	Axis x—x	Axis y—y	Axis x—x	Axis y—y	Axis x—x
10 × 8	55	16·2	·40	·783	1·84	4·22	13·7	57·6	1·18e_y	0·28e_x
10 × 6	40	11·8	·36	·709	1·36	4·17	7·3	41·0	1·62e_y	0·29e_x
10 × 5	30	8·9	·36	·552	1·05	4·06	3·9	29·3	2·28e_y	0·30e_x
10 × 4½	25	7·4	·30	·505	·94	4·08	2·9	24·5	2·55e_y	0·30e_x
9 × 7	50	14·7	·40	·825	1·65	3·76	11·5	46·3	1·28e_y	0·32e_x
9 × 4	21	6·2	·30	·457	·82	3·62	2·1	18·0	2·99e_y	0·34e_x
8 × 6	35	10·3	·35	·648	1·38	3·34	6·5	28·8	1·58e_y	0·36e_x
8 × 5	28	8·3	·35	·575	1·11	3·29	4·1	22·4	2·03e_y	0·37e_x
8 × 4	18	5·3	·28	·398	·81	3·24	1·8	13·9	3·03e_y	0·38e_x
7 × 4	16	4·8	·25	·387	·84	2·89	1·7	11·3	2·81e_y	0·42e_x
6 × 5	25	7·4	·41	·520	1·11	2·44	3·6	14·6	2·02e_y	0·51e_x
6 × 4½	20	5·9	·37	·431	·96	2·43	2·4	11·6	2·45e_y	0·51e_x
6 × 3	12	3·5	·23	·377	·64	2·44	·97	7·0	3·64e_y	0·50e_x
5 × 4½	20	5·9	·29	·513	1·06	2·06	2·9	10·0	2·01e_y	0·59e_x
5 × 3	11	3·3	·22	·376	·67	2·05	·97	5·5	3·36e_y	0·60e_x
4¾ × 1¾	6·5	1·9	·18	·325	·37	1·88	·30	2·8	6·37e_y	0·67e_x
4 × 3	10	2·9	·24	·347	·67	1·63	·88	3·9	3·34e_y	0·76e_x
4 × 1¾	5	1·5	·17	·239	·36	1·58	·21	1·8	7·00e_y	0·80e_x
3 × 3	8·5	2·5	·20	·332	·70	1·23	·83	2·5	3·04e_y	0·99e_x
3 × 1½	4	1·2	·16	·249	·33	1·19	·17	1·1	6·94e_y	1·06e_x

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COMPOUND COLUMNS

Safe Loads

Reference Mark	Size $d \times b$ inches	SAFE CONCENTRIC LOADS IN TONS FOR EFFECTIVE HEIGHTS IN FEET													
		10	12	14	16	18	20	22	24	26	28	30	32	34	36
*k*332	12 × 12	337	324	309	291	272	250	229	207	187	169	152	137	124	113
*k*331	11½ × 12	296	284	270	254	236	217	197	178	160	144	130	117	106	96·6
*k*330	11 × 12	255	244	231	217	200	183	166	149	134	120	108	97·4	88·0	79·8
*k*329	10¾ × 12	234	224	212	198	182	166	150	134	120	108	96·9	87·3	78·8	
*k*328	10½ × 12	214	204	192	179	164	149	134	119	107	95·6	85·7	77·0	69·5	
*k*327	10¼ × 12	193	183	172	159	146	131	117	104	93·3	83·2	74·4	66·7	60·1	
*k*326	10 × 12	172	163	152	140	126	113	101	89·5	79·4	70·5	63·0	56·5		
		Rivets ⅞-in. diameter													
*k*318	10 × 10	192	180	167	151	136	120	105	93·2	82·2	72·9	64·8			
*k*317	9¾ × 10	175	164	151	137	122	108	95·0	83·4	73·5	65·0	57·8			
*k*316	9½ × 10	159	148	136	122	109	95·8	83·9	73·6	64·7	57·2				
*k*315	9¼ × 10	142	132	120	108	95·4	83·4	72·8	63·7	55·9	49·3				
*k*314	9 × 10	125	115	104	93·1	81·4	70·8	61·5	53·7	47·0	41·4				
*k*313	8¾ × 10	107	98·8	88·4	77·6	67·2	58·0	50·1	43·5	38·0					
		Rivets ¾-in. diameter													
*k*304	9½ × 9	132	121	108	95·5	82·8	71·4	61·8	53·7	46·8					
*k*303	9¼ × 9	117	107	95·2	83·1	71·6	61·6	53·1	46·0	40·1					
*k*302	9 × 9	102	92·5	81·5	70·4	60·3	51·6	44·3	38·3						
*k*301	8¾ × 9	86·9	77·5	67·3	57·4	48·8	41·4	35·4	30·5						
		Rivets ⅝-in. diameter													

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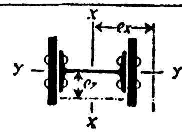
COMPOUND COLUMNS



Composition and Properties

Composed of: One Steel Joist	Composed of: Plates each Flange to form	Weight per foot in pounds	Area in square inches	Radii of Gyration: Axis y—y	Radii of Gyration: Axis x—x	Moduli of Section: Axis y—y	Moduli of Section: Axis x—x	Eccentricity Coefficients: Axis y—y	Eccentricity Coefficients: Axis x—x
9×7@50	12×1½	176	50·7	3·05	4·88	78·7	201·2	0·64e_y	0·25e_x
,,	12×1¼	156	44·7	2·99	4·73	66·7	173·9	0·67e_y	0·26e_x
,,	12×1	135½	38·7	2·91	4·57	54·7	147·3	0·71e_y	0·26e_x
,,	12× ⅞	125	35·7	2·86	4·49	48·7	134·2	0·73e_y	0·27e_x
,,	12× ¾	115	32·7	2·80	4·41	42·7	121·3	0·77e_y	0·27e_x
,,	12× ⅝	105	29·7	2·72	4·33	36·7	108·5	0·81e_y	0·27e_x
,,	12× ½	94½	26·7	2·63	4·24	30·7	95·8	0·87e_y	0·28e_x
8×6@35	10×1	105½	30·3	2·48	4·15	37·2	104·3	0·81e_y	0·29e_x
,,	10× ⅞	97	27·8	2·44	4·07	33·1	94·5	0·84e_y	0·29e_x
,,	10× ¾	88½	25·3	2·39	3·99	28·9	84·8	0·88e_y	0·30e_x
,,	10× ⅝	80	22·8	2·33	3·91	24·7	75·2	0·92e_y	0·30e_x
,,	10× ½	71½	20·3	2·25	3·82	20·6	65·8	0·99e_y	0·31e_x
,,	10× ⅜	63	17·8	2·15	3·72	16·4	56·4	1·08e_y	0·32e_x
8×5@28	9× ¾	76½	21·8	2·16	4·00	22·5	73·4	0·97e_y	0·30e_x
,,	9× ⅝	69	19·5	2·10	3·91	19·1	64·7	1·02e_y	0·30e_x
,,	9× ½	61	17·3	2·03	3·82	15·8	56·1	1·10e_y	0·31e_x
,,	9× ⅜	53½	15·0	1·93	3·72	12·4	47·6	1·21e_y	0·32e_x

of Messrs. Redpath, Brown & Co., Ltd.

CHAPTER XII

COLUMN BASES. STEEL GRILLAGES

Column Bases

THE detail required at the foot of a column is governed by clauses contained in the L.C.C. By-laws (1938) (Clauses 69 to 73, inclusive). The reader should consult these regulations; the following extracts from the regs. indicate the nature of the requirements therein. Clause 69 refers to steel columns, other than columns of solid round section, and reads as follows:

Except as provided in clause (b) of by-law 70, the foot of a steel column other than a column of solid round section, shall, after riveting up complete, be properly machined over the whole area of the foot so formed and shall have affixed thereto either:

(a) A base plate in effective contact with the whole area of the machined foot. The gusset plates, angles, cleats and stiffeners (if any) in combination with the bearing area of the machined column foot and the base plate shall be sufficient to distribute the load in accordance with these by-laws.

(b) A slab or bloom base-plate, in effective contact with the whole area of the machined column end.

When the load under the slab is uniformly distributed, the minimum thickness, in inches, of a rectangular slab shall be:

$$\sqrt{\frac{3W}{4f} \cdot \frac{(B-b)}{D}} \quad \text{or} \quad \sqrt{\frac{3W}{4f} \cdot \frac{(D-d)}{B}}$$

whichever is the greater, where

W is the total axial load in tons.

B is the length in inches of the slab measured at right angles to the web of the column.

b is the width in inches of the column measured at right angles to the web of the column.

D is the length in inches of the slab measured parallel to the web of the column.

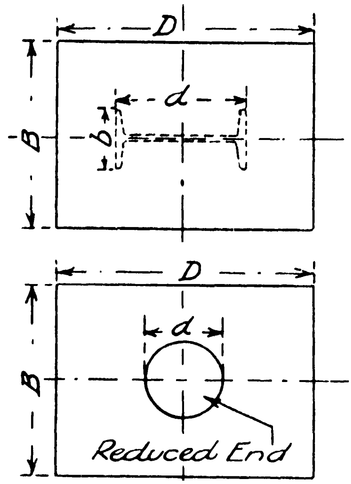


FIG. 169.—MILD STEEL SLAB BASE.

d is the width in inches of the column measured parallel to the web of the column.

f is the working stress in the steel taken at 9 *tons per square inch.*

When the load under the slab is not uniformly distributed, or where the slab is not rectangular, the specified limits of stress shall not be exceeded.

Clause 70 (*b*) refers to the ends of columns in one-storey buildings and reads: *Provided that, in buildings of one-storey, the column ends need not be machined in cases where sufficient gussets and rivets are provided to transmit the whole load to the foundations.*

For Solid Round Steel Columns the By-laws (Clause 73) state:

Solid round structural steel columns shall have properly machined shouldered ends and shall be provided with caps and bases, the bearing surfaces of which shall be properly machined after being shrunk or screwed on. When the load under the cap or base is

uniformly distributed, the minimum thickness, in inches, of a rectangular cap or base shall be :

$$\sqrt{\frac{3W}{4f} \cdot \frac{D}{B - d}}.$$

where

W is the total axial load in tons.

B is the length of the shorter side of cap or base in inches.

D is the length of the longer side of cap or base in inches.

d is the diameter of the reduced end of the column in inches.

f is the working stress in the steel taken as 9 *tons per square inch.*

When the load on the cap or under the base is not uniformly distributed, or where the cap or base is not rectangular, the specified limits of stress shall not be exceeded.

A cap or base plate shall not be less than 1·5 (d + 3) *inches in length 'B' or in diameter.*

Theoretical Consideration of Bases

The theory involved in the derivation of formulæ, such as those already quoted for slab base thicknesses, is similar in character to that usually considered to be applicable to the design of steel grillages (see later). In Fig. 170 X is a rectangular block of material symmetrically disposed relatively to another rectangular block Y, to which it transmits an axial load W. The supporting load under block Y is uniformly distributed. The usually accepted forms of the B.M. and S.F. diagrams for portion Y are as indicated in Fig. 173, and the value of the maximum B.M. for axis a-a will be given by

$$\left(\frac{W}{2} \times \frac{L}{4}\right) - \left(\frac{W}{2} \times \frac{l}{4}\right)$$

$$= \frac{WL}{8} - \frac{Wl}{8}$$

$$= \frac{W}{8}(L - l).$$

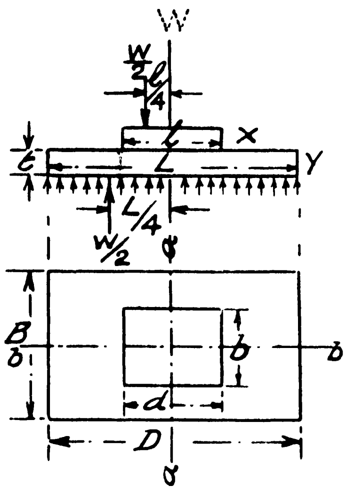


FIG. 170.

Applying the formula for the moment of resistance of a rectangular beam section to the slab Y:

$$M = \frac{fbd^2}{6}.$$

$$\frac{W}{8}(L - l) = \frac{fbd^2}{6}.$$

In this case b in the formula = B in figure, and d in formula is represented by t in figure. Also L becomes D and l is equivalent to d.

$$\therefore \frac{W}{8}(D - d) = \frac{fBt^2}{6}$$

$$\text{or } t = \sqrt{\frac{3W}{4f} \times \frac{D - d}{B}}.$$

If bending were considered about axis b-b we would similarly obtain the alternative formula

$$t = \sqrt{\frac{3W}{4f} \times \frac{B - b}{D}}.$$

B.S.S. No. 449–1937.—The formulæ for slab thicknesses given in the revised B.S.S. No. 449 are somewhat different from those already given. With the permission of the B.S.I.

these formulæ are given below. For further details the reader should consult the B.S.S. referred to.

Solid Round Steel Columns.—Clause 24 contains the following extract:

When it can be assumed that the load on the cap or under the base is uniformly distributed, the minimum thickness, in inches, of a square cap or base shall be

$$t = \sqrt{\frac{9W}{16f} \cdot \frac{D}{D - d}}, \textit{ where}$$

t is the thickness of the plate in inches.

W is the total axial load in tons.

D is the length of the side of cap or base in inches.

d is the diameter of the reduced end of the column in inches.

f is the working stress in the steel taken at 9 tons per sq. in. in the case of mild steel and 13·5 tons per sq. in. in the case of high tensile steel.

Clause 25 (ii) includes the following:

When it can be assumed that the slab distributes the loading uniformly, the minimum thickness, in inches, of a rectangular slab shall be:

$$t = \sqrt{\frac{3p}{f}\left(y^2 - \frac{y_1^2}{4}\right)}$$

where

t is the plate thickness in inches.

p is the pressure or loading on base in tons per sq. in.

f is the working stress in the steel taken at 9 tons per sq. in. in the case of mild steel and 13·5 tons per sq. in. in the case of high tensile steel.

y is the greater projection of plate over column in inches.

y_1 is the lesser projection of plate over column in inches.

EXAMPLE. *Using the L.C.C. By-law formula, calculate the necessary dimensions for a mild steel slab base to carry an axial load of 300 tons. The base rests on 1 : 2 : 4 concrete (30 tons per sq. ft. safe bearing pressure). The column size is 14″ × 12″.*

Necessary area of base $= \frac{300}{30}$ sq. ft. $=$ 10 sq. ft.

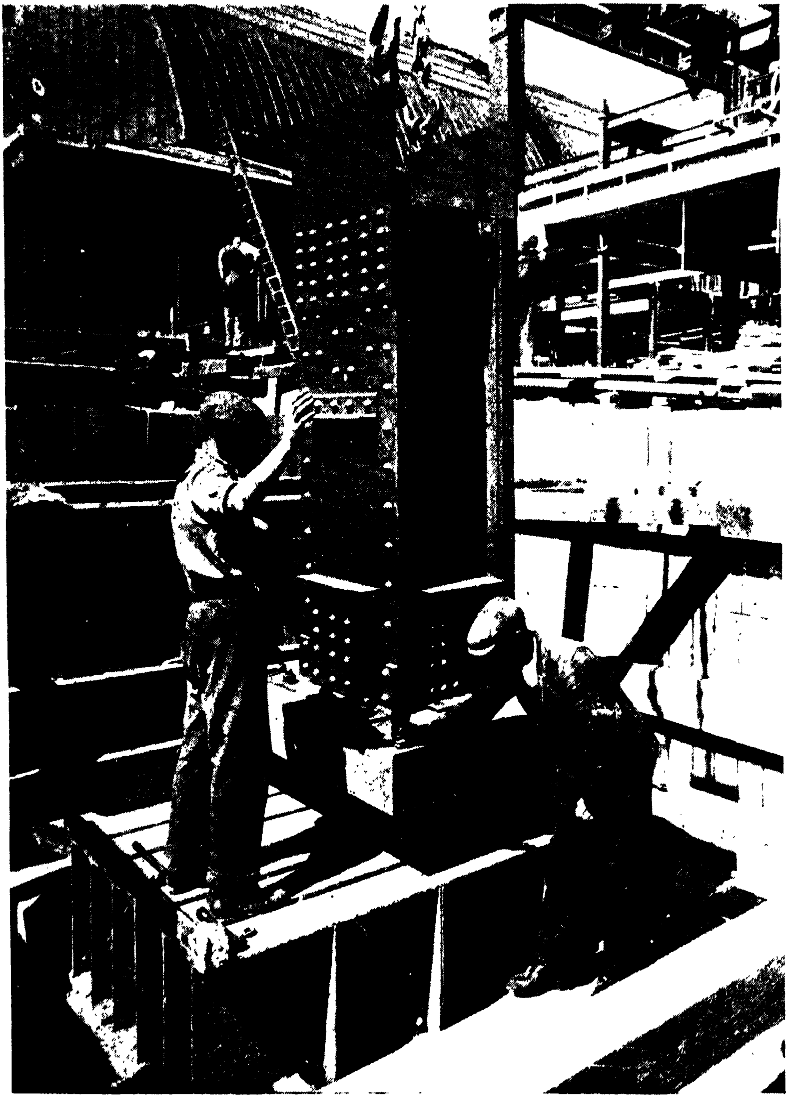


FIG. 171. - SLAB BASE AND GRILLAGE.
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A rectangular base 36″ × 40″ gives 10 sq. ft.

$$t = \sqrt{\frac{3W}{4f} \cdot \frac{B - b}{D}}.$$

Putting W = 300, f = 9, B = 36, b = 12, D = 40, d = 14

$$t = \sqrt{\frac{3 \times 300 \times 24}{4 \times 9 \times 40}} = 3{\cdot}9''.$$

Alternatively, $t = \sqrt{\frac{3W}{4f} \cdot \frac{D - d}{B}}$

$$= \sqrt{\frac{3 \times 300 \times 26}{4 \times 9 \times 36}} = 4\tfrac{1}{4}''.$$

By the B.S.S. formula, $t = \sqrt{\frac{3p}{f}\left(y^2 - \frac{y_1^2}{4}\right)}$

p = 30 tons per sq. ft. = $\frac{30}{144}$ tons per sq. in.

f = 9 tons/in.^{2}; $y = \frac{40'' - 14''}{2}$ = 13″ (i.e. the greater overhang); $y_1 = \frac{36'' - 12''}{2}$ = 12″ (i.e. the lesser overhang).

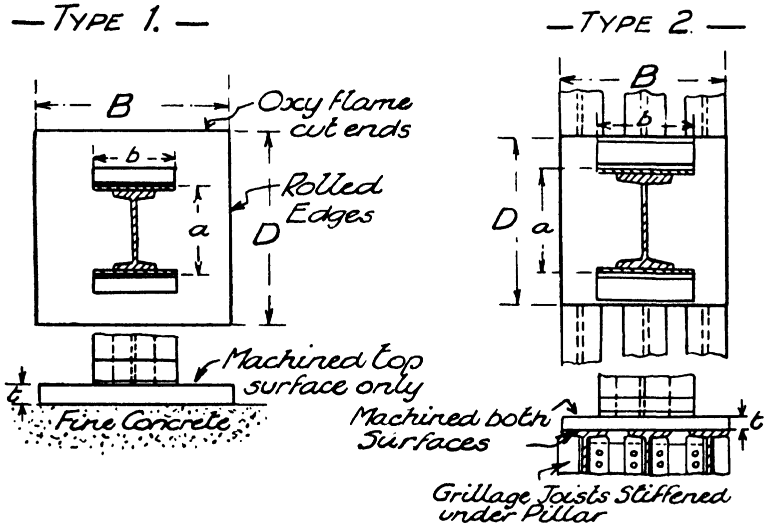
$$\therefore t = \sqrt{\frac{3 \times 30}{9 \times 144} \times \left(13^2 - \frac{12^2}{4}\right)} = \text{say } 3''.$$

The photograph of the slab base, and grillage, shown in Fig. 171, is given by the courtesy of Messrs. Dorman, Long & Co., Ltd., who have also supplied the table of mild steel slab thicknesses which appears on page 236.

Permissible Bearing Pressures on Subsoils.—As a guide to the permissible loads upon various subsoils, the L.C.C. regs. give the following values.

	Load on ground. Tons per square foot
Alluvial soil, made ground, very wet sand	½
Soft clay, wet or loose sand	1
Ordinary fairly dry clay, fairly dry fine sand, sandy clay	2
Firm dry clay	3
Compact sand or gravel, London blue or similar hard compact clay	4

MILD STEEL SLAB BASES.
Calculated in accordance with B.S.S. No 449-1937.



Column Load. Tons.	Assumed Column Size. $a \times b$ inches.	TYPE 1. Slab bearing direct on Fine Concrete at 40 tons per square foot. Size in inches. $D \times B \times t$.	Weight in lb.	TYPE 2. Slab bearing on Steel Grillages. Size in inches. $D \times B \times t$.	Weight in lb.
100	12 × 10	20 × 18 × 1¼	127		
150	12 × 10	24 × 23 × 1¾	274	19 × 15 × 1½	121
200	12 × 12	27 × 27 × 2	413	20 × 15 × 2	170
250	13 × 12	30 × 30 × 2½	637	21 × 16 × 2	190
300	14 × 12	33 × 33 × 3	926	22 × 18 × 2	224
350	15 × 14	36 × 35 × 3	1,071	24 × 24 × 2	326
400	16 × 14	39 × 37 × 3½	1,431	24 × 24 × 2½	408
450	18 × 16	42 × 39 × 3½	1,624	27 × 26 × 2½	497
500	18 × 16	43 × 42 × 3½	1,791	27 × 27 × 2½	516
600	19 × 20	45 × 48 × 4	2,448	27 × 29 × 2½	555
700	19 × 20	50 × 51 × 4½	3,251	27 × 29 × 2½	555
800	22 × 20	56 × 54 × 4½	3,855	30 × 29 × 2½	616

Compiled from data kindly supplied by Messrs. Dorman, Long and Co., Ltd.

B.S.S. 449.—*The above pressures may be exceeded by an amount equal to the weight of the material in which a foundation is bedded and which is displaced by the foundation itself, measured downward from the final finished lowest adjoining floor or ground level.*

The B.S.S. gives safe bearing pressures for concrete foundations for various mixes.

For 1 : 1 : 2 concrete, 40 tons per sq. ft. is given, and for 1 : 2 : 4, 30 tons per sq. ft.

Example of Steel Grillage Design.—*A column transmits a load of* 310 *tons to a steel grillage (which is composed of two tiers of steel beams) through a base plate* 2′ 6″ *square. The subsoil safe bearing pressure is* $2\frac{1}{2}$ *tons per sq. ft. Select suitable B.S.B.s for the grillage.*

Allowing 40 tons for the combined weight of grillage beams and concrete filling, the total load carried by the ground = 350 tons.

$$\text{Necessary area at bottom of grillage} = \frac{350}{2\cdot5} \text{ sq. ft.}$$
$$= 140 \text{ sq. ft.}$$

The grillage beams will be made 12′ long. The steel beams of a grillage are usually completely encased in concrete, and sufficient space must be left between the flange edges of adjacent beams to permit of the concrete completely filling in all internal spaces. A working stress of 12 tons/in.² is used in designing the steel beams, but certain requirements have to be fulfilled for this increased stress to be permissible. The L.C.C. By-laws require a minimum cover of 3″, but also state (Clause 82) that the foundation shall be composed of one solid block of concrete of constant horizontal cross-section throughout its whole depth.

The formula for the maximum bending moment, in a given tier of the grillage, is similar to that already given for the case of a steel slab base, viz.

$$\text{B.M. maximum} = \frac{W}{8}(L - l).$$

For the first tier of beams l = length of base plate of column (or steel slab), and L = the length of the beams in the first tier. In the case of the second tier, l = total width of beams in first

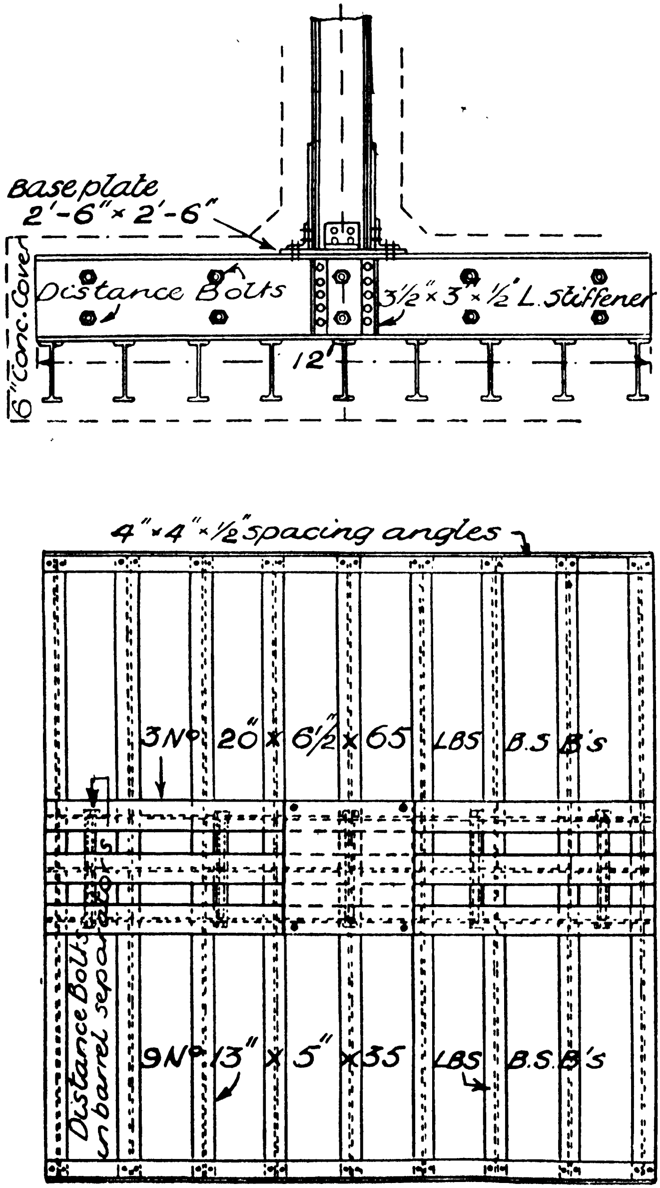


FIG. 172.—STEEL GRILLAGE

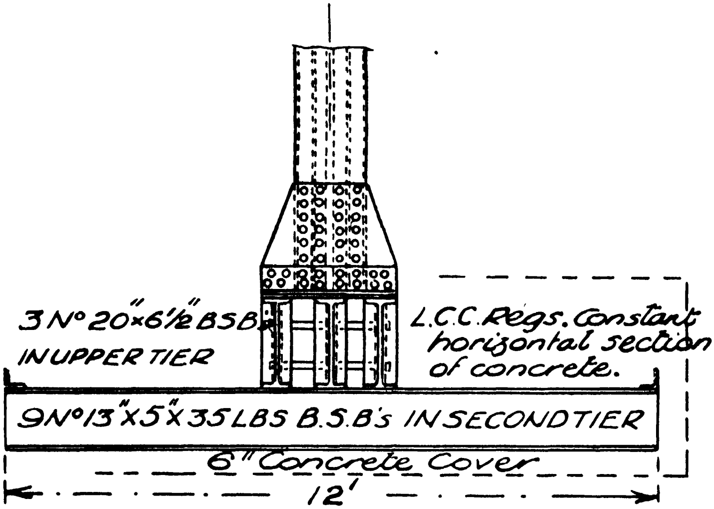


FIG. 172A.

tier and L = length of beams in second tier. For a square base plate (or steel slab) with a grillage, square in plan, the total maximum B.M. for the first and second tiers will be the same. The formula given assumes that the base plate conforms to the deflection of the beams beneath it, and it will be sufficiently accurate for steel grillage design in the usual case. Sometimes l is taken as the distance between the outer lines of rivets in the flange to base connections of the column.

In the example the load transmitted through the grillage = 310 tons.

$$\text{B.M. maximum} = \frac{W}{8}(L - l)$$

$$= \frac{310}{8}(144 - 30) \text{ tons ins.}$$

$$= 4415 \text{ tons ins.}$$

$$\text{Total section modulus required} = \frac{M}{f} = \frac{4415}{12} \text{ ins.}^3$$

$$= 367{\cdot}9 \text{ ins.}^3.$$

If there are three beams in the first tier:
Z for each beam = $122{\cdot}6$ ins.^{3}.

A 20″ × 6½″ × 65 lb. B.S.B. has this section modulus, hence it will be suitable from the point of view of flexural stress.

The webs of grillage beams should always be tested for shear stress. In calculating the area of web of a rolled beam, the full depth of the beam is taken.

Total shear area provided by 3 No. 20″ × 6½″ B.S.B.s $= (3 \times 20 \times \cdot 45)$ in.^{2} = 27 in.^{2}.

As shown in Fig. 173 the maximum shear force is at the edge of the base plate. Its value may be obtained from $\frac{W}{2}$ (= 155 tons) by proportion.

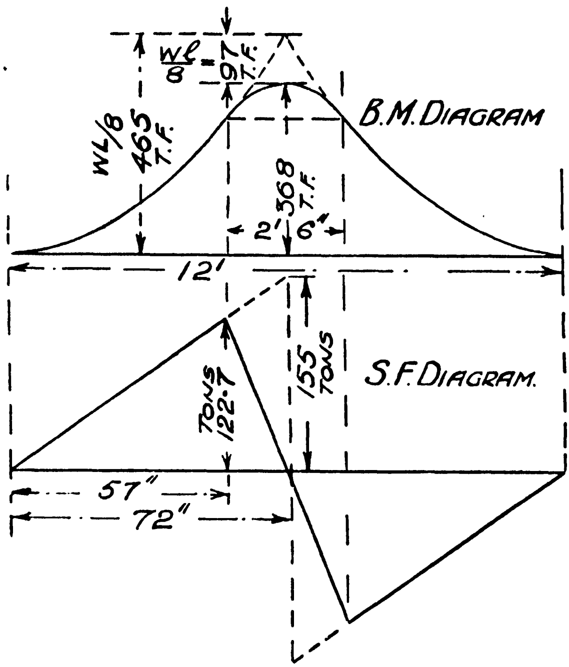


FIG. 173.—B.M. AND S.F. DIAGRAMS FOR GRILLAGE.

$$\text{Thus maximum S.F.} = \left(\frac{57}{72} \times 155\right) \text{tons}$$
$$= 122{\cdot}7 \text{ tons.}$$

$$\text{Maximum shear stress in webs} = \frac{122{\cdot}7}{27} \text{ tons/in.}^2$$
$$= 4{\cdot}55 \text{ tons/in.}^2$$

The beams in the upper tier should be provided with web stiffeners, machined to fit under the flanges, and placed, in the case of riveted construction, under the column flange gussets. Some form of distance pieces should also be provided, to prevent movement during concreting. End angles as shown in the second tier in Fig. 172 are commonly used for this purpose. For the second tier, 9 No. beams have been selected. In our example, the B.M. maximum is the same for the second as for the first tier, so that each beam must have a section modulus of at least $\frac{367 \cdot 9}{9}$ ins.^{3} = $40 \cdot 9$ ins.^{3}.

A $13'' \times 5'' \times 35$ lb. B.S.B. has a section modulus of $43 \cdot 62$ ins.^{3}, hence will be a suitable selection.

The total shear area provided $= (9 \times 13 \times \cdot 35)$ ins.^{2}

$$= 40 \cdot 95 \text{ ins.}^2.$$

$$\text{Shear stress in webs} = \frac{122 \cdot 7}{40 \cdot 95} \text{ tons/in.}^2 = 3 \text{ tons/in.}^2.$$

The distance between the flange edges in the first tier

$$= \frac{30'' - 3 \times 6\tfrac{1}{2}''}{2} = 5\tfrac{1}{4}''.$$

There is ample room for concreting in the second tier.

Webs, as already indicated, must be stiffened if liable to buckling. The calculations involved in buckling considerations are illustrated in the following example of a compound grillage.

Design and Detail of a Combined Grillage

The two stanchions to be carried have base loads of 181 tons and 113 tons, respectively, and are at 19′ 9″ centres (Fig. 174).

The first point to be remembered in dealing with combined

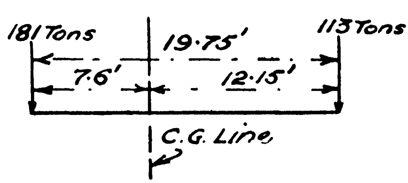


FIG. 174.

grillages is that the centre of gravity of the foundation must be in the same vertical line as the centre of gravity of the loads.

$$\text{C.G. of loads from } 181^{T} \text{ load} = \frac{113 \times 19\cdot75}{181 + 113} = 7\cdot6'.$$

The length of the grillage is determined by the least projection required (from a consideration of the connection of a stanchion base to the grillage).

This is usually about 18″, and, as the stanchion concerned is the one with the 113 tons load, the length of the grillage

$$= 2 \times (12\cdot15 + 1\cdot5) = 2 \times 13\cdot65' = 27\cdot3',$$

say 27′ 4″ (Fig. 175).

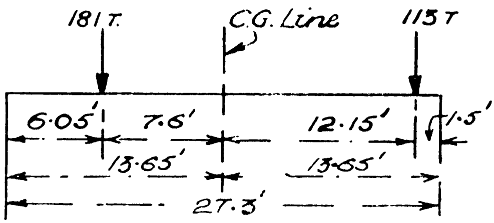


FIG. 175.

The grillage may be considered to be an inverted beam, loaded with a uniform load (the pressure under the grillage) and supported at two points (the stanchions), thus having a cantilevered portion at each end (Fig. 176).

This load system will produce cantilever moments at each end, as well as a moment between the stanchions (the span moment) of the opposite character.

The B.M. and S.F. diagrams are shown inverted in order to correspond with the previous examples of overhanging beams considered in Chapter V.

The maximum cantilever moment and the span moment must be calculated, and the grillage designed to withstand the greater of these.

$$\text{Load per ft. run of grillage} = \frac{294}{27\cdot3} = 10\cdot8 \text{ tons.}$$

$$\text{Maximum cantilever moment} = \frac{10\cdot8 \times 6\cdot05^2}{2} = 197 \text{ tons. ft.}$$

The maximum span moment occurs at the point of zero shear (see Chapter IX).

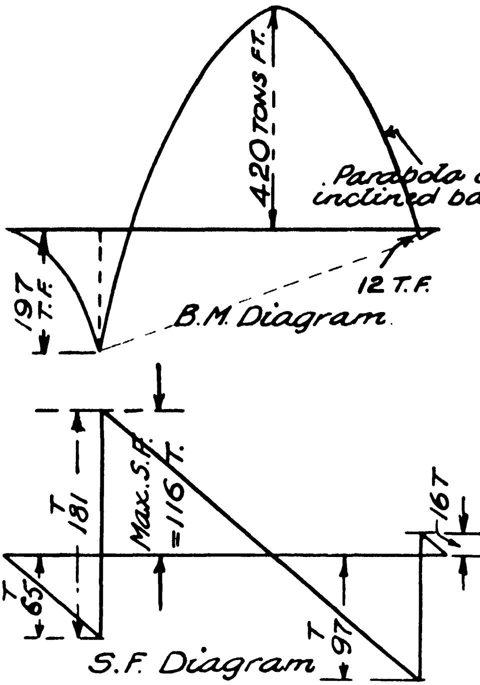


FIG. 176.- B.M. AND S.F. DIAGRAMS FOR COMBINED GRILLAGE.

Point of zero shear from l.h. end of grillage $= \frac{181}{10 \cdot 8} = 16 \cdot 75$ ft.

$\therefore$ Maximum span moment $= 181\,(16 \cdot 75 - 6 \cdot 05) - 181 \times \frac{16 \cdot 75}{2}$

$$= 181 \times 10 \cdot 7 - 181 \times 8 \cdot 38$$
$$= 181 \times 2 \cdot 32 = 420 \text{ tons ft.}$$

Assuming the grillage to be encased in concrete, as in the previous example of a simple grillage, the allowable bending stress $= 12$ tons/in.^{2}. Section modulus required for grillage $= \frac{420 \times 12}{12} = 420$ ins.^{3}. Use three No. $22'' \times 7'' \times 75$ lb. B.S.B.s, each having a modulus of $152 \cdot 4$ ins.^{3}, thus giving a total modulus of $457 \cdot 2$ ins.^{3}.

Shear.—From the shear diagram (Fig. 176) we find that the maximum shear force in the grillage $= 116$ tons.

The web thickness of a $22'' \times 7'' \times 75$ lb. B.S.B. is $\cdot 5''$,

hence the total shear area for three such beams $= (3 \times 22 \times \cdot 5)$ ins.^{2}.

Using the 50% stress increase permitted, the working stress in shear $= 7\cdot5$ tons/in.^{2}.

$\therefore$ Value in shear of the three beams

$$= (3 \times 22 \times \cdot 5 \times 7\cdot5) \text{ tons} = 247\cdot5 \text{ tons.}$$

Web Buckling.—The direct stress on the beam webs must not be more than is allowed by considering the web as a fixed-ended column of length equal to the clear distance between the flanges of the beam.

The value of a beam in web buckling is tabulated in some handbooks, and the figure given represents the safe load per inch of length of web measured along the beam.

The length of web which may be assumed to resist a load is given by B + D, where

B = the width of the stanchion parallel to the webs of the grillage beams,

D = the depth of the grillage beams.

Where more than one tier of grillage beams is used, B, for any tier—except the top tier—is the overall width of the tier immediately above the beams being considered.

If B is taken as 10″ in the example, B + D = 10″ + 22″ =32″.

$\therefore$ Load carried per inch run of each beam

$$= \frac{181}{3 \times 32} \text{ tons} = 1\cdot89 \text{ tons.}$$

The clear depth between the flanges of a 22″ × 7″ × 75 lb. B.S.B. is 18·68 ins. (obtained from section tables). Taking a column 18·68″ high, 1″ × ½″ in section, and applying the methods of the L.C.C. By-laws, we have:

Effective height = ·75 × 18·68″ = 14″ (assuming complete end restraint).

$$\therefore \frac{l}{r} = 14'' \div \frac{t''}{\sqrt{12}} = 14 \times \sqrt{12} \times 2 = 97.$$

F_1 (from tables) $= (4\cdot3 - \cdot7 \times \cdot5)$ tons/in.^{2} $= 3\cdot95$ tons/in.^{2}.
$\therefore$ Safe load on column $= (3\cdot95 \times 1 \times \frac{1}{2})$ tons $= 1\cdot97$ tons, i.e. safe load per inch length of web = 1·97 tons.

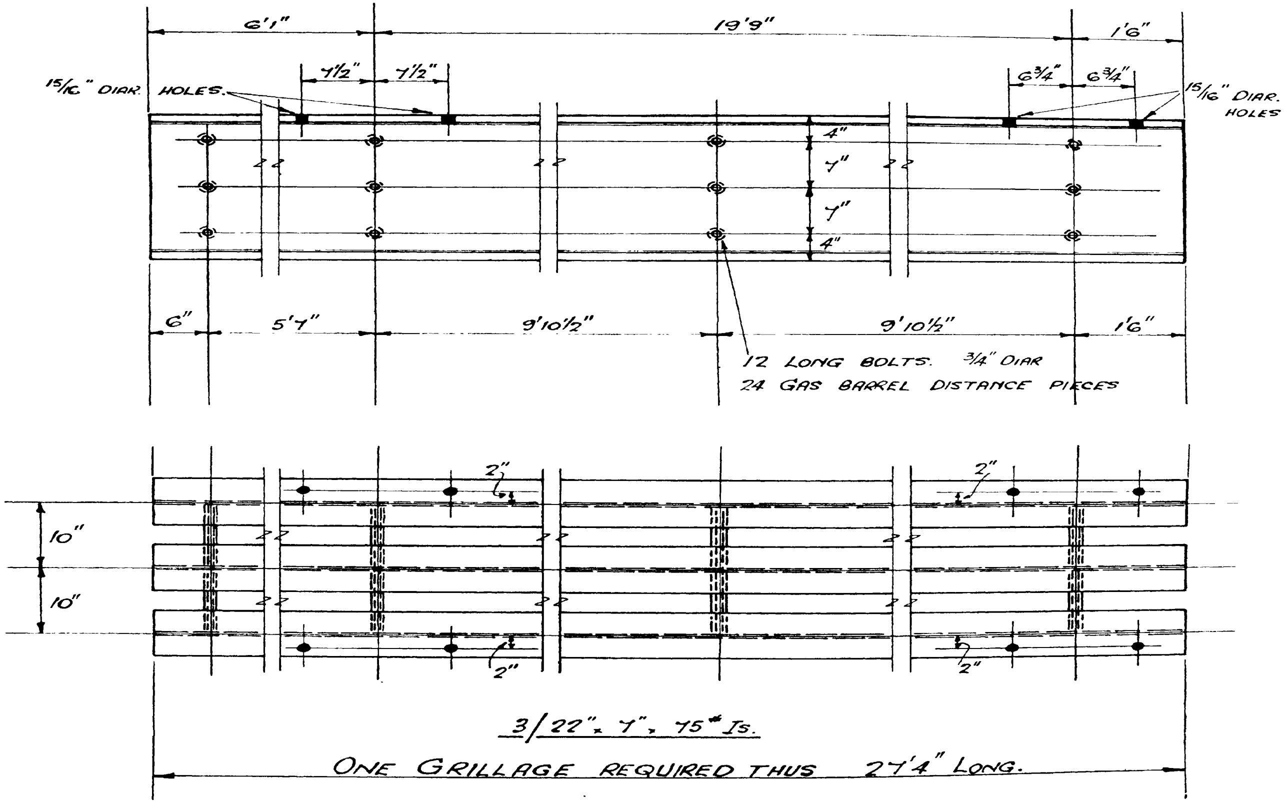


PLATE III.—COMBINED GRILLAGE.

Adopting the 50% increase in stress we get a still higher margin of strength.

No bottom tier is necessary in this grillage as the area provided by the tier already designed makes the bearing pressure on the foundation quite a low figure.

Riveting at Column Ends

There are various methods employed, and assumptions made, in the design of the riveting at the base (or top) of a column. The problem turns upon the nature of the stress distribution beneath the base plate (or on top of the cap). An axial column load can only produce uniform stress over, say, a concrete foundation, if the base plate does not yield near the edges and, by such deflection, cause the more centrally placed portion—actually under the column—to take a higher stress. If a uniform stress under a base plate be assumed, the flexural stress in the portion of base projecting beyond the *vertical leg* of the column flange base angles should be checked.

EXAMPLE. *Design a suitable column base for a* 10″ × 8″ × 55 *lb. joist column which transmits an axial load of* 70 *tons. The base plate is to rest on* 1 : 2 : 4 *concrete.*

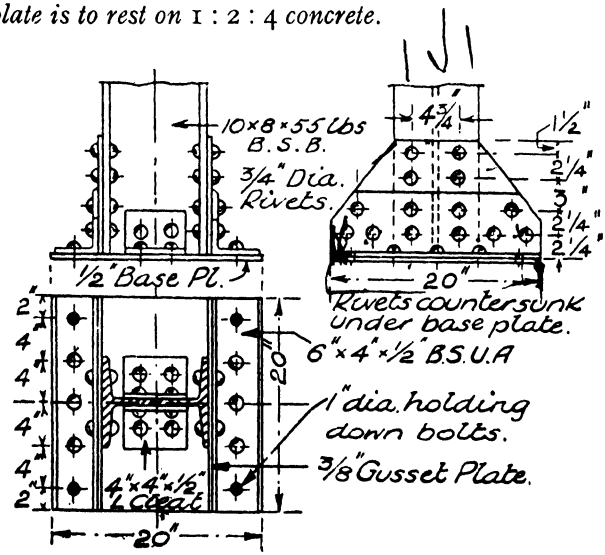


FIG. 177.—RIVETED BASE OF COLUMN.

The minimum area required in the base plate, taking 30 tons per sq. ft. safe stress for the concrete foundation, $= \frac{70}{30}$ sq. ft.

$$= \frac{70 \times 144}{30} \text{ sq. ins.} = 336 \text{ sq. ins.}$$

If we assume $6'' \times 4'' \times \frac{1}{2}''$ base angles for the column flanges ($4''$ horizontal), and $\frac{3}{8}''$ gusset plates, the minimum width will be $(10'' + \frac{3}{4}'' + 8'') = 18\frac{3}{4}''$, say $20''$ square (see Fig. 177).

Average stress under base plate $= \frac{70 \times 144}{400}$ tons per sq. ft.

$$= 25 \cdot 2 \text{ tons per sq. ft.}$$

The overhang of the base plate beyond the vertical leg of the $6'' \times 4''$ angle $= 4\frac{1}{8}''$. The total upward thrust on the strip of base plate $4\frac{1}{8}''$ wide $\times$ $20''$ long, at $25 \cdot 2$ tons per sq. ft.

$$= \left(\frac{4 \cdot 125 \times 20}{144} \times 25 \cdot 2\right) \text{ tons} = 14 \cdot 5 \text{ tons.}$$

The strip acts as a cantilever, as illustrated in Fig. 178.

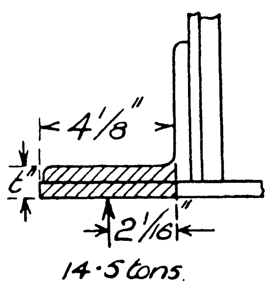


FIG. 178.

The thickness of the cantilever is the combined angle and base thickness. Let t'' = the minimum combined thickness.

Maximum B.M. $= \left(14 \cdot 5 \times \frac{4 \cdot 125}{2}\right)$ tons ins. $= 29 \cdot 9$ tons ins.

$$M = \frac{f \times b \times d^2}{6}.$$

In this case $f = 9$ tons/in.^{2}; $b = 20''$; $d = t''$ (the thickness required).

$$29 \cdot 9 = \frac{9 \times 20 \times t^2}{6}.$$

$$t = 1''.$$

As angle is $\frac{1}{2}''$ thick, the base plate should be $\frac{1}{2}''$.

The web cleats are often regarded as connecting cleats, assisting only in the distribution of the load uniformly to the base plate. Usual practice requires a sufficient number of rivets to carry 60% of the total load.

Sixty per cent. of 70 tons = 42 tons. The rivets in the flange connection are in single shear and bearing (in the gusset plate, or in flange of column).

In either case the single shear value will be the lesser.

Assuming $\frac{3}{4}''$ diameter rivets, the S.S. value of one rivet at 6 tons/in.^{2} = 2·65 tons. Number of rivets required = $\frac{42}{2 \cdot 65}$ = 16, i.e. 8 in each flange.

The extracts from B.S.S. No. 449–1937 in this chapter have been made by permission of the British Standards Institution, 28 Victoria Street, London, S.W.1 (see Appendix I), from whom official copies of the specification may be obtained, price 2*s.* 2*d.*, post free.

EXERCISES 12

(1) A load of 150 tons is transmitted by a column $12'' \times 10''$ to a mild steel slab base, $27'' \times 27''$, the base resting on concrete. Calculate a suitable thickness for the base (*a*) by first principles, (*b*) by employing the L.C.C. By-law formula. Take f as 9 tons/in.^{2}.

(2) A load of 120 tons is to be carried by a two-tier steel grillage. The base slab of the column is 21″ square. The safe bearing pressure on the foundation is $2\frac{1}{4}$ tons per sq. ft. Assuming the weight of grillage plus concrete filling to be 20 tons, work out suitable details for the grillage.

(3) The footing of a wall is composed of a concrete slab 3′ thick. The projection of the footing is 18″ beyond the edge of the wall. The total loading is such as to produce a *net* upward pressure on the footing of 2 tons per sq. ft. Calculate the maximum tensile stress in the concrete.

(Find B.M. at edge of wall, taking 1 ft. *length* of footing, and treat as a cantilever with rectangular cross-section, 1 ft. wide × 3 ft. deep.)

(4) A square base, $4' \times 4'$, carries an eccentric load of 32 tons, the eccentricity being $1'$ with respect to one of the axes of symmetry. Calculate the maximum foundation pressure under the base.

(Maximum pressure $= \frac{W}{A}\left(1 + \frac{6x}{b}\right)$ where $x =$ eccentricity and $b =$ breadth of base, parallel to direction of eccentricity.)

(5) Find, by means of the formula given in B.S.S. 449–1937, the necessary thickness for a mild steel slab base, $24'' \times 23''$, which carries a load of 150 tons, transmitted through a column $12'' \times 10''$ (a concrete of safe bearing pressure 40 tons/ft.^{2} is assumed). Check the value by means of the table given on page 236.

(6) Two stanchions $12'\ 6''$ apart carry loads of 141 tons and 198 tons respectively. It is required to carry these two stanchions on a single grillage, top tier having 3 B.S.B.s. If the grillage is solidly embedded in concrete, determine the section required for each of the top-tier beams. Assume that the minimum overhang beyond a stanchion centre line is $1'\ 6''$. Investigate the grillage from the points of view of shear and web buckling, given that the dimension of both stanchions, in the direction of the grillage length, is $12''$.

CHAPTER XIII

ENCASEMENT OF STEELWORK. FIRE-RESISTING FLOORS

Fire-resistance Regulations

THE steelwork of buildings of more than one storey has to be suitably encased, as a protection against fire. Certain exceptions are permitted; roof trusses of open frame need not be encased. The casing may be of solid brickwork, concrete or other fire-resisting material. As the detail of the casing may affect the stresses used in design, some of the regulations which are concerned with the question of fire resistance will now be considered.

The following extract from the L.C.C. By-laws (Clause 68) deals with the encasing of pillars and beams:

(*a*) A steel column or beam wholly or partly in an external wall or wholly or partly within a recess in a party wall shall be completely encased and protected from the action of fire with brickwork, terra-cotta, concrete, stone, tiles or other similar incombustible materials (or suitable combination of such incombustible materials) at least four inches in thickness in compliance with this by-law.

Provided that the casing on the underside of such a beam, and to the edges of the flanges thereof and of plates and angles connected therewith, may be of any thickness not less than two inches.

(*b*) Any other steel column or beam shall be completely encased and protected from the action of fire with brickwork, terra-cotta, concrete, stone, tiles or other similar incombustible materials or any suitable combination thereof approved by the district surveyor at least two inches in thickness in compliance with this by-law.

Provided that the casing on the upper surface of the upper flange of such a beam, and on other parts (such as projecting

cleats, projecting rivet-heads and the like) of such a column or beam, may be of any thickness not less than one inch.

This requirement shall not apply in the case of a building which comprises only one storey and is no more than twenty-five feet in height.

(c) All casing required for compliance with this by-law shall be executed with Portland cement, and shall be bedded close up to the steel without any intervening cavities. All joints in such casing shall be made full and solid.

Compressive Stresses in Beams

It will be recalled that the compressive working stress in a beam flange depends upon the particular circumstances of the beam. A stress of 8 tons per sq. in. on the gross section may be used :

(i) If the compression flange is embedded in a concrete floor, or otherwise laterally secured.

(ii) If the laterally unsupported length L of a beam is less than 20 times the width b of the compression flange.

If the unsupported length of a beam exceed 20 times b the expression $11{\cdot}0 - 0{\cdot}15\frac{L}{b}$ must be used to determine the working stress (in tons per sq. in.). In the case of properly encased beams, the value of b in the formula may be taken as the width of the compression flange plus the lesser side concrete cover beyond the edge of the flange on one side only, with a maximum of 4″.

Under no consideration may the ratio $\frac{L}{b}$ exceed 50.

EXAMPLES

(1) *A 9″ × 4″ × 21 lb. B.S.B. is used as an uncased beam. Calculate the safe uniformly distributed load for the beam for an effective span of (a) 6 ft., (b) 10 ft. No lateral support.*

(a) $20 \times b = 20 \times 4'' = 80'' = 6'\ 8''$. Hence a stress of 8 tons/in.^{2} may be used in compression (and tension).

$$\frac{Wl}{8} = fZ.$$

$$\frac{W \times 6 \times 12}{8} = 8 \times 18{\cdot}03.$$

$$W = 16 \text{ tons}.$$

(*b*) As the unsupported length of beam exceeds $20b$, the working stress in compression is given by $11{\cdot}0 - 0{\cdot}15\,\frac{L}{b}$

$$= \left(11{\cdot}0 - 0{\cdot}15 \times \frac{10 \times 12}{4}\right) \text{ tons/in.}^2 = 6{\cdot}5 \text{ tons/in.}^2$$

$$\frac{Wl}{8} = fZ.$$

$$\frac{W \times 10 \times 12}{8} = 6{\cdot}5 \times 18{\cdot}03.$$

$$W = 7{\cdot}8 \text{ tons.}$$

(2) *A 12″ × 6″ × 54 lb. B.S.B. is solidly encased in concrete, the cover being the minimum required in the regulations referred to. Obtain the working stresses in compression, assuming an effective span of* (*a*) 18 *ft.*, (*b*) 36 *ft. The beam is laterally unsupported.*

(*a*) $L = 18$ ft. $= 216''$.

Breadth of beam (*b* plus concrete cover) $= 6'' + 2'' = 8''$.

$$\text{Working compressive stress} = \left(11{\cdot}0 - 0{\cdot}15 \times \frac{216}{8}\right) \text{ tons/in.}^2$$

$$= 6{\cdot}95 \text{ tons/in.}^2$$

(*b*) Maximum permissible unsupported length $= 50 \times b = 50 \times 8'' = 400'' = 33'\ 4''$. Hence a span of 36 ft. is inadmissible.

Fire-resisting Floors

A solid, unreinforced, concrete floor possesses the necessary properties for fire-resistance, but such floors are heavy and tend to transmit noise. The use of steel reinforcement reduces the necessary floor thickness, and strong, light floors are obtained by employing the T-beam principle in their design.

Several firms specialise in floors of this description, and have their own patent methods of developing the T-beam principle. By the courtesy, and with the permission, of the firms concerned, two well-known types of such floors are briefly described and illustrated in this chapter.

Floor Loads

The L.C.C. By-laws, Clause 4, contain particulars of

the floor loads to be used in the construction of steel-framed buildings. The loads are given in lb. per sq. ft. of floor area, excluding any allowance for possible partitions. Where such partitions are not definitely located, an additional load of 20 lb. per sq. ft. should be taken to cover their effect.

For the purpose of calculating the total load to be carried on foundations, columns, piers and walls in buildings of more than two storeys in height, if the floor load does not exceed 100 lb. per sq. ft., the regs. permit certain reductions varying from 10% to 50% according to the position of the floor.

The scheduled beam floor loads range from 40 lb. per sq. ft. for rooms used for domestic purposes, to 200 lb. per sq. ft. for warehouses, book stores, etc. Theatres, cinemas and restaurants are listed at 100 lb. per sq. ft. in B.S.S. 449.

The values given must be regarded as minimum values, and it must be noted that they apply to the general design of the steelwork only. Design of floor slabs will normally require higher values, which for any given case should be determined by reference to the by-laws.

Clause 4 (*c*) of the By-laws gives a list of alternative loads which floor detail shall be capable of carrying. This section of the clause is inserted to ensure the floors being satisfactorily designed, even with the possibility of having to support heavy localised concentrated loads. Such loads depend upon whether floor slabs or floor beams are being considered. Floor beams spaced not more than 2′ 6″ apart take the tabulated slab loads (see L.C.C. By-laws, Clause 4 (*c*)).

Filler Floor Beams

Solid unreinforced concrete floors are supported between main steel beams by having *filler joists* embedded in the concrete. Fig. 179 illustrates the construction of such a floor. Two methods of calculation are in use for finding the strength of a filler joist concrete floor. One method regards the joists as simple beams, but a higher than normal working stress is used in the steel. The second method is to treat the problem as one in reinforced concrete, neglecting the strength of the concrete in tension.

EXAMPLE. *Fig. 179 shows an existing solid concrete floor supported by 5″ × 2½″ × 9 lb. B.S.B.s (a section not in the revised list and having an I value of 10·9 ins.⁴). Assuming a span of 10 ft. for the filler joists, calculate the safe uniformly distributed load for the floor (including the self-weight of the floor). Use a working stress of 9 tons/in.² in the filler joists.*

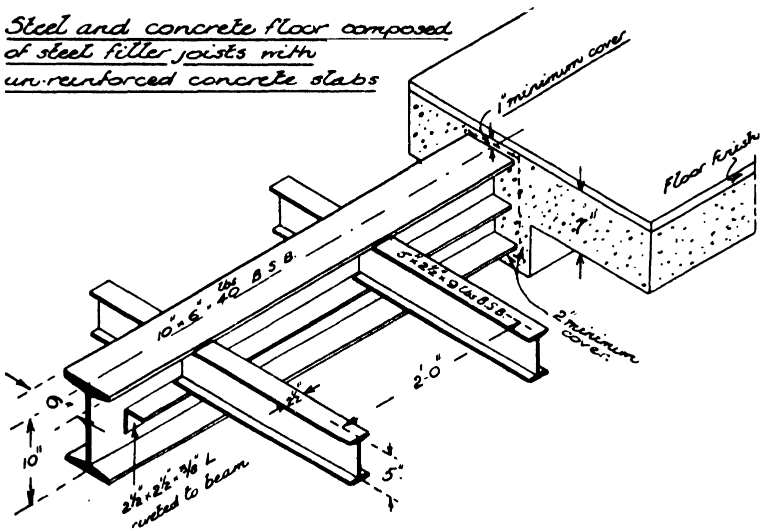


FIG. 179.—FILLER JOIST FLOOR.

Let W lb./ft.² = safe total U.D. load.

Area supported by one filler joist = 10′ × 2′ = 20 sq. ft.

∴ Total load carried = 20W lb.

$$\text{B.M. maximum in joists} = \frac{20W \times 10 \times 12}{8 \times 2240} \text{ tons ins.}$$

$$M = \frac{fI}{y}.$$

$$\frac{20W \times 10 \times 12}{8 \times 2240} = \frac{9 \times 10{\cdot}9}{2{\cdot}5}.$$

$$W = 292 \text{ lb./ft.}^2.$$

Readers acquainted with the theory of reinforced concrete will be able to follow a solution of the problem based upon the method referred to in the regulations affecting filler floor beams.

The L.C.C. By-law No. 83, reads: *The strength of filler floor*

beams of structural steel entirely encased in a concrete floor slab may be estimated on the basis of the combined moment of inertia of the steel and surrounding concrete calculated as in reinforced concrete, neglecting the strength of concrete in tension and taking the limit of flexural stress in the steel at 9 *tons per sq. in.*

Fig. 180 shows the problem reduced to one in reinforced concrete. The solution will be given briefly, using the usual symbols.

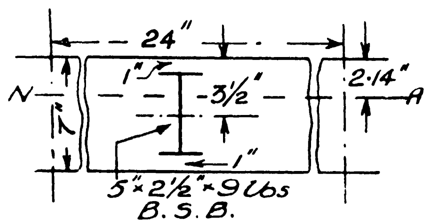


FIG. 180.

$$\text{Area of joist} = 2{\cdot}64 \text{ ins.}^2$$
$$I_{xx} = 10{\cdot}9 \text{ ins.}^4$$
$$d = 3\tfrac{1}{2}''.$$
$$b = 24''.$$
$$r = \frac{A}{bd} = \frac{2{\cdot}64}{24 \times 3{\cdot}5} = {\cdot}0315.$$

$$\frac{n}{d} = \sqrt{2mr + m^2r^2} - mr$$
$$= \sqrt{2 \times 15 \times {\cdot}0315 + 15^2 \times {\cdot}0315^2} - 15 \times {\cdot}0315$$
$$= {\cdot}61.$$
$$\therefore\ n = {\cdot}61 \times 3{\cdot}5 = 2{\cdot}14''.$$
$$I_e = \frac{bn^3}{3} + mI_{xx} + mA(d-n)^2$$
$$= \frac{24 \times 2{\cdot}14^3}{3} + (15 \times 10{\cdot}9) + (15 \times 2{\cdot}64 \times 1{\cdot}36^2).$$

(*Note*: $(d - n) = (3{\cdot}5 - 2{\cdot}14)$ ins.)

$$I_e = 314 \text{ ins.}^4$$

$$\text{Moment of Resistance} = \frac{C \times I_e}{n} = \frac{600 \times 314}{2{\cdot}14} \text{ lb. ins.}$$
$$= 88000 \text{ lb. ins.}$$

(Check on steel stress. d_s = depth to bot. fibre.

$$88000 = \frac{t}{m}\left(\frac{I_e}{d_s - n}\right)$$

$$t = \frac{15 \times 88000 \times 3{\cdot}86}{314} \text{ lb./in.}^2 = 16000 \text{ lb./in.}^2\text{.)}$$

As before, $\dfrac{20W \times 10 \times 12}{8} = 88000.$

$$W = 293 \text{ lb./ft.}^2.$$

The results are in close agreement in this case.

Examples of Patent Hollow-block Floors

By the courtesy and permission of the firms indicated, two patent forms of fire-resisting floors, complying with the current L.C.C. regulations for fire-resisting floors, will be briefly referred to.

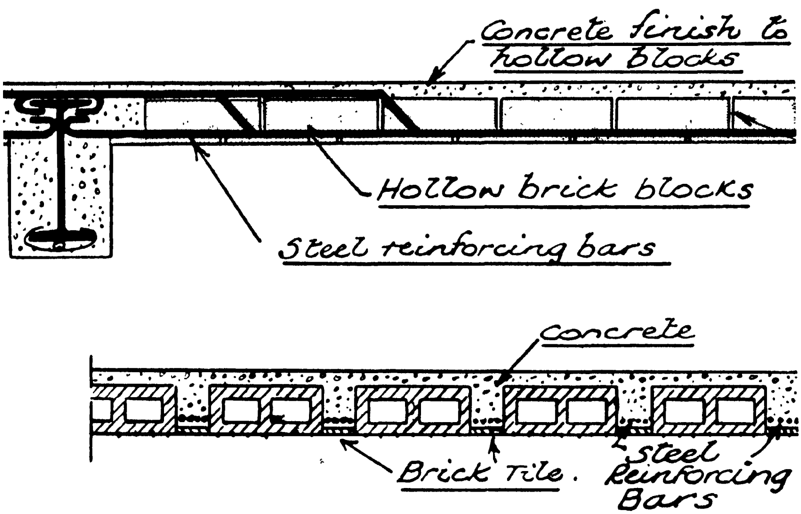


FIG. 181.—KLEINE FLOOR.

(*a*) *Kleine Hollow Brick Floors.*—Fig. 181 shows typical longitudinal and transverse sections of a 'Kleine Floor.' In Fig. 182 the steel reinforcement between adjacent blocks is shown, together with the patent separators.

The illustrations given in Figs. 183 and 184 (kindly supplied by the firm) show the method of construction of these floors.

The ends of the hollow blocks are mortar jointed with a 2 : 1 mix of sand and cement mortar before being placed in position—joints left open mean less compressive strength,

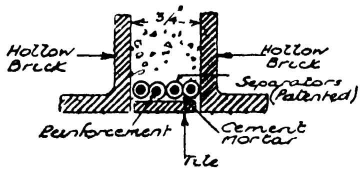


FIG 182

and less effectiveness in fire-resistance. Between all courses of hollow bricks, burnt clay tiles are laid, equal in width to the concrete joints. This forms a key for the plaster (Fig. 183).

To ensure correct positioning of the reinforcing rods, the rods are threaded with patent steel washer separators.

Before concreting is commenced, a thick mixture of sand and cement mortar without stones is poured into the joints, to a sufficient depth to cover the reinforcing rods. This is done to ensure a dense covering, and absence of voids (Fig. 184). 3 : 1 concrete (aggregate $\frac{3}{8}$ in. down) is then filled in between, and over, the hollow bricks

(*b*) *Caxton Floors.*—The Caxton Floor is another type of

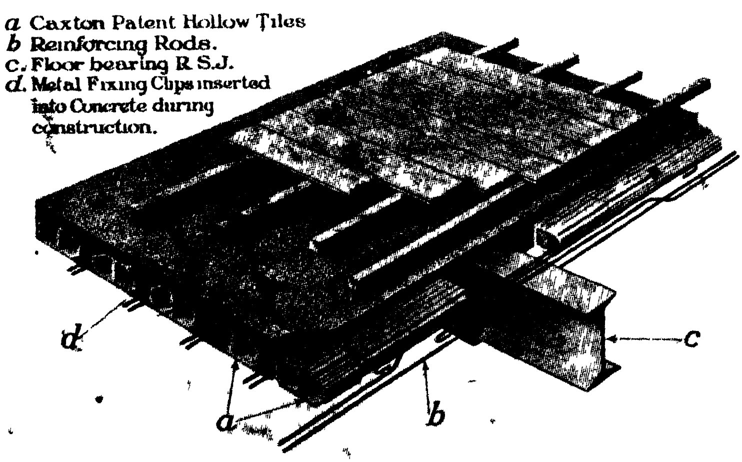


FIG. 185.

FIG. 183.

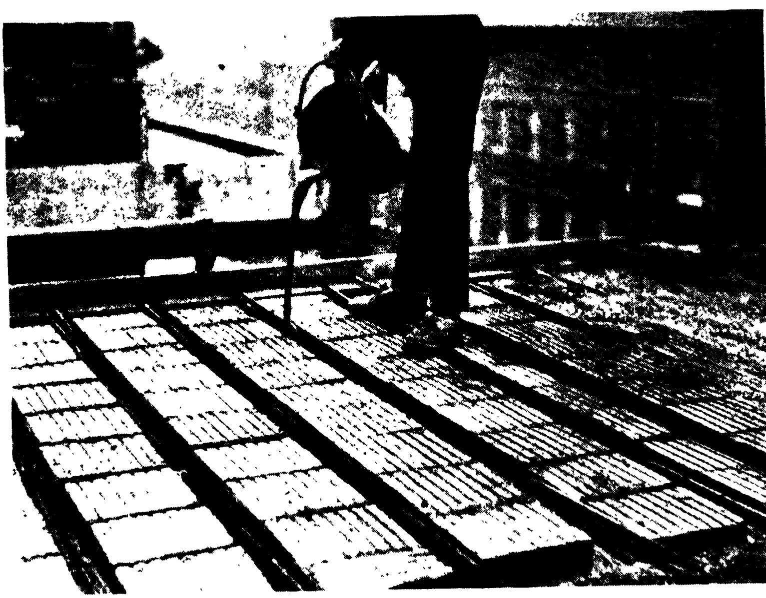


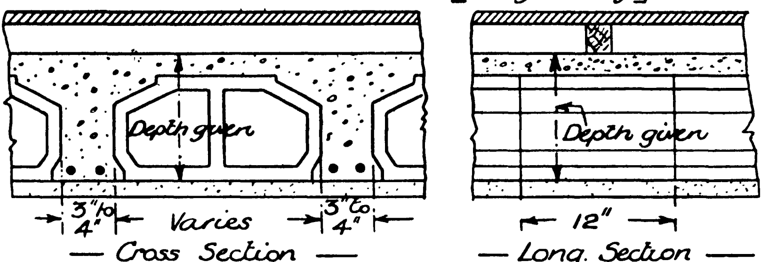
FIG. 184.

Figs. 181, 182, 183 and 184 reproduced by permission and courtesy of Messrs. The Kleine Company, Ltd.

patent hollow tile reinforced concrete floor. The features of this floor are shown in the illustrations (Figs. 185 and 186). By permission of Messrs. Caxton Floors Ltd., a table of ‘ floor thicknesses in the London area ’ (Fig. 186), and the following explanatory notes are given.

The floors are designed to carry the superimposed load

— Hollow Tile R.C. Ribs [Single Way] —



Self Dead Weight of Floor.

Depth in ins	5"	5½"	6¼"	7¼"	8¼"	9¼"	10"	11"	12"	13"
Weight in lbs/□'	40	43	44	47	55	61	66	69	73	78

In cases marked * an additional 20 lbs/□' is allowed for partitions

Super Load Lbs/□'	Spans in feet and depths in inches.									
	8	10	12	14	16	18	20	22	24	26
30	5	5	5½	6¼	7¼	8¼	9¼	10	11	12
40	5	5	5½	6¼	7¼	8¼	9¼	10	11	12
50	5	5	5½	6¼	7¼	8¼	9¼	10	11	12
*50 : 70	5	5	5½	6¼	7¼	8¼	9¼	10	11	12
80	5	5	5½	6¼	7¼	8¼	9¼	10	11	12
*80 : 100	5	5	6¼	7¼	7¼	8¼	10	10	11	12
200	5½	5½	6¼	7¼	8¼	9¼	10	11	12	13

FIG. 186.

Figs. 185 and 186 reproduced by permission and courtesy of Messrs. Caxton Floors Ltd.

stipulated, plus self-weight and an allowance of 20 lb. per sq. ft. for dead weight of finishings. Apart from the L.C.C. regulations, as published, certain additional requirements are sometimes enforced in connection with ‘ excess cube ’ or other buildings requiring special sanction. Such special requirements might modify the design of the floors. The thicknesses given represent average design. Slab strength depends on both

thickness and percentage reinforcement—the figures are therefore susceptible to a certain modification.

For fullest economy in design intermediate bays would be of about equal span, with a reduction in span in end bays.

In some cases, changes in economic thickness occur at intermediate points between the spans in the table.

Exercises 13

(1) Give diagrams showing the minimum concrete cover required in the following cases: (*a*) columns in external walls, (*b*) columns not in external walls, (*c*) beams not in external walls.

(2) Write down the value of the working compressive stress in beams under the following conditions:

(*a*) Uncased beams having L greater than 20*b*.

(*b*) Uncased beams having L less than 20*b*. Explain the meanings of 'L' and '*b*' respectively, and give any modifications in the value of the latter—due to encasement.

(3) An 8″ × 6″ × 35 lb. B.S.B. is used, without encasement or lateral support, to carry a uniformly distributed load for an effective span of 12′. Find the maximum permissible stress in compression, and evaluate the safe total uniformly distributed load. [$Z = 28 \cdot 76$ ins.^{3}.]

(4) A 10″ × 8″ × 55 lb. B.S.B. [$Z = 57 \cdot 74$ ins.^{3}] is used to carry a central load, the effective span being 16′. The beam is solidly encased in concrete, the cover on the vertical sides being 2″. Deduce the working compressive stress and the safe load. [Allow 1 ton U.D. for weight of beam and casing.]

(5) Calculate the safe total uniformly distributed load for a 12″ × 5″ × 32 lb. B.S.B. which has an effective span of 20′, the compression flange being embedded in a concrete floor. [$Z = 36 \cdot 84$ ins.^{3}.]

(6) A filler joist floor (similar to that shown in Fig. 179) has 5″ × 3″ × 11 lb. B.S.B.s spaced at 20″ centres. The span of the joists = 11′. Using a working stress of 9 tons/in.^{2}, find the total safe load per sq. ft. of floor. [Z for B.S.B. $= 5 \cdot 47$ ins.^{3}.]

CHAPTER XIV

INTRODUCTORY PRINCIPLES OF THE METAL ARC WELDING OF STEELWORK

Welding of Structural Steelwork

AN alternative method to the connecting together of structural members by riveting, is to effect the joint by means of a *weld*. Welding has been employed in America for the fabrication of structural steelwork for some time. Of late years, the use of welding has found considerable favour amongst English engineers, and welding seems destined to play a big part in structural engineering practice in this country. In 1934 a British Standard Specification was published dealing with *Metal Arc Welding as Applied to Steel Structures* (B.S.S. No. 538–1934). This specification has now been revised. The title of the new specification is *Metal Arc Welding in Mild Steel as Applied to General Building Construction* (B.S.S. No. 538–1940).

In this chapter it is proposed to discuss briefly, and in a simple introductory manner, the underlying principles involved in the estimation of the strength of simple welds. More detailed information may be obtained by referring to the clauses contained in B.S.S. No. 538 and in the regulations issued by the London County Council.

Methods of Welding.—There are two methods of welding in common use, (i) oxy-acetylene welding and (ii) metal arc welding. In the former process a flame of high temperature is produced by burning a mixture of acetylene and oxygen.

The gases are obtained from cylinders and mixed and ignited in a special form of blowpipe. The hot flame fuses the steel pieces at their point of contact and, by means of steel welding rods, a *fusion weld* is obtained.

In the metal arc process electricity is employed, and the metal, required for deposition at the weld, itself forms part of

the electrical circuit. The future development of steel constructional welding is likely to involve this method of welding, and a general introduction to its principles is given below.

FIG. 187.
*Reproduced by permission and courtesy of the Institution of Structural Engineers.**

Metal Arc Welding

The principle of metal arc welding is illustrated in the simple diagram given in Fig. 188.

The two given steel plates are to be joined by a weld, to form a T-section. C is the source of electrical energy, giving

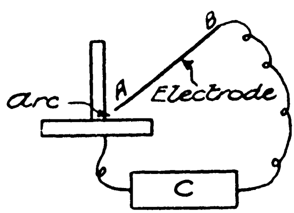


FIG. 188.

either alternating or direct current—both being suitable for the welding process. AB is the *electrode*, which is a metallic conductor used up in the production of the weld. By rubbing

* From 'Report on the Treatment of Welded Structures by the Metallic Arc Process.'

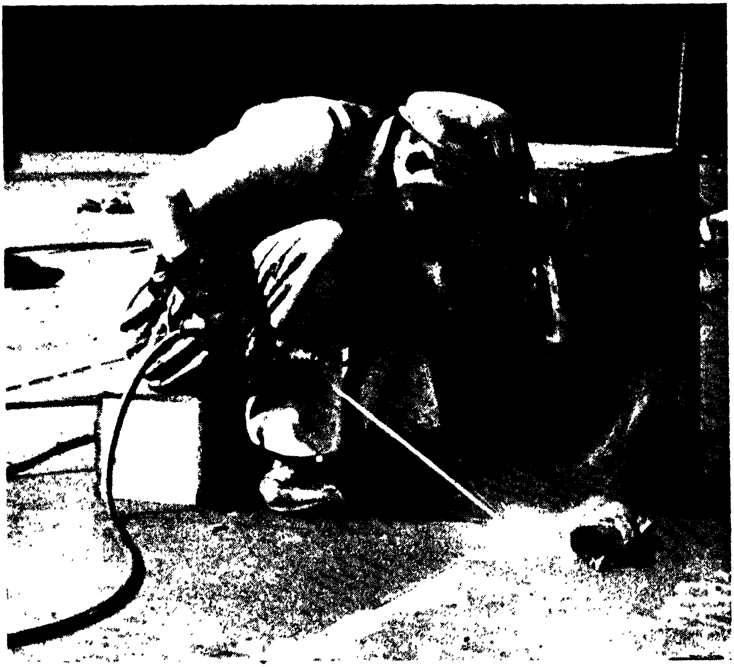


FIG. 189.

FIG. 190.

Figs. 189 *and* 190 *reproduced by permission and courtesy of Messrs. The Quasi-Arc Company, Limited.*

end A on the lower steel plate the circuit is closed; the end may then be slightly withdrawn from the plate, an electric arc being maintained by the ionised gases produced in the volatilisation of the material forming the electrode.

Electrodes may be bare mild steel wire, or may be steel covered with substances possessing certain necessary physical and chemical properties.

The temperature of the steel at the weld in the welding operation is such that iron oxide tends to form, by the combination of iron with the oxygen of the air. To prevent this chemical combination, the electrode covering is made of material which forms a *slag*. The slag covers the welding metal immediately after deposition has taken place, and protects it from the air. The slag is removed, subsequently, by gently tapping with a hammer, followed by wire brushing. In addition to the property of suitable slag formation, the covering should be composed of substances which possess a higher affinity for oxygen than has iron. Certain firms specialise in the manufacture of electrodes, using their own patented coverings.

The photographs shown in Figs. 189 and 190, kindly supplied by Messrs. The Quasi-Arc Co., Ltd., show welding being employed to join steel plates (Fig. 189) and to strengthen an existing girder (Fig. 190). The method of holding the electrode, and the use of the protective light shield for the eyes, are clearly shown.

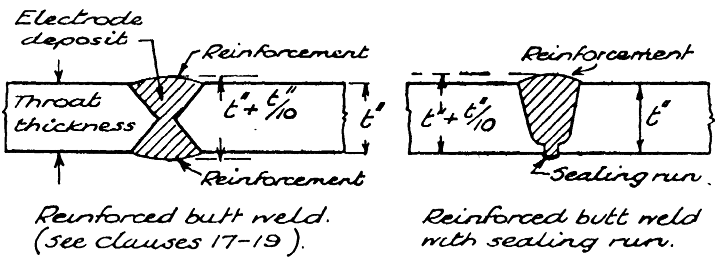


FIG. 191.—BUTT WELDS.

Forms of Welds

The two types of welds are (i) *Butt welds* (Fig. 191), (ii) *Fillet welds* (Fig. 192). There are nine forms of butt welds. Details of these are given later.

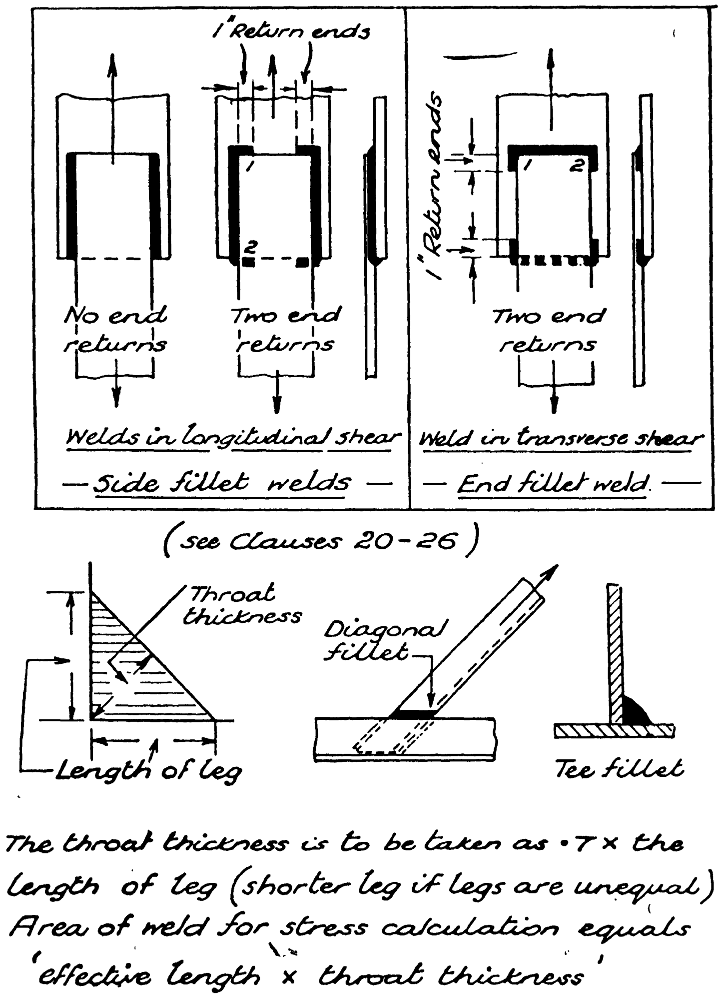


FIG. 192.—FORMS OF FILLET WELDS.

Fillet welds are distinguished as *end fillets, side fillets, diagonal fillets* and *tee-fillets.* These are described in Clauses 23 to 26 on page 268.

L.C.C. Regulations for Metal Arc Welding

Regulations were made by the London County Council on December 7th, 1937, with respect to the use of metal arc

welding within the Administrative County of London. It should be realised that welding is permitted in the Administrative County of London only with the Council's consent and that the particulars, given in the regulations, are given only as a general guide to assist applicants applying for modifications or ' waivers ' of the building by-laws. The title of the regulations is ' Welding—Applications for Modifications or Waivers.' The regulations relate to the modifications or waivers of certain building by-laws so as to permit the use of electric (metal) arc welding instead of riveting, bolting or lapping.

Readers interested in welding should purchase a copy of the regulations, which costs threepence. By permission of the L.C.C., certain extracts from the clauses follow:

Clause 6

The Council will determine in each case the maximum permissible stresses, the detail arrangement of connections and such other restrictions as the Council may deem proper for the use of such welding in the manner proposed.

The following table may, however, be taken as a general indication of the probable maximum stresses which will be permitted by the Council:

Classification of stress in welded connections.	*Maximum permissible stress, in tons per square inch.*
Tension and compression in butt welds	8
Shearing in butt welds in webs of plate girders and joists	6
Shearing in butt welds other than webs of plate girders and joists	5
Stress in end fillet welds	6
Stress in side fillet welds, diagonal fillet welds and tee fillet welds	5

Butt Welds

Clauses 16–19

16. Where steel parts of different thicknesses are butt welded, and the surfaces of the steel parts are one quarter of an inch or more out of plane, the thicker part should be bevelled so that the slope of the surface from one part to the other is not more steep than one in five (*see* Fig. 10).*

Alternatively, the weld metal should be built up at the junction with the thicker part to a thickness at least 25 per cent. greater than the thickness of the thinner part.

* Clause fig. number (*see* page 267).

17. (*a*) Single V, U, J or bevel butt welds should be reinforced wherever practicable by depositing a run of weld metal on the back of the joint. Where this is not done, the maximum stress in the weld should be (except as provided in paragraph (*b*) of this clause) not more than one-half of the corresponding stress indicated in Clause 6.

(*b*) Where it is not practicable to deposit a run of weld metal on the back of the joint, then, provided another steel part is in contact with the back of the joint, and provided also the steel parts are bevelled to an edge with a gap of at least one-eighth of an inch to ensure fusion into the bottom of the V and the steel part at the back of the joint, and provided further that the first run is made with an electrode not larger than No. 8 (S.W.G.), the working stress should not exceed that indicated in Clause 6.

18. (*a*) A butt weld should be reinforced so that the thickness at the centre of the weld is at least 10 per cent. more than the thickness of the steel parts joined.

(*b*) Where a flush surface is required, the butt weld should be first reinforced as in paragraph (*a*) of this clause, and then dressed flush.

(*c*) Where a butt weld is dressed flush in accordance with paragraph (*b*) of this clause, the working stress in the weld metal should not exceed that specified in Clause 6.

19. The throat thickness of a butt weld should be taken as the thickness of the thinner of the steel parts joined.

Description of Butt Welds

The Council may include in the conditions upon which a waiver or modification is granted the following requirements :

(i) Electrodes for welding should comply with the requirements for Class A electrodes in the British Standard Specification No. 639—1935.

(ii) Butt welds should be made in one of the following forms :

(*a*) Square butt joint, as shown in Fig. 1 ;
(*b*) Single V butt joint, as shown in Fig. 2 ;
(*c*) Double V butt joint, as shown in Fig. 3 ;

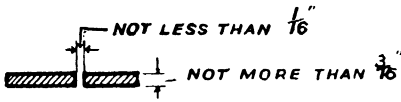


FIG. 1. SQUARE BUTT WELD

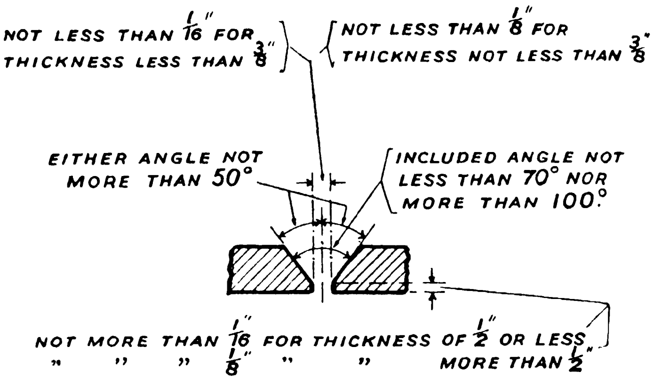


FIG. 2. SINGLE V. BUTT WELD

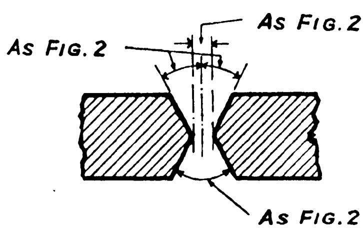


FIG. 3. DOUBLE V. BUTT WELD

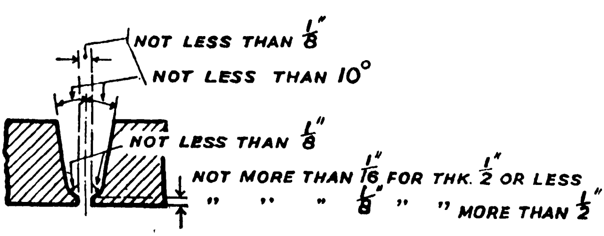


FIG. 4. SINGLE U. BUTT WELD

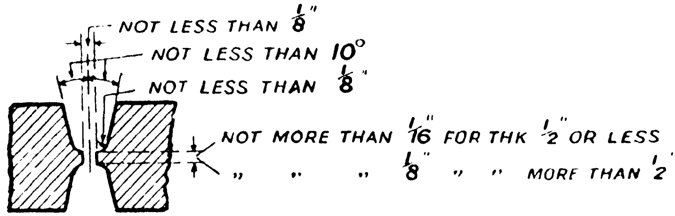


FIG. 5. DOUBLE U. BUTT WELD

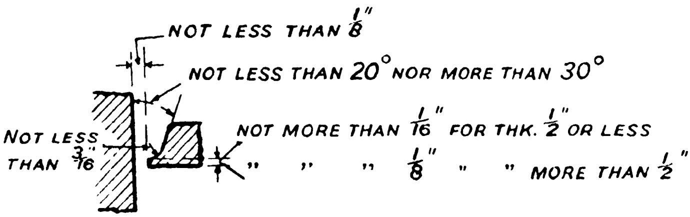


FIG. 6. SINGLE J. BUTT WELD

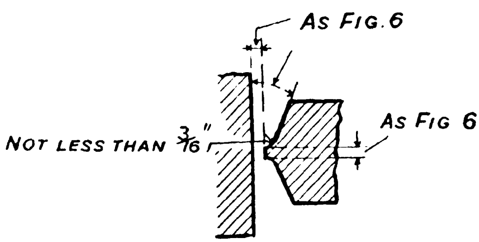


FIG. 7. DOUBLE J. BUTT WELD

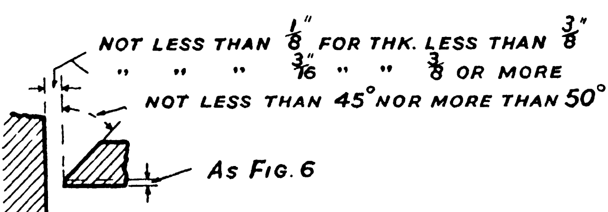


FIG. 8. SINGLE BEVEL BUTT WELD

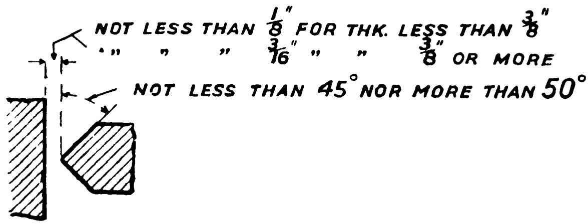


FIG. 9. DOUBLE BEVEL BUTT WELD

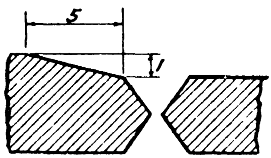


FIG. 10.

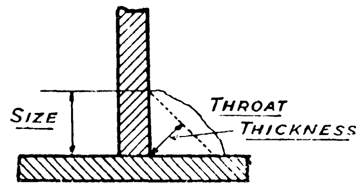


FIG. 11.

(*d*) Single U butt joint, as shown in Fig. 4;
(*e*) Double U butt joint, as shown in Fig. 5;
(*f*) Single J butt joint, as shown in Fig. 6;
(*g*) Double J butt joint, as shown in Fig. 7;
(*h*) Single bevel butt joint, as shown in Fig. 8;
(*i*) Double bevel butt joint, as shown in Fig. 9.

Fillet Welds

Clauses 20–22

20. The size of a fillet weld should be specified by the length of the shorter leg (*see* Fig. 11).

The throat thickness of a fillet weld should be not less than 0·7 of the size (*see* Fig. 11).

21. The strength of a fillet weld should be calculated on a dimension of 0·7 of the size.

The effective length of a fillet weld (for the purpose of stress calculation) should be deemed to be the overall length of the weld minus twice the weld size.

22. The minimum effective length of a fillet weld required

to transmit loading should be not less than two inches nor less than six times the size of the weld.

Fillet welds connecting steel parts the surfaces of which form an angle less than 60 degrees or more than 110 degrees should not be relied upon to transmit loading.

Description of Fillet Welds

Clauses 23–26

23. A side fillet weld is a fillet weld stressed in longitudinal shear—i.e. a fillet weld the axis of which is parallel with the direction of the applied load.

24. An end fillet weld is a fillet weld stressed in transverse shear—i.e. a fillet weld the axis of which is at right angles to the direction of the applied load.

25. A diagonal fillet weld is a fillet weld inclined to the direction of the applied load.

26. A tee fillet weld is a fillet weld joining two steel parts, the end or edge of one part butting on a surface of the other part.

Fig. 192 illustrates the two important types of fillet welds, the *side fillet* and the *end fillet*. The L.C.C. regulations recommend that in end connections a single end fillet should not be used without side fillet welds. In the case of two or more end fillet welds, used without side fillets, the welds should be returned as side welds for a length of at least one inch. No extra strength may be claimed for these returns but the full length of the end weld may be taken without the deduction referred to in Clause 21.

The method of computation of fillet weld area is shown in Fig. 192 (page 262).

Examples

(1) *Calculate the throat thickness to be taken in the calculation of weld strength in each of the following cases: (a) $\frac{1}{4}''$ fillet weld, (b) a fillet weld having one leg dimension $\frac{3}{8}''$ and the other $\frac{1}{2}''$, (c) a butt weld joining two $\frac{1}{2}''$ plates, (d) a butt weld joining two plates, one being $\frac{1}{2}''$ thick and the other $\frac{9}{16}''$.*

(*a*) Throat thickness $= \cdot 7 \times$ specified size of leg $= \cdot 7 \times \cdot 25'' = \cdot 175.''$

(*b*) Throat thickness = ·7 × length of shorter leg = ·7 × ·375″ = ·2625″.

(*c*) Throat dimension = plate thickness = $\frac{1}{2}''$.

(*d*) Throat dimension = thickness of thinner plate = $\frac{1}{2}''$.

(2) *Obtain the throat area to be taken in the case of a $\frac{3}{8}''$ end fillet of 6 inches overall length, (a) if no return side fillets are used, (b) if return fillets 1″ long are employed at both ends of the weld.*

(*a*) In this case we have to take $(6'' - 2 \times \frac{3}{8}'')$ as the effective fillet length = 5·25″.

The throat dimension = ·7 × fillet size = $\cdot 7 \times \frac{3}{8}'' = \cdot 2625''$.

Effective throat area = (5·25 × ·2625) sq. ins. = 1·378 sq. ins.

(*b*) The full 6″ may be taken in this case ∴ throat area = 6″ × ·2625″ = 1·575 sq. ins.

(3) *Calculate the safe load for the butt-welded mild steel tie-bars shown in Fig. 193. Each bar is 4″ wide × $\frac{7}{16}''$ thick.*

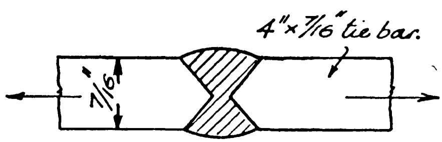


FIG. 193.—BUTT WELD.

The throat thickness is $\frac{7}{16}''$ and the throat area = $4'' \times \frac{7}{16}''$ = 1·75 sq. ins.

At 8 tons per sq. inch the safe load = 8 × 1·75 = 14 tons.

4. *Find the safe value of P, in the example shown in Fig. 194, from the point of view of the side fillet welds.*

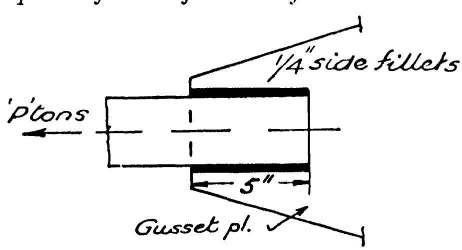


FIG. 194.—SIDE FILLET CONNECTION.

Effective length of each side fillet

$= (5'' - 2 \times \text{fillet size}) = 5'' - (2 \times \frac{1}{4}'') = 4{\cdot}5''$.

Throat thickness = $\cdot 7 \times \cdot 25'' = \cdot 175''$.

∴ Total throat area = $(2 \times 4{\cdot}5 \times \cdot 175)$ sq. ins. = $1{\cdot}575$ sq. ins.

∴ Safe load at 5 tons/in.^{2} = $5 \times 1{\cdot}575 = 7{\cdot}875$ tons.

[For the effect of returning the ends see example (6).]

(5) *Fig.* 195 *shows a connection with two end welds both having* 1″ *return side fillets. Calculate the strength of the welded joint.*

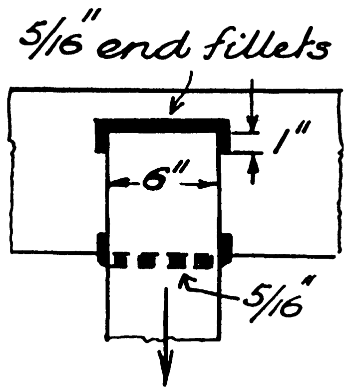


FIG. 195.—END FILLET CONNECTION.

Effective length of each fillet = actual length, as the ends are returned as side fillets.

∴ Effective length = 6″ for each fillet.

Throat thickness = $\cdot 7 \times \frac{5}{16}'' = \cdot 7 \times \cdot 3125'' = \cdot 218''$.

∴ Throat area = $2 \times 6'' \times \cdot 218'' = 2{\cdot}616$ sq. ins.

Safe load at 6 tons/in.^{2} = $6 \times 2{\cdot}616 = 15{\cdot}7$ tons.

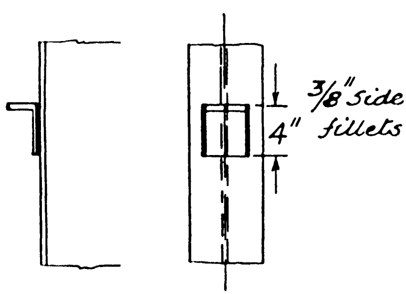


FIG. 196.—SIMPLE BRACKET CONNECTION.

(6) *In Fig.* 196 *a welded bracket connection is shown. Calculate the safe load for the bracket (assuming no bending moment on*

the welds). Find also the safe load if the side fillets shown had returned ends at the top and bottom of the bracket.

Assuming no returned ends the effective length of each side fillet $= (4'' - 2 \times \frac{3}{8}'') = 3{\cdot}25''$.

$\therefore$ Total length of side fillet $= 6{\cdot}5''$.

Throat thickness $= {\cdot}7 \times {\cdot}375 = {\cdot}2625''$.

$\therefore$ Effective throat area $= 6{\cdot}5'' \times {\cdot}2625'' = 1{\cdot}706$ sq. ins.

$\therefore$ Safe load at 5 tons/in.^{2} $= 5 \times 1{\cdot}706 = 8{\cdot}53$ tons.

If the ends are returned at top and bottom no deduction need be made in the overall fillet length as the end *crater* of the welds will be in the returned portions (which are omitted from the calculations).

$\therefore$ Effective length of side fillet $= 8''$.

$\therefore$ Safe load $= (8 \times {\cdot}2625 \times 5)$ tons

$= 10{\cdot}5$ tons.

This result may be checked by inspection of the safe load tables for simple brackets given in Fig. 197. Looking across horizontally from the fillet size, $\frac{3}{8}''$, until the column headed '4' is reached we find the value $10{\cdot}5$ tons.

For no returned ends we have to deduct twice the end factor value, i.e. $2 \times {\cdot}98 = 1{\cdot}96$ tons.

$\therefore$ Safe load when the returned ends are omitted, top and bottom $= (10{\cdot}5 - 1{\cdot}96)$ tons $= 8{\cdot}54$ tons.

(7) *A simple bracket connection of the form shown in Fig.* 196 *has side fillet welds and a top fillet weld so that the sides and top form one continuous weld. The side fillets are returned at the bottom of the bracket. Calculate the vertical shear load this bracket can safely take, given the following particulars:*

Depth of angle bracket $= 6''$.
Breadth of angle bracket $= 5''$.
Size of both fillet welds $= \frac{1}{4}''$.

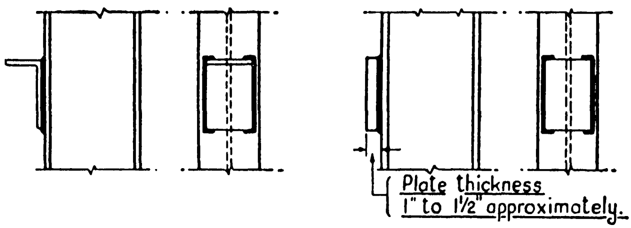
Length of top end fillet $= 5''$. This is the effective length as there will be no end craters to allow for.

$\therefore$ Effective area of throat $= (5 \times {\cdot}7 \times {\cdot}25)$ sq. ins. $= {\cdot}875$ sq. ins.

Weld strength at 6 tons/in.^{2} $= 6 \times {\cdot}875 = 5{\cdot}25$ tons.

As the side fillets are returned at the ends the effective length of each $= 6''$.

SIMPLE BRACKETS
Connected by
SIDE FILLET WELDS



TYPICAL BRACKETS

LOAD IN TONS FOR VARIOUS SIZES OF WELDS CARRYING VERTICAL SHEAR ONLY

Size of Fillet Welds in inches	Depth of Bracket in inches							Return End Factor tons
	3	3½	4	4½	5	6	7	
$\frac{3}{16}$	4·0	4·6	5·3	5·9	6·6	7·9	9·2	·25
$\frac{1}{4}$	5·3	6·2	7·0	7·9	8·8	10·6	12·3	·44
$\frac{3}{8}$	7·9	9·2	10·5	11·8	13·1	15·7	18·3	·98
$\frac{1}{2}$	10·5	12·3	14·0	15·8	17·5	21·0	24·5	1·75
$\frac{5}{8}$	13·1	15·3	17·5	19·7	21·9	26·3	30·7	2·74
$\frac{3}{4}$	15·8	18·4	21·0	23·7	26·3	31·6	36·8	3·95

The above values are for brackets having returned end welds as illustrated in the diagrams. Where the welds cannot be returned at the top, deduct the return end factor value from the tabulated value. If returned ends are omitted at top and bottom, deduct twice the end factor value.

Note. In using plate brackets special attention must be given to the buckling strength of the web of the supported beam.

FIG. 197.
Reproduced by permission and courtesy of the Institute of Welding.

(*continued*)

Total length of side fillet = 12″.

Effective area of throat = (12 × ·7 × ·25) sq. ins. = 2·1 sq. ins.

Weld strength at 5 tons/in.² = 2·1 × 5 = 10·5 tons.

$\therefore$ Safe shear load for bracket $= (5{\cdot}25 + 10{\cdot}5)$ tons $= 15{\cdot}75$ tons.

Welded Structures.—The examples given illustrate the method of computation of the strength of a simple weld. The theory involved in the design of welded structures, such as a welded steel frame, will be generally concerned with the mechanics of rigid structures. For the nature of the detailed calculations which welded structures entail, the reader is recommended to consult the 'Handbook for Welded Structural Steelwork' (5*s*. 6*d*. post free), published by the Institute of Welding, 2 Buckingham Palace Gardens, London, S.W.1. The simple bracket tables given in Fig. 197 are taken from the handbook by kind permission of the Institute. Those especially interested in the welding processes should write to the Secretary for particulars of the activities of the Institute.

Safe Load Tables.—It is convenient to evaluate and tabulate the strength in tons *per lineal inch of weld* for the various types of welds and for the usual sizes. We have only then to multiply the 'strength per inch' by the 'effective weld length in inches' to obtain the total strength of the weld.

Assuming, for example, a $\frac{1}{4}''$ end fillet, the throat area per lineal inch of weld $= {\cdot}7 \times {\cdot}25'' \times 1'' = {\cdot}175$ sq. ins. At 6 tons/in.^{2} the strength per inch $= 6 \times {\cdot}175 = 1{\cdot}05$ tons.

Similarly for a $\frac{1}{4}''$ side fillet, the strength per lineal inch of weld would be ${\cdot}175 \times 5 = {\cdot}875$ tons.

The table given in Fig. 198 is built up by calculations similar to the foregoing. A few only of weld sizes are shown.

Practical Considerations in Welding

The production of a successful weld is an operation demanding skilled and trained workmanship. Such training is now being provided by welding firms themselves, and also in technical institutions.

Welding operations have to be carried out in horizontal, vertical and overhead positions, each possessing a special technique to ensure a successful weld. It is essential, of course, that the electrode should deposit high-grade material to build up the weld, but many other considerations are involved. The

SAFE LOADS PER LINEAL INCH OF FILLET WELDS

Working Stresses: End Fillets 6 tons/in.^{2}
Side Fillets 5 tons/in.^{2}

Size of Fillet.	End Fillet.		Side Fillet.	
Ins.	Calculation.	Tons.	Calculation.	Tons.
$\frac{1}{8}$	$\cdot 7 \times \frac{1}{8} \times 6$	0·525	$\cdot 7 \times \frac{1}{8} \times 5$	0·437
$\frac{3}{16}$	$\cdot 7 \times \frac{3}{16} \times 6$	0·788	$\cdot 7 \times \frac{3}{16} \times 5$	0·656
$\frac{1}{4}$	$\cdot 7 \times \frac{1}{4} \times 6$	1·050	$\cdot 7 \times \frac{1}{4} \times 5$	0·875
$\frac{3}{8}$	$\cdot 7 \times \frac{3}{8} \times 6$	1·575	$\cdot 7 \times \frac{3}{8} \times 5$	1·312
$\frac{1}{2}$	$\cdot 7 \times \frac{1}{2} \times 6$	2·100	$\cdot 7 \times \frac{1}{2} \times 5$	1·750
$\frac{9}{16}$	$\cdot 7 \times \frac{9}{16} \times 6$	2·362	$\cdot 7 \times \frac{9}{16} \times 5$	1·969
$\frac{5}{8}$	$\cdot 7 \times \frac{5}{8} \times 6$	2·625	$\cdot 7 \times \frac{5}{8} \times 5$	2·187
$\frac{3}{4}$	$\cdot 7 \times \frac{3}{4} \times 6$	3·150	$\cdot 7 \times \frac{3}{4} \times 5$	2·625

FIG. 198.

electrical current used is a material factor. Welds are usually made in more than one *run*, and the current required may differ for different runs. Anything, such as plate thickness, which affects the heat conditions of the welding operation, has to be taken into account in fixing the welding current. Important data relating to the physical and metallurgical aspects of welding will be found in the handbooks of firms which specialise in welding and in the reports issued as a result of practical research.

The extracts in this chapter from L.C.C. regulations have been made by permission of the London County Council. Certain data has been kindly supplied by the Institute of Welding. B.S.S. No. 538–1940 may be obtained, price 2*s.* 2*d.* post free, from the British Standards Institution, 28 Victoria Street, London, S.W.1. The report of the Institution of Structural Engineers referred to may be obtained from the Institution, 11 Upper Belgrave Street, London, S.W.1, price 5*s.*

EXERCISES 14

(1) Distinguish between a '*butt weld*' and a '*fillet weld.*' Give simple diagrams illustrating the various forms of fillet welds.

Explain the meaning of the terms '*throat thickness*' and '*effective weld length.*'

(2) Write down the working stresses given in L.C.C. regulations for (i) end fillets, (ii) side fillets. Under what circumstances may the tensile and compressive stresses respectively in butt welds be taken at 8 tons/in.^{2}? State the importance of '*returned ends*' in fillet welds.

(3) Find (i) the throat thickness for a $\frac{5}{16}''$ fillet weld, (ii) the throat area for a $\frac{7}{16}''$ fillet weld which has an effective length of 8″, (iii) the effective length of a $\frac{1}{2}''$ fillet weld, 6″ long, which has no returned ends.

(4) Calculate the safe load per lineal inch for (i) a $\frac{1}{4}''$ side fillet weld, (ii) a $\frac{3}{8}''$ end fillet weld, (iii) a $\frac{5}{8}''$ butt weld, reinforced and sealed.

(5) A flat tie-bar laps on to a gusset plate to which it is connected by two side fillets each 9″ long. Assuming $\frac{3}{8}''$ fillets, find the safe axial load for the tie-bar from the point of view of its welded end connection. Each fillet has two returned ends.

(6) Two lengths of a mild steel tie member, each 6″ wide × $\frac{1}{2}''$ thick, are joined by a double-V butt weld which is suitably reinforced. Calculate the safe axial load for the welded member.

(7) A simply supported beam 'AB' is supported at 'A' and 'B' by similar welded angle bracket connections. The angles are 5″ deep and have $\frac{1}{4}''$ side fillets which are returned at the bottom ends only. Assuming the welds to be subjected to simple vertical shear, calculate the safe total uniformly distributed load for the beam 'AB,' from the point of view of its end supports.

(8) The flange of a plate girder is joined to the web by two $\frac{5}{16}''$ fillet welds. Calculate the maximum permissible horizontal shear, in tons per inch run of girder, at the level of the junction of flange and web.

CHAPTER XV

PLATE AND LATTICE GIRDERS. THEORY AND PRACTICAL DESIGN

M.R. of Plate Girder Section

SOMETIMES, in steel beam design, the large span and heavy loads carried demand deeper sections than can be supplied by compounding standard sections together. In these cases, plates and angles may be riveted together to obtain a section which will satisfy the requirements of both strength and stiffness (see Fig. 199). The relative dimensions of plate girder details are governed by theoretical considerations, and by regulations which are the result of practical experience.

The true M.R. of a plate girder section may be derived from its moment of inertia $\left(\text{M.R.} = \frac{f\text{I}}{y}\right)$ or from its section modulus (M.R. $= f\text{Z}$). The building up of a girder section to give the required modulus is not a difficult matter, especially if section books be employed to give the I-value of flange plates at stated distance apart. In any case, a trial section can easily be tested for suitability.

Another method commonly employed is to use a formula for the M.R. which involves the 'flange area' and 'depth of the girder' (*flange-area* method). The formula is approximate, but sufficiently accurate for practical design.

In Fig. 200 F represents the resultant compressive and tensile forces acting, respectively, in the top and bottom flanges of the girder. D is the distance between the lines of action of these forces, i.e. the 'arm' of the couple resisting bending. The M.R. of the girder section is clearly very nearly given by the product $\text{F} \times \text{D}$; an error on the safe side arises by neglecting the relatively small contribution of the web. If $f =$ the average stress in the flange, and $\text{A} =$ the flange area, $\text{F} = f\text{A}$. The expression for M.R. is therefore $f.\text{A}.\text{D}$.

FIG. 199.—PLATE GIRDERS USED TO SUPPORT A FLOOR OF A GARAGE.

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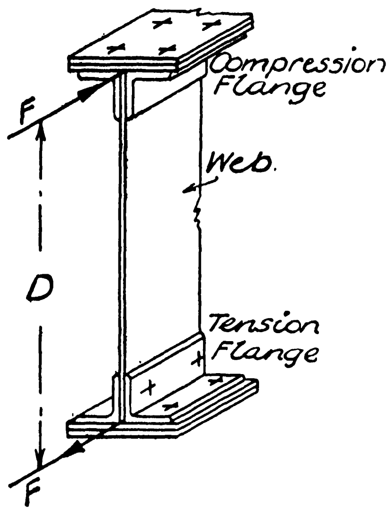

FIG. 200.—COUPLE RESISTING BENDING IN PLATE GIRDER SECTION.

Writing M for moment of resistance (and for the applied bending moment) we have the general expression **M = *f*.A.D.** A slight error—on the wrong side—occurs if f is regarded as the maximum skin stress in the steel.

Flange Area.—The constitution of the flange area is variously taken by different designers. Three methods are common. The flange area is assumed to be made up of :

(i) *Horizontal legs of angles only, plus the flange plates.*

(ii) *Whole angle area, plus the flange plates.*

(iii) *Whole angle area and flange plates, plus a portion (usually* $\frac{1}{8}$*th) of the web.*

Rivet holes are allowed for in tension flanges, but gross areas are taken in compression flanges.

Depth of Girder.—D is the distance between the centres of gravity of the respective flanges. This has to be estimated from the chosen web depth and is frequently taken as the web depth itself. It should not be taken greater than the web depth.

EXAMPLE. *A plate girder of 40′ effective span is to carry the concentrated load system given in Fig. 201. Calculate suitable flange details for the girder, assuming the compression flange to*

be embedded in a concrete floor. The overall depth of the girder—measured over the flange plates—is not to exceed 3′ (limitation due to headroom requirement).

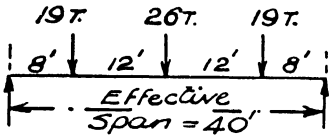

FIG. 201.

B.M. maximum for concentrated loads :

$$R_A = R_B = \frac{19 + 26 + 19}{2} \text{ tons} = 32 \text{ tons.}$$

B.M. max. $= (32 \times 20) - (19 \times 12)$ tons ft. $= 412$ tons ft.

B.M. maximum due to self-weight of girder, etc. :

Assuming girder + casing to weigh 12 tons,

$$\text{B.M. maximum} = \frac{12 \times 40}{8} = 60 \text{ tons ft.}$$

Total B.M. maximum $= 412 + 60 = 472$ tons ft.

Flange area required :

$$M = f.A.D.$$

$f = 8$ tons/in.^{2}, D $= 33''$ (assuming a tentative value for web depth).

$$A = \frac{M}{f.D.} = \frac{472 \times 12}{8 \times 33} \text{ ins.}^2$$
$$= 21{\cdot}46 \text{ ins.}^2.$$

Compression Flange.—As the girder is shallow, the horizontal limbs only of the main angles will be taken in the flange area.

Assuming $6'' \times 6'' \times \frac{5}{8}''$ angles, the horizontal limbs give a gross area of $(2 \times 6 \times \frac{5}{8})$ ins.$^2 = 7{\cdot}5$ ins.^{2}.

∴ Area to be supplied by the flange plates

$= (21{\cdot}46 - 7{\cdot}5)$ ins.$^2 = 13{\cdot}96$ ins.^{2}.

If the flange plates be taken $14''$ wide, the necessary total thickness of plates $= \dfrac{13{\cdot}96''}{14} = 1''$, say $2 - \frac{1}{2}''$ plates.

Tension Flange.—Taking two rivet holes $\frac{13}{16}''$ diameter (for $\frac{3}{4}''$ rivets) from the gross angle area, the net area supplied by angles $= (7{\cdot}5 - 2 \times \frac{13}{16} \times \frac{5}{8})$ ins.$^2 = 6{\cdot}5$ ins.^{2}.

The plates will have to supply a net area of $(21{\cdot}46 - 6{\cdot}5)$ ins.^{2} $= 14{\cdot}96$ ins.^{2}.

The effective solid width of the flange plates

$$= (14 - 2 \times \tfrac{13}{16}) \text{ ins.} = 12{\cdot}38 \text{ ins.}$$

$$\therefore \text{ necessary total plate thickness} = \frac{14{\cdot}96''}{12{\cdot}38} = 1{\cdot}21''.$$

$2 - \frac{5}{8}''$ plates would be suitable.

When the same working stress is used for tension and compression flanges, it is good practice to design the tension flange, and then make the gross area of compression flange equal to it, i.e. have similar detail for both flanges (see B.S.S. 449–1937).

Design and Detail of a Plate Girder

In designing the girder, the following points will be taken into consideration :

(1) The depth of the girder should be within the limits $\frac{L}{10} - \frac{L}{14}$, where L is the length of the girder. The depth of the girder may be less than the minimum given by this rule, provided that the deflection is calculated and proved to be not excessive. Shallow girders lead to high web stress and riveting difficulties.

(2) It is common practice to camber plate girders. If a girder be made without camber, the deflection of the girder when fully loaded is marked, and in fact appears to the eye to be greater than actually is the case. The camber is therefore made equal to the maximum calculated deflection, so that under full load the girder appears substantially horizontal.

(3) The width of the girder should be within the limits $\frac{L}{40} - \frac{L}{50}$. The maximum width given by this rule may be exceeded—if made necessary by a consideration of the unsupported length of flange plates (see Note 5)—provided the flange plates are adequately stiffened.

(4) At least $33\frac{1}{3}\%$ of the gross flange area should be provided by the flange angles.

(5) The maximum stresses to be used are as follows :

(i) In tension (on the net area of the flange), 8 tons/in.^{2}.

(ii) In compression (on the gross area of the flange, if L exceeds $20b$), $11{\cdot}0 - 0{\cdot}15\frac{L}{b}$ tons/in.^{2}, where L = the unsupported length of flange plates and b = width of girder.

The unsupported length of the flange may be taken as the distance between lateral connections, in the case of a girder carrying a series of point loads. For a girder supporting a uniformly distributed floor load the compression flange is considered to be adequately restrained laterally.

The width of the girder (if solidly encased) may be taken as the width of the compression flange plus the lesser side concrete cover beyond the edge of the flange, on one side only, with a maximum of 4″ (see Chapter XIII).

(iii) In shear (on the gross area of the web), 5 tons/in.^{2}.

Particulars of Girder.—We will consider the design of a plate girder for the balcony of a small cinema. The girder is 60′ span between the centres of bearings, and the loads come on to it from the main balcony rakers. These are at 10′ centres, and provide a maximum reaction of 16 tons each. The rakers pass over the top of the girder.

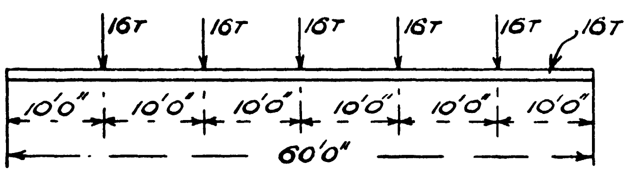

FIG. 202.

Own weight of girder (no casing), say, 16 tons.

Total load on girder = 96 tons.

∴ Each reaction = $\frac{96}{2}$ = 48 tons.

Maximum B.M. = $(48 \times 30 - 16 \times 10 - 16 \times 20 - 8 \times 15)$ tons ft. = 840 tons ft.

Note.—The weight of a girder is given approximately by $\frac{WL}{530}$, where W = equivalent distributed load on girder in tons.

L = length of girder in feet;

or by $\frac{\text{B.M. (tons ft.)}}{65}$.

In this case $\frac{\text{B.M.}}{65} = \frac{840}{65} = 13$ tons.

The assumed weight of 16 tons is, therefore, on the right side.

Web Plate.—Taking $\frac{L}{14}$ for depth of girder, depth $= \frac{60 \times 12}{14}$ ins. $= 51 \cdot 4''$. Assume web 48″ deep.

Maximum shear = 48 tons.

Thickness of web $= \frac{48}{48 \times 5} = \cdot 2''$.

The minimum thickness of web that may be used $= \frac{3}{8}''$, $\therefore$ make web $48'' \times \frac{3}{8}''$.

Flanges.—As the maximum unsupported length of flange is only 10′, we can make the plates as narrow as convenient.

Make plates 16″ wide $= \frac{L}{45}$ (see Note 3), and use $6'' \times 6'' \times \frac{1}{2}''$ angles.

The overhang of the plate beyond the flange angles should not exceed 2″ to 3″ (see page 52).

$$\text{Actual overhang} = \frac{16''}{2} - \frac{3''}{16} - 6'' = 1\tfrac{13}{16}''.$$

Tension Flange.

$$M = f.A.D.$$

D is assumed to be $47\frac{1}{2}''$.

$f = 8$ tons/in.^{2} on net flange area.

$$840 \times 12 = 8 \times A \times 47 \cdot 5.$$

$$A = \frac{840 \times 12}{8 \times 47 \cdot 5} = 26 \cdot 53 \text{ ins.}^2.$$

The gross area of a $6'' \times 6'' \times \frac{1}{2}''$ angle is $5 \cdot 75$ ins.^{2}. The net area supplied by two angles, allowing one $\frac{15}{16}''$ hole in each horizontal leg and one $\frac{15}{16}''$ hole in each vertical leg,

$$= (2 \times 5 \cdot 75) - (4 \times \tfrac{15}{16} \times \tfrac{1}{2})$$
$$= 11 \cdot 5 - 1 \cdot 9 = 9 \cdot 6 \text{ ins.}^2.$$

The plates will therefore have to supply $(26 \cdot 53 - 9 \cdot 6)$ ins.^{2}
$= 16 \cdot 93$ ins.^{2}.

Effective width of plates $= (16 - 2 \times \frac{15}{16}'') = 14 \cdot 12''$.

$\therefore$ Necessary total thickness $= \frac{16 \cdot 93''}{14 \cdot 12} = 1 \cdot 2''$.

Use $2/\frac{5}{8}''$ plates. Net flange area $= 27 \cdot 27$ in.^{2}.

Compression Flange.—In this case the unsupported length (10′) is less than $20 \times b$, i.e. $20 \times 16''$, hence the working compressive stress may be 8 tons/in.². Theoretically, as there is no rivet allowance to be made, the section of the compression flange could be made smaller than that of the gross tension flange. As previously mentioned, it is good design to make the gross compression flange area at least equal to the gross tension flange area, whatever working stresses are used (B.S.S. 449).

Gross tension flange area in this case

$$= (2 \times 5{\cdot}75) + (16 \times 1\tfrac{1}{4}) = 31{\cdot}5 \text{ in.}^2.$$

It will be seen that the flange angles provide $\dfrac{11{\cdot}5}{31{\cdot}5} \times 100$ $= 36{\cdot}5\%$ of the gross flange area.

Riveting.—Horizontal shear/foot $= \dfrac{S}{D}$. Taking D as the

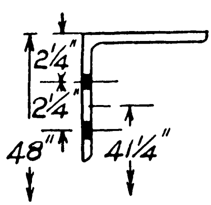

FIG. 203.

average depth between the rivet lines in top and bottom flanges, $D = 48'' - 2\,(2\tfrac{1}{4} + 1\tfrac{1}{8})'' = 41\tfrac{1}{4}''$ (Fig. 203).

$\therefore$ at the ends of the girder the horizontal shear/foot

$$= \frac{48 \times 12}{41{\cdot}25} = 14 \text{ tons.}$$

As there are two rows of rivets in each angle leg, we have two rivet values per pitch length.

Value of $1 - \tfrac{7}{8}''$ diameter rivet in a $\tfrac{3}{8}''$ plate $= 3{\cdot}937$ tons.

$\therefore$ No. of rivets required $= \dfrac{14}{3{\cdot}937} = 4$ per ft.

$\therefore$ a 6″ pitch will be suitable.

The maximum pitch normally used in structural work being 6″, the pitch need not be varied throughout the length of the

girder. We have therefore: *2 rows at 6″ pitch in the vertical legs of the angles and 4 rows at 6″ pitch in the horizontal legs.*

Stiffeners.—Stiffeners are usually placed at the ends of plate girders and under point loads, and at equal distances between the point loads—the spacing being determined mainly by the distances apart of the loads. The stiffeners should not, however, be farther apart than the depth of the web plate. In this example, stiffeners will be placed at the ends, under the point loads and at 3′ 4″ centres between the loads. They will be in pairs, one on either side, and will be $6'' \times 4'' \times \frac{3}{8}''$ Ls.

Curtailment of Flange Plates.—This is best done by drawing the B.M. diagram and superimposing upon it the resistance moment diagrams of the various sections composing the girder. It is usual practice to run the plate next to the angles for the full length, and also to place the thicker plates nearer the flange angles.

In the example we will have $1 - 16'' \times \frac{5}{8}''$ plate full length and $1 - 16'' \times \frac{5}{8}''$ stopped off.

Construction of B.M. diagram.

B.M. at centre $= 840$ tons ft.

B.M. at 20′ from ends $= (48 \times 20 - 16 \times 10 - 5{\cdot}33 \times 10)$ tons ft.

$= 960 - 213{\cdot}3$

$= 746{\cdot}7$ tons ft.

B.M. at 10′ from ends $= (48 \times 10 - 2{\cdot}67 \times 5)$ tons ft.

$= 466{\cdot}7$ tons ft.

These values are used in constructing the B.M. diagram shown in Fig. 204.

Resistance moment of girder with one $\frac{5}{8}''$ plate on each flange:

Taking A as the net flange area,

$$A = (16 \times \tfrac{5}{8}) + (2 \times 5{\cdot}75) - 2 \times \tfrac{15}{16} \times \tfrac{9}{8} - 2 \times \tfrac{15}{16} \times \tfrac{1}{2}$$

$$= 10 + 11{\cdot}5 - 2{\cdot}11 - {\cdot}94 = 18{\cdot}45 \text{ in.}^2.$$

$D = 47\frac{1}{4}''$ (taken slightly less owing to the removal of outer plates).

$f = 8$ tons/in.^{2}.

$$M = f.A.D = 8 \times 18{\cdot}45 \times 47{\cdot}25 \div 12 \text{ tons ft.}$$

$$= 580 \text{ tons ft.}$$

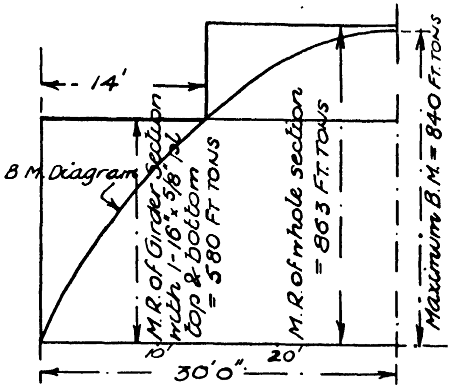

FIG. 204.—CURTAILMENT OF FLANGE PLATE DIAGRAM.

For whole girder section:

$A = 27{\cdot}27$ in.^{2} (net area).

$M = f.A.D$

$= 8 \times 27{\cdot}27 \times 47{\cdot}5 \div 12$ tons ft.

$= 863$ tons ft.

The length of a curtailed plate, as determined by the B.M. diagram method, should be extended by about a foot at each end. Some designers make calculations to determine the number of rivets which would be required *in the extension* to equal the whole or part of the plate strength (see B.S.S. No. 449–1937).

Splices.—*Web.*—The web plate will be spliced at the centre of the girder, at which point there is theoretically little shear.

Angles.—About 40′ is the usual maximum length for angles, without splicing, though longer angles may be used if necessary. The rivet strength, in single shear, each side of the splice, should equal the strength of the cut angle, at the working flange stress, the cover being composed of a *bosom* angle.

Taking the case of the tension flange angle:

Net area of one $6'' \times 6'' \times \frac{1}{2}''$ L $= [5{\cdot}75 - 2 \times \frac{15}{16} \times \frac{1}{2}]$ ins.^{2}

$= 5{\cdot}75 - {\cdot}94 = 4{\cdot}81$ ins.^{2}.

Strength of L at 8 tons/in.^{2} $= 4{\cdot}81 \times 8 = 38{\cdot}48$ tons.

∴ No. of rivets required in cover, each side of splice,

$$= \frac{38{\cdot}48}{3{\cdot}6} = 11 \text{ rivets.}$$

∴ Use 6 rivets in each vertical and horizontal leg.

The covers will be formed by $6'' \times 6'' \times \frac{1}{2}''$ bosom angles, one on each side of the flange. A similar joint will be made in the compression flange angles, and the two joints will be symmetrically disposed with respect to the centre line of girder. They will be positioned respectively about halfway between the flange plate splice and the web splice.

Joint in Inner $16'' \times \frac{5}{8}''$ *Flange Plates.*—There will be one joint in each flange, symmetrically placed with respect to the centre of girder.

Considering the joint in the tension flange, as the splice occurs more than 14 ft. from the end of the girder (i.e. at a point where both $\frac{5}{8}''$ flange plates are required) a $\frac{5}{8}''$ cover plate will be used. This represents common practice, though some regulations require a slightly thicker cover than simply the cut plate thickness itself.

Sufficient rivets should be placed each side of the splice to equal the strength of the cut plate.

Net area of $16'' \times \frac{5}{8}''$ plate $= (16 - 2 \times \frac{15}{16}) \times \frac{5}{8}$ ins.^{2}
$= 8{\cdot}83$ ins.^{2}.

Strength of plate at 8 tons/in.^{2} $= (8{\cdot}83 \times 8)$ tons
$= 70{\cdot}64$ tons.

$1 - \frac{7}{8}''$ diameter rivet in single shear $= 3{\cdot}6$ tons.

∴ No. of rivets required $= \dfrac{70{\cdot}64}{3{\cdot}6} = 20.$

As there are four rows of rivets we require five rivets in each row. At 6″ pitch this would require about 2′ 6″ length each side of the splice. Cover plate will be made 5′ long.

Length of outer $\frac{5}{8}''$ plate (in each flange) will be made

$$60' - 2 \times 12'\ 6'' = 35 \text{ ft.}$$

(See detail on elevation of girder, Plate IV.)

Camber.—The deflection of a plate girder designed to maintain $\dfrac{M}{I}$ constant, i.e. to make the moment of inertia of the section vary directly as the changing B.M. throughout the span, is given by $\dfrac{ML^2}{8EI}$ (see page 130).

The I-value for the designed section will be found to be approximately 35000 ins.[4].

$$\therefore \text{ Maximum deflection} = \frac{840 \times 12 \times 60 \times 12 \times 60 \times 12}{8 \times 13000 \times 35000}$$

$$= 1\tfrac{1}{2}''.$$

Hence the camber should be made $1\frac{1}{2}''$.

The detail shown at the plate girder end (hole positions, etc.) was for a strap joint to two columns, in the original design. The girder is supported by a joist column, the diagonal hatching lines indicating countersinking of rivets.

Design and Detail of a Lattice Girder

This lattice girder is one of a series erected in a London garage, the dimensions of which are $200' \times 90'$. There is no internal support of any description, the floor space being wholly unrestricted.

The loads from the roof trusses are shown in Fig. 206, which comprises the space diagram, the force diagram and the tabulated forces obtained from the force diagram.

Design

Main Compression Boom.—Maximum force in boom (members 5H and 6J) $= 52{\cdot}0^{T}$ compression.

Length of member $= 11'\ 3''$ (see Fig. 206).

Each length of compression member, in a lattice girder of this type, may be considered as being adequately restrained, in both position and direction, at each end, the effective length then being taken at ·75 actual length.

It is usual in designing roof truss and latticed frame members, to make an allowance for the fact that the maximum allowable stress cannot operate over the whole section, owing to the distance of the extreme tips of outstanding legs of angle sections (which are mainly used in this class of work), from the line of application of the load—which is the line of riveting.

This allowance is made by deducting, from the gross area of the section, an area equal to half the area of the outstanding leg or legs.

This is standard practice for tension members.

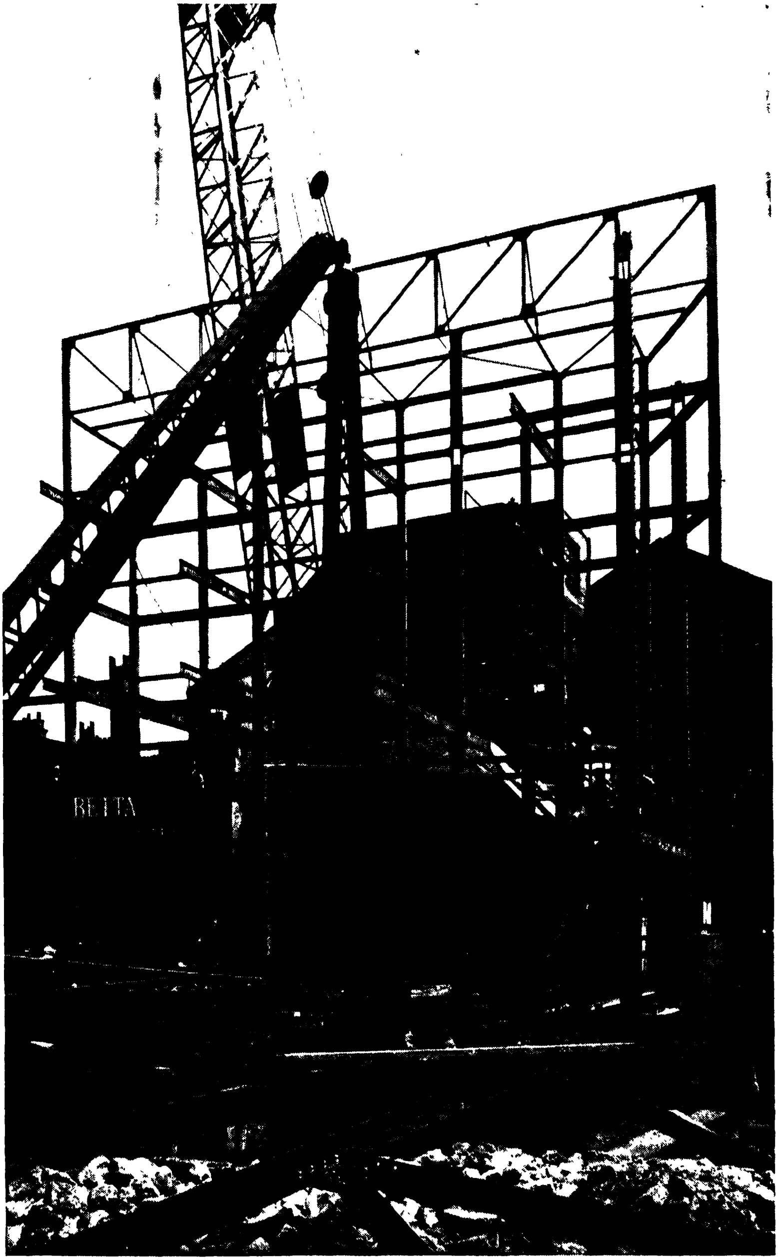

FIG. 205.—EMPLOYMENT OF LATTICE GIRDERS IN FACTORY BUILDING.
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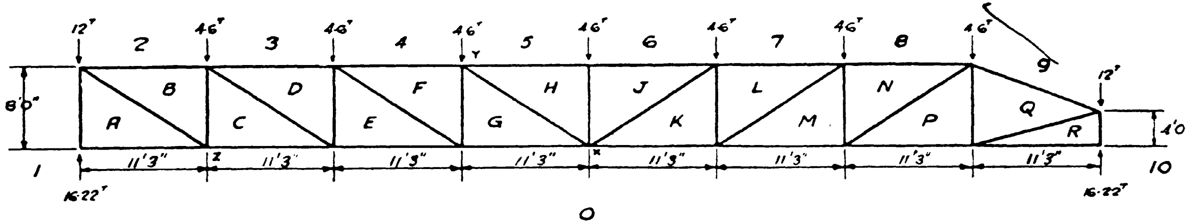
REACTIONS FROM TRUSSES = 4·35 T

WEIGHT OF GIRDER = 2·0 T = ·25 T AT EACH PANEL POINT

TOTAL LOAD = 4·60 T " " " "

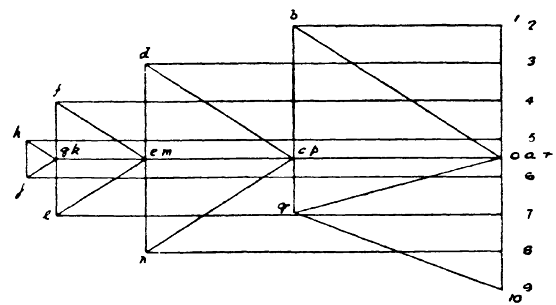


SPACE DIAGRAM

MEMBER	FORCE	MEMBER	FORCE	MEMBER	FORCE	MEMBER	FORCE
2B	228 Tons C	OA	0	AB	279 Tons T	JK	43 Tons T
3D	391 " C	OC	228 Tons T	BC	162 " C	KL	69 " C
4F	486 " C	OE	391 " T	CD	198 " T	LM	118 " T
5H	520 " C	OG	486 " T	DE	115 " C	MN	114 " C
6J	520 " C	OK	486 " T	EF	118 " T	NP	198 " T
7L	486 " C	OM	391 " T	FG	69 " C	PQ	66 " C
8N	391 " C	OP	228 " T	GH	43 " T	QR	238 " T
9Q	249 " C	OR	0	HJ	46 " C		

TABLE OF FORCES.

C REPRESENTS COMPRESSION T REPRESENTS TENSION



FORCE DIAGRAM

FIG. 206.

Use 2$\underline{|s}$ $6'' \times 4'' \times \frac{5}{8}''$. Gross area of section $= 11{\cdot}72$ in.^{2}.
Least radius of gyration $= 1{\cdot}51''$.

$$\frac{\text{Effective length of member}}{\text{Least radius of gyration}}\cdot\left(\frac{l}{r}\right) = \frac{{\cdot}75 \times 135}{1{\cdot}51} = 67.$$

Allowable stress (L.C.C. By-laws (1938)) $= 5{\cdot}56^T/\text{in.}^2$.

Net area of section $= 11{\cdot}72 - 3\frac{3}{8} \times \frac{5}{8} = 11{\cdot}72 - 2{\cdot}11$
$= 9{\cdot}61$ in.^{2}

Actual stress $= \dfrac{52{\cdot}0}{9{\cdot}61} = 5{\cdot}41^T/\text{in.}^2$.

$\therefore$ the section chosen is sufficient.

Riveting

Use $\frac{7}{8}''$ diameter rivets and $\frac{1}{2}''$ thick gusset plates.
Value of $\frac{7}{8}''$ diameter rivet in single shear $= 3{\cdot}6$ tons.
Value of $\frac{7}{8}''$ diameter rivet in bearing in $\frac{1}{2}''$ plate $= 5{\cdot}25$ tons.

$\therefore$ Number of $\frac{7}{8}''$ rivets required in S.S. $= \dfrac{52}{3{\cdot}6} = 15$ rivets.

" " " in bearing $= \dfrac{52}{5{\cdot}25} = 10$ rivets.

Actual riveting used (see detail)
$8/\frac{7}{8}''$ rivets in S.S. $= 8 \times 3{\cdot}6 = 28{\cdot}8$
$5/\frac{7}{8}''$ rivets in bearing $= 5 \times 5{\cdot}25 = 26{\cdot}2$
$= 55{\cdot}0$ tons.

The girder is delivered in two halves for convenience of handling and therefore a field joint will be made at the centre.

The forces in the members in the end bays are so much smaller than those in the remaining bays that it is economical to change the section of the main compression boom at the panel points, 11′ 3″ from either end.

Design of end compression members

Maximum load (member 9Q) $= 24{\cdot}9^T$ compression.
Length of member 12′.
Use 2$\underline{|s}$ $6'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$. Gross area $= 6{\cdot}84$ in.^{2}.
Least radius of gyration $= 1{\cdot}23''$.

$$\frac{\text{Effective length}}{\text{Least radius of gyration}} = \frac{{\cdot}75 \times 144}{1{\cdot}23} = 88.$$

Allowable stress $= 4{\cdot}44^T/\text{in.}^2$.

Net area of section $= 6{\cdot}84 - 3\frac{1}{8} \times \frac{3}{8} = 6{\cdot}84 - 1{\cdot}17$
$= 5{\cdot}67$ in.².

Actual stress $= \dfrac{24{\cdot}9}{5{\cdot}67} = 4{\cdot}4^{T}/\text{in.}^2$.

∴ Section chosen is sufficient.

Actual riveting used

$6/\frac{7}{8}''$ rivets in S.S. $= 6 \times 3{\cdot}6 = 21{\cdot}6$
$4/\frac{7}{8}''$ rivets in bearing $= 4 \times 5{\cdot}25 = 21{\cdot}0$
$42{\cdot}6$ tons.

This riveting is adequate to take the force in the member 8N which is jointed to the member 9Q.

Where the section of the frame is unbroken over a panel point, the number of rivets which connect the gusset plate to the unbroken member need not be more than is sufficient to take the difference between the forces in the portions of the section, on either side of the panel point.

Main Tension Boom

Maximum force in boom (members OG and OK) $= 48{\cdot}6^{T}$ tension.

Net area required $= \dfrac{48{\cdot}6}{8} = 6{\cdot}08$ in.².

Use 2∟s $6'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$. Gross area $= 9{\cdot}0$ in.².

Net area $= 9{\cdot}0 - 3{\cdot}0 \times {\cdot}5 - 2 \times \frac{15}{16} \times \frac{1}{2}$ (deduction for rivet holes)
$= 9{\cdot}0 - 1{\cdot}5 - {\cdot}94$
$= 9{\cdot}0 - 2{\cdot}44$
$= 6{\cdot}56$ in.².

∴ Section chosen is sufficient.

Riveting.—As for compression boom. Value $= 55{\cdot}0$ tons.

As there is no force in the end members OA and OR, the main section may be reduced at the panel points, 11′ 3″ from each end, where 2∟s $3'' \times 3'' \times \frac{3}{8}''$ will be used.

Riveting required at end of members OC and OP

Force in OC and OP $= 22{\cdot}8^{T}$.

Riveting supplied:

$= 4/\frac{7}{8}''$ rivets in S.S. $= 4 \times 3{\cdot}6 = 14{\cdot}4$
$3/\frac{7}{8}''$ rivets in bearing $= 3 \times 5{\cdot}25 = 15{\cdot}8$
$30{\cdot}2$ tons.

Intermediate Members

Compression members

Maximum load (member BC) = 16·2 tons.
Length of members = 8′.

Use 2 L̲s 3″ × 3″ × $\frac{3}{8}$″ ⅃⌐ Gross sectional area = 4·22 in.^{2}.
Least radius of gyration = 1·13″.

$$\frac{\text{Effective length}}{\text{Least radius of gyration}} = \frac{\cdot 75 \times 96}{1 \cdot 13} = 64.$$

Allowable stress = 5·71^{T}/in.^{2}.

Net area of section = 4·22 − 2$\frac{5}{8}$ × $\frac{3}{8}$ = 4·22 − ·99
= 3·23 in.^{2}.

$$\text{Actual stress} = \frac{16 \cdot 2}{3 \cdot 23} = 5 \cdot 02^{T}/\text{in.}^2$$

Riveting

Member BC. Number of $\frac{7}{8}$″ rivets in S.S.

$$= \frac{16 \cdot 2}{3 \cdot 6} = 5 \text{ rivets.}$$

Members DE and MN. Number of $\frac{7}{8}$″ rivets in S.S.

$$= \frac{11 \cdot 5}{3 \cdot 6} = 4 \text{ rivets.}$$

Members FG, HJ, KL and PQ. Number of $\frac{7}{8}$″ rivets in S.S.

$$= \frac{6 \cdot 9}{3 \cdot 6} = 2 \text{ rivets.}$$

Tension members

Members QR and AB. Maximum force = 27·9 tons.

$$\text{Area required} = \frac{27 \cdot 9}{8} = 3 \cdot 5 \text{ in.}^2$$

Use 2 L̲s 4″ × 4″ × $\frac{3}{8}$″. Gross area = 5·72 in.^{2}.

Net area of section = 5·72 − 3$\frac{5}{8}$ × $\frac{3}{8}$ − 2 × $\frac{15}{16}$ × $\frac{3}{8}$
= 5·72 − 1·36 − ·71 = 5·72 − 2·07
= 3·65 in.^{2}.

Riveting used

7/$\frac{7}{8}$″ rivets in bearing = 7 × 5·25 = 36·8 tons.

Remaining tension members.

Maximum load (member CD) = 19·8 tons.

$$\text{Area required} = \frac{19 \cdot 8}{8} = 2 \cdot 48 \text{ in.}^2$$

Use 2 L̲s 3″ × 3″ × $\frac{3}{8}$″. Gross area = 4·22 in.^{2}.

$$\text{Net area} = 4{\cdot}22 - 2\tfrac{5}{8}'' \times \tfrac{3}{8}'' - 2 \times \tfrac{15}{16}'' \times \tfrac{3}{8}''$$
$$= 4{\cdot}22 - {\cdot}99 - {\cdot}71$$
$$= 4{\cdot}22 - 1{\cdot}7$$
$$= 2{\cdot}52 \text{ in.}^2.$$

Riveting

Members CD and NP. Number of $\frac{7}{8}''$ rivets in bearing

$$= \frac{19{\cdot}8}{5{\cdot}25} = 4 \text{ rivets.}$$

Members EF and LM. Number of $\frac{7}{8}''$ rivets in bearing

$$= \frac{11{\cdot}8}{5{\cdot}25} = 3 \text{ rivets.}$$

Members GH and JK. Number of $\frac{7}{8}''$ rivets in bearing

$$= \frac{4{\cdot}3}{5{\cdot}25} = 1 \text{ rivet.}$$

Method of Sections

The method of sections is an analytical method of determining the forces in the members of a frame. It is sometimes used instead of drawing a stress diagram. With complicated loading on frames, however, the process becomes cumbersome, and it will be found that the stress diagram method is at once the speedier and more simple.

Typical calculations for obtaining the forces by the method of sections are given below.

Force in 5H (see Fig. 206).

Take moments of external forces about the point X and divide by the perpendicular distance between X and the member 5H.

Force in member 5H

$$= \frac{16{\cdot}22 \times 45{\cdot}0 - {\cdot}12 \times 45{\cdot}0 - 4{\cdot}6 \times 33{\cdot}75 - 4{\cdot}6 \times 22{\cdot}5 - 4{\cdot}6 \times 11{\cdot}25}{8}$$

$$= \frac{730 - 316}{8} = \frac{414}{8} = 51{\cdot}8 \text{ tons.}$$

This compares with 52·0 tons from the stress diagram.

Force in OG

Take moments about the point Y.

Force in member OG

$$= \frac{16{\cdot}22 \times 33{\cdot}75 - {\cdot}12 \times 33{\cdot}75 - 4{\cdot}6 \times 22{\cdot}5 - 4{\cdot}6 \times 11{\cdot}25}{8}$$

$$= \frac{548 - 160}{8} = \frac{388}{8} = 48{\cdot}5 \text{ tons.}$$

This compares with 48·6 tons from the stress diagram.

Force in BC

The force in a vertical member is equal to the shear force at the point under consideration.

Shear force at Z = 16·22 − ·12 = 16·1 tons.

This compares with 16·2 tons from the stress diagram.

Force in AB

Considering the point Z, it follows that the vertical component of the force in AB must equal the vertical component (i.e. the force, since the member is vertical) of the force in BC.

$$\therefore \text{Force in AB} = \text{Force in BC} \times \frac{\text{Length of AB}}{\text{Length of BC}} = 16{\cdot}1 \times \frac{13{\cdot}8}{8}$$

$$= 27{\cdot}8 \text{ tons.}$$

This compares with 27·9 tons from the stress diagram.

CHAPTER XVI

DESIGN OF A STEEL FRAME FOR A SMALL WAREHOUSE BUILDING, WITH TYPICAL DETAILS

Introduction

It is intended in this chapter to demonstrate how the calculations for a composite steel frame may be set out and arranged in a form convenient for practical application. Points of theory not already explained in previous chapters are dealt with as they arise.

The building, as will be seen by reference to the architect's plans (Plate XII) has four floors and a pitched roof, and is one actually erected for a commercial firm.

The superimposed load to be allowed for on all floors is 3 cwts. per square foot, and this heavy loading will necessitate the use of a filler joist floor.

In the design of the steel frame the L.C.C. By-laws (1938) will be referred to, and the allowable pressures on soil and concrete (mixture 1 : 2 : 4) are taken at 3 tons per square foot and 30 tons per square foot, respectively.

On the data sheet it will be noticed that the various floor and wall loads are each brought to a fraction of a ton—a step which simplifies and quickens the ensuing calculations.

The following abbreviations are made use of in the calculations :

E.D.L. = Equivalent Distributed Load.

O. W. and C. = Own Weight and Casing.

R/L and R/R (in tables) = Left and right end reactions, respectively.

The weights of the lift motors and machinery are taken as centrally applied point loads on Beams R51, R53 and R91 (see Roof Steel plan), the actual joists required for these loads not being shown.

The ground floor is assumed to be laid on a hardcore filling,

which is placed on the soil, and therefore is not a 'suspended' floor, i.e. its weight is not taken by a frame, and consequently does not enter into the calculations.

Data Sheet

Flat roof over lifts

5" reinforced concrete = 60
$\frac{3}{4}$" asphalt = 9
69
Superimposed load = 30

99 lb./ft.^{2} = $\frac{1}{22 \cdot 6}$ tons/ft.^{2}.

Floor to lift-motor room

5" reinforced concrete = 60 lb./ft.^{2} = $\frac{1}{37 \cdot 4}$ tons/ft.^{2}.

Floors

6" concrete = 72
1½" finish = 18
Fillers = 6
96
Superimposed load = 336

432 lb./ft.^{2} = $\frac{1}{5 \cdot 18}$ tons/ft.^{2}.

Stairs and landings

Construction, say, = 100
Superimposed load = 100

200 lb./ft.^{2} = $\frac{1}{11 \cdot 2}$ tons/ft.^{2}.

Walls

9" wall at 120 lb./cu. ft. = 90 lb./ft.^{2} = $\frac{1}{25 \cdot 0}$ tons/ft.^{2}.

Lifts

10 tons each.

Design of Trusses (Fig. 207)

The usual assumption is made (for trusses supported by steel columns) that the horizontal reactions at each end are

equal; i.e. in this case, each equals 1 ton $\div 2 = \cdot 5^T$. In finding R_R (vertical) the correct proportion of each vertical load is taken and, in addition, the value of R_R required to resist the moment about R_L of the horizontal force of 1 ton acting at 4·33 ft. ($\frac{1}{2}$ truss height) above, is taken into account.

Compression Boom

Maximum load = 2·6 tons, compression.

Length = 5′ 9″.

Use 1 $\llcorner$ 3″ × 3″ × $\frac{1}{4}$″. Gross area = 1·44 in.².

Least r = ·59″.

Taking effective strut length as ·75 × actual length

$$\frac{L}{r} = \frac{\cdot 75 \times 69}{\cdot 59} = 88. \quad \text{Allowable stress} = 4\cdot 44 \text{ tons/in.}^2$$

It is good practice in roof truss design, when dealing with angles riveted along one leg only, to take in only half the area of the outstanding leg, both for compression and tension.

Effective area $= 1\cdot 44 - \frac{1}{2} \times 2\frac{3}{4}'' \times \frac{1}{4}'' = 1\cdot 44 - \cdot 35 = 1\cdot 09$ in.².

Actual stress $= \dfrac{2\cdot 6}{1\cdot 09} = 2\cdot 39$ tons/in.². Use 2/$\frac{3}{4}$″ rivets.

Tension Boom

Maximum load = 2·4 tons, tension.

Area required $= \dfrac{2\cdot 4}{8} = \cdot 3$ in.².

Use 1 $\llcorner$ 2″ × 2″ × $\frac{1}{4}$″. Effective net area

$$\begin{aligned} &= \cdot 937 - \tfrac{1}{2} \times 1\tfrac{3}{4}'' \times \tfrac{1}{4}'' - \tfrac{11}{16}'' \times \tfrac{1}{4}'' \\ &= \cdot 937 - \cdot 219 - \cdot 172 \\ &= \cdot 937 - \cdot 391 \\ &= \cdot 546 \text{ in.}^2 \end{aligned}$$

Use 2/$\frac{5}{8}$″ rivets.

Intermediate Members

Maximum compression = 1·2 tons.

Maximum tension = 1·5 tons.

Use 1 $\llcorner$ 2″ × 2″ × $\frac{1}{4}$″ throughout with 2/$\frac{5}{8}$″ rivets.

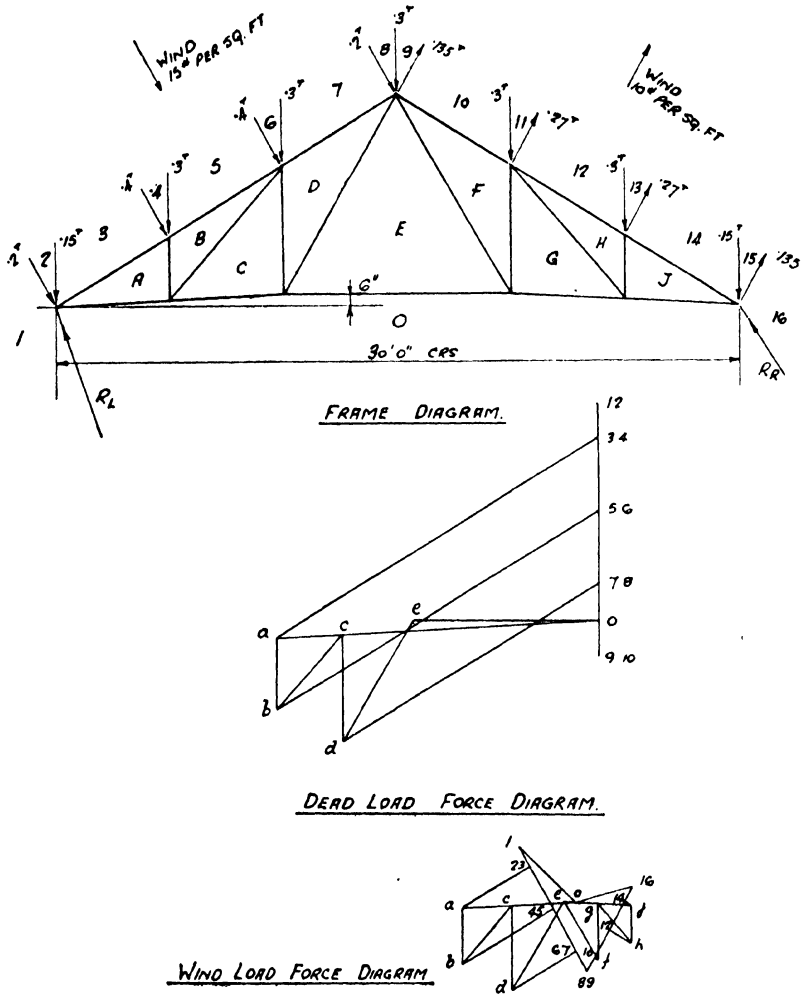


FIG. 207.

Note.—Height of truss = 15′ × tan 30° = 8·66′
Vertical R_R due to wind = $\cdot 26^T - \cdot 525^T + \cdot 145^T$
= $\cdot 12^T$ downward.
Horizontal ,, ,, ,, = $\cdot 5^T$ to left.
These components enable 16–0 to be drawn in Wind Force Diagram.

B.S.S. 449 requires the wind loads to be taken separately 'with or without' suction. L.C.C. By-laws require suction to be considered independently of windward force. The various cases are left as exercises for the reader.

CALCULATIONS

DEAD LOAD

	Slope		*Flat*
Tiles	4		
Boarding	4		
Purlins	2		
	10	=	11·5
Truss		=	2·5
		Total	14·0 lb./ft.^{2}.

$$\text{Total Load} = \frac{30 \times 10 \times 14}{2240} = 1{\cdot}8^T \; (= {\cdot}3^T \text{ per panel point}).$$

WIND LOADS

$$\text{Windward} = \frac{17{\cdot}3 \times 10 \times 15}{2240} = 1{\cdot}2^T \; (= {\cdot}4^T \text{ per panel point}).$$

Pitch of Roof = 30°.

$\therefore$ Horizontal component = $\cdot 6^T$ (i.e. $1\cdot2 \cos 60°$).

Vertical component = $1\cdot04^T$ downward.

$$\text{Leeward} = \frac{17{\cdot}3 \times 10 \times 10}{2240} = {\cdot}8^T \; (= {\cdot}27^T \text{ per panel point}).$$

Horizontal component = $\cdot 4^T$

Vertical component = $\cdot 7^T$ upward.

REACTIONS (*taking positive and negative wind loads*)

Horizontal

Total horizontal force = $\cdot6^T + \cdot4^T = 1\cdot0^T$. $\therefore R_L = R_R = \cdot5^T$.

Vertical

Total vertical force = $1\cdot8^T + 1\cdot04^T - \cdot7^T = 2\cdot14^T$.

$$R_R = \frac{1{\cdot}8}{2} + \frac{1{\cdot}04}{4} - \frac{{\cdot}7 \times 3}{4} + \frac{1{\cdot}0 \times 4{\cdot}33}{30}$$

$$= {\cdot}9 + {\cdot}26 - {\cdot}525 + {\cdot}145 = {\cdot}78^T \text{ upward.}$$

$$\therefore R_L = 2{\cdot}14 - {\cdot}78 = 1{\cdot}36^T \text{ upward.}$$

MEMBER.	D.L.		W.L.		W.R.		TOTAL.	
	C	T	C	T	C	T	C	T
3A, 14J .	1·7		·7			·1	2·4	
5B, 12H .	1·7		·9			·2	2·6	
7D, 10F .	1·3		·7			·1	2·0	
OA, OJ .		1·4		1·0	·6			2·4
OC, OG .		1·2		·6	·3			1·8
OE . .		·9		·1		·1		1·0
AB, HJ .	·3		·5			·3	·8	
BC, GH .		·5		·7	·5			1·2
CD, FG .	·5		·7			·5	1·2	
DE, EF .		·6		·9	·6			1·5

TABLE OF FORCES.

Wind Bracing

Horizontal load per truss = 1·0 ton.

Each pair of wind braces is assumed to take all the horizontal load out of the truss, and transfer it to the stanchions.

∴ Horizontal load in each wind brace = ·5 tons.

The actual load in the wind brace (which is in the plane of the rafter) is given by dividing the actual length of the wind brace by its horizontal flat projection and multiplying by the horizontal load.

$$\therefore \text{Load in wind brace} = \cdot 5 \times \frac{11\cdot 5}{5\cdot 0} \times \frac{5\cdot 75}{5\cdot 0} = 1\cdot 32 \text{ tons tension.}$$

Use 1∟ 2″ × 2″ × $\frac{1}{4}$″ with 2/$\frac{5}{8}$″ rivets.

Purlins

Dead load per square foot of slope = 10
Wind load per square foot of slope = 15

25 lb./ft.^{2}.

$$\therefore \text{Load per purlin } (4' \text{ centres}) = \frac{10 \times 4 \times 25}{2240} = \cdot 446 \text{ tons.}$$

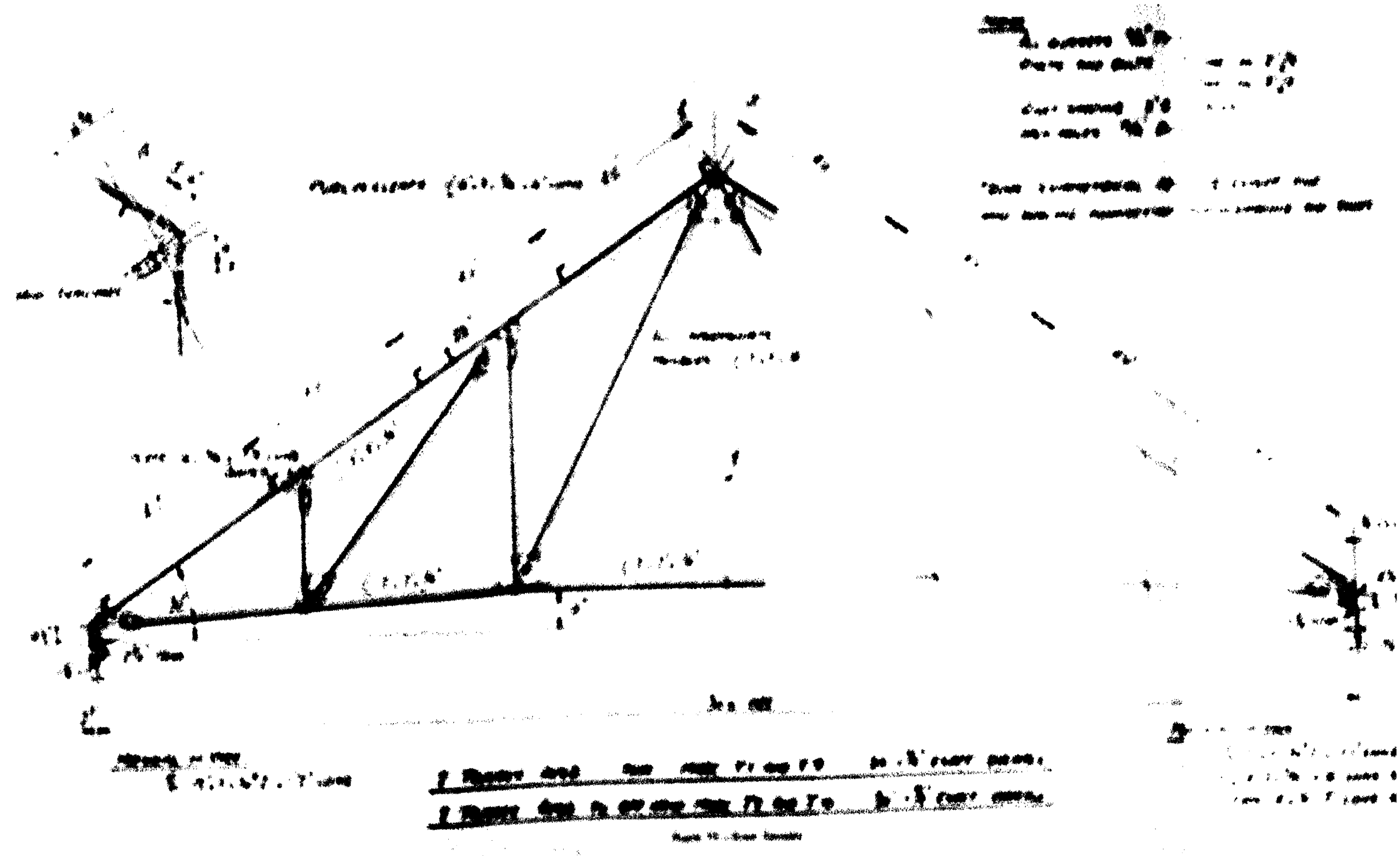
Maximum B.M. on purlin = $\frac{Wl}{10}$, because the purlins are continuous.

$$\therefore \text{B.M.} = \frac{\cdot 446 \times 10}{10} = \cdot 446 \text{ tons ft.}$$

Modulus required = 1·5 × ·446 = ·669 in.^{3}.
(Z = 1·5 × B.M. in tons ft., see Chapter VI.)
Use 1∟ 3$\frac{1}{2}$″ × 3″ × $\frac{1}{4}$″. Modulus supplied = ·75 in.^{3}.

Design of Roof Steel (see Plate VIII, page 318)

The load values used in the calculations are obtained from the data sheet. The left and right end reactions are tabulated for the purpose of the stanchion calculations later. The reader should identify the particular beam section being considered, in the steelwork plans (Plate VIII) by means of the reference numbers in the last column of the calculation sheet.



Calculation Sheet

Motor Roof Steel

[Figures thus (24·5) beneath section chosen indicate the modulus actually provided.]

Loads and Calculations.	R/L. Tons.	R/R. Tons.	Section.	Beam Ref.
Span 10′ 0″. Ties			6″ × 3″ × 12 lb. I	51. 91.
Span 20′ 0″. Roof $\frac{20 \times 5}{22\cdot6} = 4\cdot4$ O.W. & C. $= 1\cdot2$ $5\cdot6^{T}$ Z reqd. $= \frac{5\cdot6 \times 20}{5\cdot33} = 21\cdot0$ in.^{3} [$Z = \frac{W \times \text{span in ft.}}{5\cdot33}$, see Chap. VI]	2·8	2·8	10″ × 4½″ × 25 lb. I (24·5)	52. 54.
Span 20′ 0″. Wall $\frac{20 \times 2}{25\cdot0} = 1\cdot6$ 1·8 O.W. $= \cdot2$ 1·0 $1\cdot8^{T}$ 2·8 Pt. load at centre from T5 $= \frac{1\cdot4 \times 20}{30} = \cdot93^{T}$, say $1\cdot0^{T}$ ∴ E.D.L. $= 1\cdot8 + 2\cdot0 = 3\cdot8^{T}$ ∴ Z reqd. $= \frac{3\cdot8 \times 20}{5\cdot33} = 14\cdot3$ in.^{3} Maximum deflection $= \frac{5 \times 1\cdot8 \times 240^3}{384 \times 13000 \times 81\cdot1}$ $+ \frac{1\cdot0 \times 240^3}{48 \times 13000 \times 81\cdot1}$ $= \cdot307 + \cdot273$ $= \cdot58''$ $\frac{L}{325} = \frac{240}{325} = \cdot74''$ (By-law value for maximum deflection) ∴ The section is adequate.	1·4	1·4	9″ × 4″ × 21 lb. I (18·0)	54 × Low Level.

Roof Steel

Loads and Calculations.	R/L. Tons.	R/R. Tons.	Section.	Beam Ref.
Span 20′ 0″. Wall $\frac{20{\cdot}0 \times 5{\cdot}05 \text{ (aver. height)}}{25{\cdot}0} = 4{\cdot}1$ Roof $\frac{20 \times 5{\cdot}0 \times 14}{2240} = {\cdot}7$ O.W. & C. $= 1{\cdot}2$ $6{\cdot}0^{\mathrm{T}}$ Z reqd. $= \frac{6{\cdot}0 \times 20}{5{\cdot}33} = 22{\cdot}5$ in.^{3} *Note.*—Roof load = 14 lb./ft.^{2} (see page 297).	3·0	3·0	10″ × 4½″ × 25 lb. I (24·5)	11. 31. 131.151.
Span 20′ 0″. O.W. & C. $= 1{\cdot}2^{\mathrm{T}}$ 1·2 Point load at centre from trusses $\} = 2{\cdot}8^{\mathrm{T}}$ 2·8 4·0 E.D.L. $= 1{\cdot}2 + 5{\cdot}6 = 6{\cdot}8^{\mathrm{T}}$ Z reqd. $= \frac{6{\cdot}8 \times 20}{5{\cdot}33} = 25{\cdot}5$ in.^{3}	2·0	2·0	10″ × 5″ × 30 lb. I (29·2)	22. 102.
Span 20′ 0″. Wall $= \frac{20 \times 5{\cdot}77}{2 \times 25{\cdot}0} = 2{\cdot}3$ Roof as 11 etc. $= {\cdot}7$ O.W. & C. $= 1{\cdot}2$ 4·2 + 2·0 $4{\cdot}2^{\mathrm{T}}$ 6·2 Point load at centre $= 2{\cdot}0^{\mathrm{T}}$ E.D.L. $= 4{\cdot}2 + 4{\cdot}0 = 8{\cdot}2^{\mathrm{T}}$ Z reqd. $= \frac{8{\cdot}2 \times 20}{5{\cdot}33} = 30{\cdot}8$ in.^{3}	3·1	3·1	12″ × 5″ × 32 lb. I (36·8)	21. 141.

Roof Steel—*continued*

Loads and Calculations.	R/L. Tons.	R/R. Tons.	Section.	Beam Ref.
Span 20′ 0″. O.W. & C. = $1\cdot2^{T}$ 1·2 Point load at centre = $2\cdot4^{T}$ 2·4 3·6 E.D.L. = $1\cdot2+4\cdot8 = 6\cdot0^{T}$ Z reqd. = $\frac{6\cdot0\times20}{5\cdot33} = 22\cdot5$ in.^{3}	1·8	1·8	10″ × 4½″ × 25 lb. I (24·5)	62.
Span 20′ 0″. O.W. & C. = $1\cdot2^{T}$ 1·2 Point load at centre = $1\cdot4^{T}$ 1·4 2·6 E.D.L. = $1\cdot2+2\cdot8 = 4\cdot0^{T}$ Z reqd. = $\frac{4\cdot0\times20}{5\cdot33} = 15\cdot0$ in.^{3}	1·3	1·3	9″ × 4″ × 21 lb. I (18·0)	12. 42. 82. 92. 122.
Span 10′ 0″. Roof $\frac{10\cdot0\times5\cdot0\times14}{2240}$ = ·4 Wall $\frac{10\cdot0\times8\cdot0}{25\cdot0}$ = 3·2 3·8 O.W. = ·2 5·0 $3\cdot8^{T}$ 8·8 Point load at centre from lift = 5^{T} E.D.L. = $3\cdot8+10\cdot0 = 13\cdot8^{T}$ Z reqd. = $\frac{13\cdot8\times10}{5\cdot33} = 25\cdot9$ in.^{3}	4·4	4·4	10″ × 6″ × 40 lb. I (40·9) It is usual to make beams carrying lift gear larger than necessary, to allow for vibration.	51. 91.
Span 10′ 0″. Wall as 51 = 3·2 3·4 O.W. = ·2 10·0 $3\cdot4^{T}$ 13·4 Point load at centre = $10\cdot0^{T}$ E.D.L. = $3\cdot4+20\cdot0 = 23\cdot4^{T}$ Z reqd. = $\frac{23\cdot4\times10}{5\cdot33} = 43\cdot9$ in.^{3}	6·7	6·7	12″ × 6″ × 44 lb. I (52·5)	53.

Roof Steel—*continued*

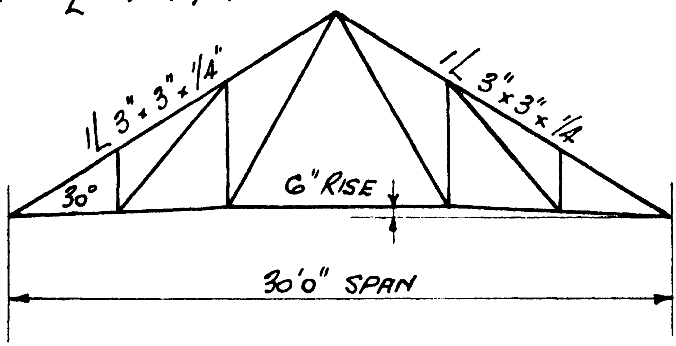
Loads and Calculations.	R/L. Tons.	R/R. Tons.	Section.	Beam Ref.
Span 20′ 0″. Floor $\frac{20 \cdot 0 \times 5 \cdot 0}{37 \cdot 4} = 2 \cdot 7$ Wall $\frac{20 \cdot 0 \times 8 \cdot 0}{25 \cdot 0} = 6 \cdot 4$ O.W. $= \cdot 4$ $9 \cdot 5$ $9 \cdot 5^{\text{T}}$ $6 \cdot 7$ Point load at centre $= 6 \cdot 7^{\text{T}}$ $16 \cdot 2$ E.D.L. $= 9 \cdot 5 + 13 \cdot 4 = 22 \cdot 9^{\text{T}}$ Z reqd. $= \frac{22 \cdot 9 \times 20}{5 \cdot 33} = 86 \cdot 0$ in.^{3}	8·1	8·1	16″ × 6″ × 62 lb. I (90·7)	52.
Span 20′ 0″. Floor as 52 $= 2 \cdot 7$ Wall $\frac{20 \times 5 \cdot 75}{25 \cdot 0} = 4 \cdot 6$ O.W. $= \cdot 4$ $7 \cdot 7$ $6 \cdot 7$ $7 \cdot 7^{\text{T}}$ Point load at centre $= 6 \cdot 7^{\text{T}}$ $14 \cdot 4$ E.D.L. $= 7 \cdot 7 + 13 \cdot 4 = 21 \cdot 1^{\text{T}}$ Z reqd. $= \frac{21 \cdot 1 \times 20}{5 \cdot 33} = 79 \cdot 0$ in.^{3}	7·2	7·2	16″ × 6″ × 62 lb. I (90·7)	54.

Design of Floor Steel (see Plate VIII)

The floors are designed in order from the fourth floor downwards. The second floor steel is the same as the third floor steel.

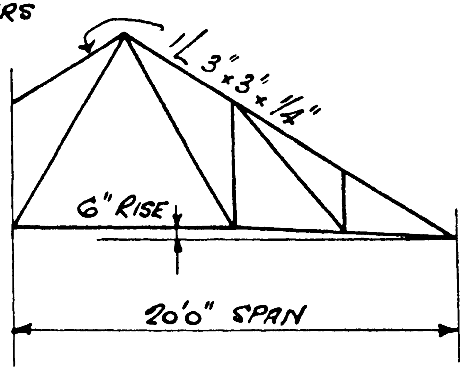
The reader should carefully read the notes at the bottom of Plate VIII. The identification letters for the various floors must be noted. Stanchions marked with a cross, thus '6 ×,' are supported by beams (the particular stanchion resting on beam D61), as will be seen by reference to stanchion schedule and the note on 4th floor steel plan. It is essential that the architect's and steel plans be carefully considered before working through the calculation sheets.

REMAINING MEMBERS
ALL 1∠ 2" × 2" × 1/4".



7 OFF THUS MARKED T1, T2, T4, T6, T8, T9 AND T10.

REMAINING MEMBERS
ALL 1∠ 2" × 2" × 1/4".



3 OFF THUS MARKED T3, T5, AND T7.

3/4" DIAR. RIVETS IN 3" ∠s.

5/8" DIAR. RIVETS IN 2" ∠s.

2 RIVETS TO EACH MEMBER.

PLATE VII.—TRUSS SCANTLINGS.

Calculation Sheet

Fourth Floor Steel

Loads and Calculations.	R/L. Tons.	R/R. Tons.	Section.	Beam Ref.
Span 10′ 0″. Suppose fillers to be spaced at S′ apart. Load per filler $= \frac{10 \times S}{5{\cdot}18}$ $= 1{\cdot}93S$ tons. Z reqd. $= \frac{1{\cdot}93S \times 10 \times 12}{8 \times 9 \text{ tons/in.}^2}$ $= 3{\cdot}22S$ in.^{3} If S = 1′ 8″ Z reqd. $= 3{\cdot}22 \times 1{\cdot}67 = 5{\cdot}41$ in.^{3}. Use 5″×3″×11 lb. I's at 1′ 8″ centres.			5″×3″×11 lb. I (5·47)	Fillers.
Span 20′ 0″. Wall $20 \times 9 = 180$ less $2 \times 7 \times 4 = 56$ $124 \times \frac{1}{25} = 5{\cdot}0$ Floor $= \frac{20 \times 5}{5{\cdot}18} = 19{\cdot}3$ O.W. $= \frac{20 \times 55 \text{ lb.}}{2240} = {\cdot}5$ $24{\cdot}8^T$ Z reqd. $= \frac{24{\cdot}8 \times 20}{5{\cdot}33} = 93{\cdot}0$ in.^{3}	12·4	12·4	18″×6″×55 lb. I (93·5)	11. 21. 31. 141. 151.
Span 20′ 0″. Floor $= \frac{20 \times 10}{5}$ 38·6 O.W. & C. $= 20\left(\frac{89 + 22 \times 11{\cdot}5}{2240}\right)$ $= 3{\cdot}0$ $41{\cdot}6^T$ Z reqd. $= \frac{41{\cdot}6 \times 20}{5{\cdot}33} = 156$ in.^{3}	20·8	20·8	20″×7½″×89 lb. I (167)	13. 23. 33. 63. 71. 73. 103. 111. 113.

Fourth Floor Steel—*continued*

Loads and Calculations.	R/L. Tons.	R/R. Tons.	Section.	Beam Ref.
Span 10′ 0″. Floor $\frac{10 \times 10}{5{\cdot}18} = 19{\cdot}3$ O.W. & C. $= {\cdot}7$ $20{\cdot}0^T$ Z reqd. $= \frac{20{\cdot}0 \times 10}{5{\cdot}33} = 37{\cdot}5$ in.^{3}	10·0	10·0	13″ × 5″ × 35 lb. I (43·5)	55. 97.
Span 20′ 0″. Wall as 11 etc. $= 5{\cdot}0$ 5·8 O.W. $= {\cdot}8$ 20·8 $5{\cdot}8^T$ 26·6 Point load at centre $= 20{\cdot}8^T$ E.D.L. $= 5{\cdot}8 + 41{\cdot}6 = 47{\cdot}4^T$ Z reqd. $= \frac{47{\cdot}4 \times 20}{5{\cdot}33} = 178$ in.^{3}	13·3	13·3	24″ × 7½″ × 95 lb. I (211·1)	12. 42. 82. 122.
Span 20′ 0″. O.W. & C. $= 3{\cdot}4^T$ 3·4 Point load at centre $= 30{\cdot}8^T$ 30·8 34·2 E.D.L. $= 3{\cdot}4 + 61{\cdot}6 = 65{\cdot}0^T$ Z reqd. $= \frac{65{\cdot}0 \times 20}{5{\cdot}33} = 244$ in.^{3}	17·1	17·1	24″ × 7½″ × 95 lb. I + 2/9″ × ⅜″ Pls (258)	62. 102.
Span 20′ 0″. O. W. & C. $= 4{\cdot}0^T$ 4·0 Point load at centre $= 41{\cdot}6^T$ 41·6 45·6 E.D.L. $= 4{\cdot}0 + 83{\cdot}2 = 87{\cdot}2^T$ Z reqd. $= \frac{87{\cdot}2 \times 20}{5{\cdot}33} = 327$ in.^{3}	22·8	22·8	24″ × 7½″ × 95 lb. I + 2/10″ × ¾″ Pls (344)	22. 32. 72. 112.

Fourth Floor Steel—*continued*

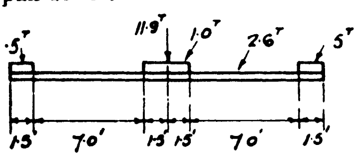
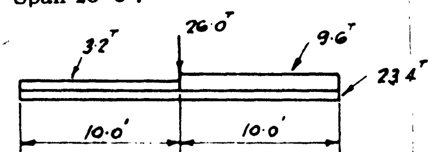
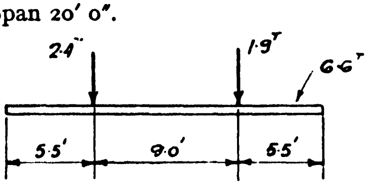
<table>
<tr><th>Loads and Calculations.</th><th>R/L. Tons.</th><th>R/R. Tons.</th><th>Section.</th><th>Beam Ref.</th></tr>
<tr><td>Span 10′ 0″.
Wall $\frac{10 \times 9\cdot0}{25\cdot0} = 3\cdot6$
O.W. $= \cdot2$
$3\cdot8^{T}$
Z reqd. $= \frac{3\cdot8 \times 10}{5\cdot33} = 7\cdot2$ in.^{3}</td><td>1·9</td><td>1·9</td><td>7″×4″×16 lb. I
(11·3)</td><td>53.</td></tr>
<tr><td>Span 20′ 0″.
Wall $\frac{20 \times 90}{25\cdot0} = 7\cdot2$ 7·6
O.W. $= \cdot4$ 1·9
$7\cdot6^{T}$ 9·5
Point load at centre $= 1\cdot9^{T}$
E.D.L. $= 7\cdot6 + 3\cdot8 = 11\cdot4^{T}$
Z reqd. $= \frac{11\cdot4 \times 20}{5\cdot33} = 42\cdot8$ in.^{3}</td><td>4·8</td><td>4·8</td><td>13″×5″×35 lb. I
(43·5)</td><td>52.</td></tr>
<tr><td>Span 20′ 0″.


FIG. 208.
·5
·5
1·0
O.W. & C. $= 2\cdot6^{T}$ 2·6
End piers $= \frac{1\cdot5 \times 8\cdot0}{25\cdot0} = \cdot5^{T}$ ea.   11·9
16·5
Central pier $= \frac{3\cdot0 \times 8\cdot0}{25\cdot0} = 1\cdot0^{T}$
Point load at centre $= 11\cdot9^{T}$
Z reqd.
$= 1\cdot5[8\cdot3 \times 10\cdot0 - \cdot5 \times 9\cdot25 - \cdot5 \times \cdot75 - 1\cdot3 \times 5\cdot0]$
$= 1\cdot5[83 - 4\cdot62 - \cdot38 - 6\cdot5]$
$= 1\cdot5[83\cdot0 - 11\cdot5]$
$= 1\cdot5 \times 71\cdot5 = 107\cdot3$ in.^{3}</td><td>8·3</td><td>8·3</td><td>20″×6½″×65 lb. I
(122·6)</td><td>54.</td></tr>
</table>

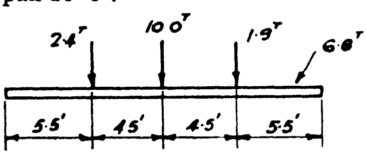
Fourth Floor Steel—*continued*

Loads and Calculations.	R/L. Tons.	R/R. Tons.	Section.	Beam Ref.
Span 20′ 0″. Floor as 13, etc. = 38·6 42·6 O.W. & C. = 4·0 6·7 42·6T 49·3 Point load at centre from S6× (see page 328) = 6·7T E.D.L. = 42·6 + 13·4 = 56·0T Z reqd. $= \frac{56·0 \times 20}{5·33} = 210$ in.³	24·7	24·7	24″×7½″×95 lb. I (211·1)	61. 101.
Span 20′ 0″.  Fig. 209. Floor (i) $= \frac{20 \times 5}{5·18} = 19·3$ 3·2 26·0 9·6 O.W. & C. = 4·1 23·4 23·4T 62·2 Floor (ii) $= \frac{10 \times 5}{5·18} = 9·6$T Wall $= \frac{10 \times 8·0}{25·0} = 3·2$T Point load at centre = 17·7 from S5× (see page 328) 8·3 from D54 26·0T $\therefore R_L = \frac{23·4}{2} + \frac{26·0}{2} + \frac{9·6}{4} + \frac{3·2 \times 3}{4}$ = 11·7 + 13·0 + 2·4 + 2·4 = 29·5T. $\therefore R_R$ = 62·2 − 29·5 = 32·7T $\therefore Z$ reqd. = 1·5[29·5 × 10·0 − 3·2 × 5·0 − 11·7 × 5·0] = 1·5[295·0 − 16·0 − 58·5] = 1·5[295·0 − 74·5] = 1·5 × 220·5 = 330·8 in.³	29·5	32·7	24″×7½″×95 lb. I + 2/10″×¾″ Pls (344)	51.

Fourth Floor Steel—*continued*

Loads and Calculations.	R/L. Tons.	R/R. Tons.	Section.	Beam Ref.
Span 9′ 0″. Stairs $\frac{9.0 \times 2.12}{11.2} = 1.7$ O.W. & C. $= \cdot 3$ 2.0^T Z reqd. $= \frac{2.0 \times 9}{5.33} = 3.4$ in.^{3}	1·0	1·0	6″ × 3″ × 12 lb. I (7·0)	Raker 94.
Span 10′ 0″. Landing $\frac{10.0 \times 2.75}{11.2} = 2.4$ 1·0 O.W. & C. $= \cdot 4$ 2·8 2.8^T 3·8 Point load at centre $= 1.0^T$ E.D.L. $= 2.8 + 2.0 = 4.8^T$ Z reqd. $= \frac{4.8 \times 10}{5.33} = 9.0$ in.^{3}	1·9	1·9	7″ × 4″ × 16 lb. I (11·3)	93.
Span 10′ 0″. Landing & O.W. & C. as 93 $= 2.8^T$ 2·8 Point load at centre $= 2.0^T$ 2·0 4·8 E.D.L. $= 2.8 + 4.0 = 6.8^T$ Z reqd. $= \frac{6.8 \times 10}{5.33} = 12.8$ in.^{3}	2·4	2·4	8″ × 4″ × 18 lb. I (13·9)	95. Low Level
Span 20′ 0″.  FIG. 210. 2·4 1·9 6·6 10·9	5·6	5·3	13″ × 5″ × 35 lb. I (43·5)	92.

Fourth Floor Steel—*continued*

Loads and Calculations.	R/L. Tons.	R/R. Tons.	Section.	Beam Ref.
Wall $= 20\times9 \quad = 180$ less $2/3{\cdot}0\times4{\cdot}0 = \underline{24}$ $156\times\frac{1}{25{\cdot}0}=6{\cdot}2$ O.W. $= \underline{{\cdot}4}$ $6{\cdot}6^T$ $R_L = \frac{6{\cdot}6}{2}+1{\cdot}9+\frac{{\cdot}5\times14{\cdot}5}{20}$ $= 3{\cdot}3+1{\cdot}9+{\cdot}4 = 5{\cdot}6^T$ $\therefore R_R = 10{\cdot}9 - 5{\cdot}6 \quad = 5{\cdot}3^T$ Zero shear from L.H. End $= \frac{5{\cdot}6-2{\cdot}4}{6{\cdot}6}\times20$ $= \frac{3{\cdot}2\times20}{6{\cdot}6} = 9{\cdot}7'$. $\therefore$ Z reqd. $=1{\cdot}5[5{\cdot}6\times9{\cdot}7-2{\cdot}4\times4{\cdot}2-3{\cdot}2\times4{\cdot}85]$ $=1{\cdot}5[54{\cdot}3 - 10{\cdot}1 - 15{\cdot}5]$ $=1{\cdot}5[54{\cdot}3 - 25{\cdot}6] = 1{\cdot}5\times28{\cdot}7$ $=43{\cdot}05$ in.^{3}			$13''\times5''\times35$ lb. I (43·5)	92 *cont.*
Span 20′ 0″.  FIG. 211. $2{\cdot}4$ $10{\cdot}0$ $1{\cdot}9$ $\underline{6{\cdot}8}$ Wall $= 20\times9 = 180$ less $4\times7 \quad = \underline{28}$ $\quad 21{\cdot}1$ $152\times\frac{1}{25{\cdot}0} = 6{\cdot}1$ O.W. $= \underline{{\cdot}7}$ $6{\cdot}8^T$ $R_L = 3{\cdot}4+5{\cdot}0+1{\cdot}9+\frac{{\cdot}5\times14{\cdot}5}{20}$ $= 10{\cdot}3+{\cdot}4 = 10{\cdot}7^T$ $\therefore R_R = 21{\cdot}1 - 10{\cdot}7 \quad = 10{\cdot}4^T$ $\therefore$ Z reqd. $= 1{\cdot}5[10{\cdot}7\times10{\cdot}0-2{\cdot}4\times4{\cdot}5$ $- 3{\cdot}4\times5{\cdot}0]$ $= 1{\cdot}5[107-10{\cdot}8-17{\cdot}0]$ $= 1{\cdot}5[107 - 27{\cdot}8]$ $= 1{\cdot}5\times79{\cdot}2 = 118{\cdot}8$ in.^{3}	10·7	10·4	$20''\times6\frac{1}{2}''\times65$ lb. I (122·6)	96.

Fourth Floor Steel—*continued*

Span 20′ 0″.

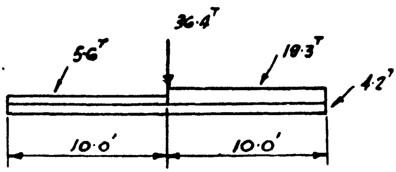


FIG. 212.

$$\text{O.W. \& C.} = 4{\cdot}2^{T}$$

$$\text{Floor}\ \frac{10{\cdot}0 \times 10{\cdot}0}{5{\cdot}18} = 19{\cdot}3^{T}$$

$$\begin{array}{r} 5{\cdot}6 \\ 36{\cdot}4 \\ 19{\cdot}3 \\ 4{\cdot}2 \\ \hline 65{\cdot}5 \end{array}$$

$$\text{Landing}\ \frac{10 \times 2{\cdot}75}{11{\cdot}2} = 2{\cdot}4$$

$$\text{Wall}\ \frac{10 \times 8{\cdot}0}{25{\cdot}0} = 3{\cdot}2$$

$$\overline{5{\cdot}6^{T}}$$

Point load at centre

$$\begin{aligned} \text{from S9}\times &= 17{\cdot}7 \\ \text{from D54} &= 8{\cdot}3 \\ \text{from D96} &= 10{\cdot}4 \\ \hline & 36{\cdot}4^{T} \end{aligned}$$

$$R_L = \frac{4{\cdot}2}{2} + \frac{36{\cdot}4}{2} + \frac{5{\cdot}6 \times 3}{4} + \frac{19{\cdot}3}{4}$$

$$= 2{\cdot}1 + 18{\cdot}2 + 4{\cdot}2 + 4{\cdot}9$$

$$= 29{\cdot}4^{T} \quad \therefore R_R = 65{\cdot}5 - 29{\cdot}4 = 36{\cdot}1^{T}$$

Z reqd.

$$= 1{\cdot}5[29{\cdot}4 \times 10{\cdot}0 - 5{\cdot}6 \times 5{\cdot}0 - 2{\cdot}1 \times 5{\cdot}0]$$

$$= 1{\cdot}5[294 - 28 - 10{\cdot}5]$$

$$= 1{\cdot}5[294 - 38{\cdot}5]$$

$$= 1{\cdot}5 \times 255{\cdot}5 = 383{\cdot}3 \text{ in.}^3$$

Loads and Calculations.	R/L. Tons.	R/R. Tons.	Section.	Beam Ref.
65·5	29·4	36·1	24″ × 7½″ × 95 lb. I + 2/10″ × 1″ PLS (395)	91.

Fourth Floor Steel—*continued*

Span 20′ 0″.

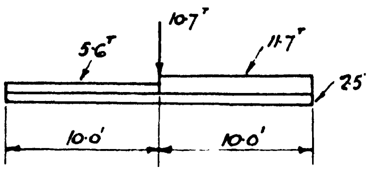


FIG. 213.

O.W. & C. $= 2·5^T$

Floor $\dfrac{10·0 \times 5·0}{5·18} = 9·6$

Wall $10·0 \times 8 = 80$

less $7·0 \times 4 = 28$

$52 \times \dfrac{1}{25·0} = 2·1$

$11·7^T$

Landing $\dfrac{10·0 \times 2·75}{11·2} = 2·4$

Wall $\dfrac{10·0 \times 8·0}{25·0} = 3·2$

$5·6^T$

$R_L = 5·35 + 1·25 + \dfrac{5·6 \times 3}{4} + \dfrac{11·70}{4}$

$= 6·6 + 4·2 + 2·9$

$= 13·7^T \quad \therefore R_R = 30·5 - 13·7$

$= 16·8^T$

Z reqd.

$= 1·5[13·7 \times 10·0 - 5·6 \times 5·0 - 1·25 \times 5·0]$

$= 1·5[137 - 28·0 - 6·2]$

$= 1·5[137 - 34·2] = 1·5 \times 102·8$

$= 154·2$ in.^{3}

Loads and Calculations.	R/L. Tons.	R/R. Tons.	Section.	Beam Ref.
5·6 10·7 11·7 2·5 30·5	13·7	16·8	20″ × 7½″ × 89 lb. I (167·0)	131.

Third Floor Steel

Loads and Calculations.	R/L. Tons.	R/R. Tons.	Section.	Beam Ref.
Beams Nos. 13, 23, 33, 61, 71, 63, 73, 101, 111, 103, 113, as 13 at 4th floor 22, 32, 72, 112 as 22 at 4th floor 55, 97 as 55 at 4th floor 62, 102 as 62 at 4th floor Rakers 94, 96 as Raker 94 at 4th floor 93, 95 low level as 95 at 4th floor				
Span 20′ 0″. Wall $20 \times 11 = 220$ less $2 \times 7 \times 4 = 56$ $164 \times \frac{1}{25{\cdot}0} = 6{\cdot}6$ Floor $\frac{20 \times 5}{5{\cdot}18} = 19{\cdot}3$ O.W. $= {\cdot}7$ $26{\cdot}6^{T}$ Z reqd. $= \frac{26{\cdot}6 \times 20}{5{\cdot}33} = 100$ in.^{3}	13·3	13·3	20″ × 6½″ × 65 lb. I (122·6)	11. 21. 31, 141. 151.
Span 10′ 0″. Wall $\frac{10 \times 11}{25{\cdot}0} = 4{\cdot}4$ O.W. $= {\cdot}2$ $4{\cdot}6^{T}$ $Z = \frac{4{\cdot}6 \times 10}{5{\cdot}33} = 8{\cdot}7$ in.^{3}	2·3	2·3	7″ × 4″ × 16 lb. I (11·3)	53.
Span 20′ 0″. Wall as 11, etc. $= 6{\cdot}6$ O.W. $= {\cdot}8$ $7{\cdot}4^{T}$ $7{\cdot}4 + 20{\cdot}8 = 28{\cdot}2$ Point load at centre $= 20{\cdot}8^{T}$ E.D.L. $= 7{\cdot}4 + 41{\cdot}6 = 49{\cdot}0^{T}$ Z reqd. $= \frac{49{\cdot}0 \times 20}{5{\cdot}33} = 184$ in.^{3}	14·1	14·1	24″ × 7½″ × 95 lb. I (211·1)	12. 42. 82. 122.

Third Floor Steel—*continued*

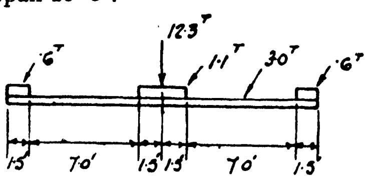
<table>
<tr><th>Loads and Calculations.</th><th>R/L. Tons.</th><th>R/R. Tons.</th><th>Section.</th><th>Beam Ref.</th></tr>
<tr><td>
Span 20′ 0″.
Wall $\frac{20\times 11}{25\cdot0}$ = 8·8   9·4
O.W. ·6   2·3
9·4T   11·7
Point load at centre = 2·3T
E.D.L. = 9·4 + 4·6 = 14·0T
Z reqd. = $\frac{14\cdot0 \times 20}{5\cdot33}$ = 52·5 in.³
</td><td>5·9</td><td>5·9</td><td>14″ × 6″ × 46 lb. I
(63·2)</td><td>52.</td></tr>
<tr><td>
Span 20′ 0″.


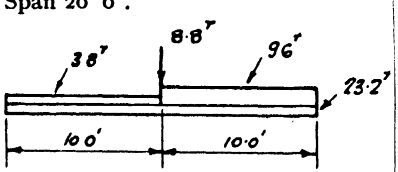
FIG. 214.
O.W. & C. = 3·0T   ·6
End piers   ·6
= $\frac{1\cdot5\times9\cdot5}{25\cdot0}$ = ·6T each   1·1
   3·0
Central pier   12·3
= $\frac{3\cdot0\times9\cdot5}{25\cdot0}$ = 1·1T   17·6
Z reqd.
= 1·5[8·8 × 10·0 − ·6 × 9·25
− ·55 × ·75 − 1·5 × 5·0]
= 1·5[88·0 − 5·6 − ·4 − 7·5]
= 1·5[88·0 − 13·5] = 1·5 × 74·5
= 111·8 in.³
</td><td>8·8</td><td>8·8</td><td>20″ × 6½″ × 65 lb. I
(122·6)</td><td>54.</td></tr>
<tr><td>
Span 20′ 0″.


FIG. 215.
</td><td></td><td></td><td></td><td>51.</td></tr>
</table>

Third Floor Steel—*continued*

<table>
<tr><th>Loads and Calculations.</th><th>R/L. Tons.</th><th>R/R. Tons.</th><th>Section.</th><th>Beam Ref.</th></tr>
<tr><td>Floor (i) $\frac{20\times 5}{5\cdot 18} = 19\cdot 3$ 3·8
8·8
9·6
O.W. & C. $= 3\cdot 9$ 23·2
$23\cdot 2^{T}$ 45·4
Floor (ii) $\frac{10\times 5}{5\cdot 18} = 9\cdot 6^{T}$
Wall $\frac{10\times 9\cdot 5}{25\cdot 0} = 3\cdot 8^{T}$
$R_L = 4\cdot 4+11\cdot 6+\frac{9\cdot 6}{4}+\frac{3\cdot 8\times 3}{4}$
$= 16\cdot 0+2\cdot 4+2\cdot 9 = 21\cdot 3^{T}$
$\therefore R_R = 45\cdot 4-21\cdot 3 = 24\cdot 1^{T}$
Z reqd.
$= 1\cdot 5[21\cdot 3\times 10\cdot 0-3\cdot 8\times 5\cdot 0 - 11\cdot 6\times 5\cdot 0]$
$= 1\cdot 5[213\cdot 0-19\cdot 0-58]$
$= 1\cdot 5[213\cdot 0-77\cdot 0]$
$= 1\cdot 5\times 136 = 204$ in.³</td><td>21·3</td><td>24·1</td><td>24″ × 7½″ × 95 lb. I
(211·1)</td><td>51
cont.</td></tr>
<tr><td>Span 20′ 0″.

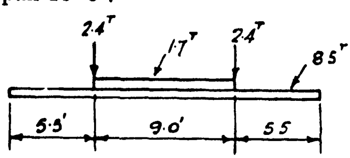
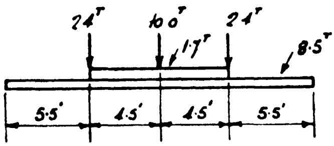
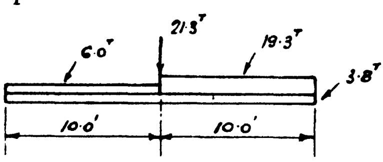
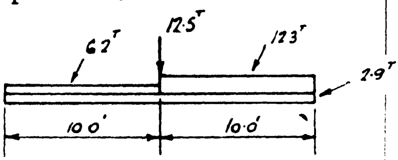



FIG. 216.
Wall 20 × 11 = 220   2·4
less 2 × 3 × 4 = 24   2·4
1·7
8·5
$196\times\frac{1}{25\cdot 0} = 7\cdot 9$ 15·0
O.W. $= \cdot 6$
$8\cdot 5^{T}$
Stairs $\frac{9\cdot 0\times 2\cdot 12}{11\cdot 2} = 1\cdot 7^{T}$
Z reqd.
$= 1\cdot 5[7\cdot 5\times 10\cdot 0-\cdot 85\times 2\cdot 25 -2\cdot 4\times 4\cdot 5-4\cdot 25\times 5\cdot 0]$
$= 1\cdot 5[75\cdot 0-1\cdot 9-10\cdot 8-21\cdot 3]$
$= 1\cdot 5[75\cdot 0-34\cdot 0]$
$= 1\cdot 5\times 41\cdot 0 = 61\cdot 5$ in.³</td><td>7·5</td><td>7·5</td><td>15″ × 6″ × 45 lb. I
(65·6)</td><td>92.</td></tr>
</table>

Third Floor Steel—*continued*

Loads and Calculations.	R/L. Tons.	R/R. Tons.	Section.	Beam Ref.
Span 20′ 0″.  FIG. 217. Wall $20 \times 11 = 220$ less $4 \times 7 = 28$ $192 \times \frac{1}{25.0} = 7.7$ O.W. $= .8$ 8.5^T Stairs as 92 $= 1.7^T$ Sum: 2.4, 2.4, 10.0, 1.7, 8.5 = 25.0 Z reqd. $= 1.5[12.5 \times 10.0 - .85 \times 2.25 - 2.4 \times 4.5 - 4.25 \times 5.0]$ $= 1.5[125.0 - 1.9 - 10.8 - 21.3]$ $= 1.5[125.0 - 34.0]$ $= 1.5 \times 91 = 136.5$ in.^{3}	12.5	12.5	22″ × 7″ × 75 lb. I (152.4)	98.
Span 20′ 0″.  FIG. 218. O.W. & C. $= 3.8^T$ Floor $\frac{10.0 \times 10.0}{5.18} = 19.3^T$ Landing $\frac{10.0 \times 2.75}{11.2} = 2.4$ Wall $\frac{10.0 \times 9.0}{25.0} = 3.6$ 6.0^T Sum: 6.0, 21.3, 19.3, 3.8 = 50.4	21.9	28.5	24″ × 7½″ × 95 lb. I + 2/9″ × ½″ PLS (281)	91.

Third Floor Steel—*continued*

Loads and Calculations.	R/L. Tons.	R/R. Tons.	Section.	Beam Ref.
Point load at centre from C54 = 8·8 „ „ „ „ from C98 = 12·5 $21{\cdot}3^T$ $R_L = 1{\cdot}9+10{\cdot}65+\frac{6{\cdot}0\times3}{4}+\frac{19{\cdot}3}{4}$ $=12{\cdot}55+4{\cdot}5+4{\cdot}85 = 21{\cdot}9^T$ $\therefore R_R = 50{\cdot}4 - 21{\cdot}9 = 28{\cdot}5^T$ Z reqd. $= 1{\cdot}5[21{\cdot}9\times10{\cdot}0-6{\cdot}0\times5{\cdot}0 - 1{\cdot}9\times5{\cdot}0]$ $= 1{\cdot}5[219{\cdot}0-30{\cdot}0-9{\cdot}5]$ $= 1{\cdot}5[219{\cdot}0-39{\cdot}5]$ $= 1{\cdot}5\times179{\cdot}5 = 269{\cdot}3$ in.^{3}			24″ × 7½″ × 95 lb. I + 2/9″ × ½″ PLs (281)	91 *cont.*
Span 20′ 0″.  FIG. 219. O.W. & C. $= 2{\cdot}9^T$ Floor $\frac{10{\cdot}0\times5{\cdot}0}{5{\cdot}18} = 9{\cdot}6$ Wall $10{\cdot}0\times9{\cdot}5=95$ less $7{\cdot}0\times4{\cdot}0=28$ $67\times\frac{1}{25{\cdot}0}=2{\cdot}7$ $12{\cdot}3^T$ 6·2 12·5 12·3 2·9 33·9 Landing $\frac{10{\cdot}0\times2{\cdot}75}{11{\cdot}2} = 2{\cdot}4$ Wall $\frac{10{\cdot}0\times9{\cdot}5}{25{\cdot}0} = 3{\cdot}8$ $6{\cdot}2^T$ $R_L = 1{\cdot}45+6{\cdot}25+\frac{12{\cdot}3}{4}+\frac{6{\cdot}2\times3}{4}$ $= 7{\cdot}7+3{\cdot}07+4{\cdot}65$ $= 15{\cdot}4^T$ $\therefore R_R = 33{\cdot}9 - 15{\cdot}4 = 18{\cdot}5^T$ Z reqd. $= 1{\cdot}5[15{\cdot}4\times10{\cdot}0-6{\cdot}2\times5{\cdot}0 - 1{\cdot}45\times5{\cdot}0]$ $= 1{\cdot}5[154-31-7{\cdot}2]$ $= 1{\cdot}5[154-38{\cdot}2]$ $= 1{\cdot}5\times115{\cdot}8 = 173{\cdot}7$ in.^{3}	15·4	18·5	24″ × 7½″ × 95 lb. I (211·1)	131.

Second Floor Steel

All Second Floor Steel is exactly as Third Floor Steel.

First Floor Steel

All First Floor Steel, except those beams calculated below, is exactly as Third Floor Steel.

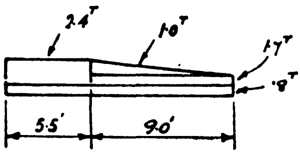
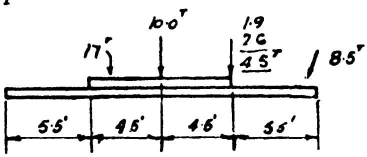
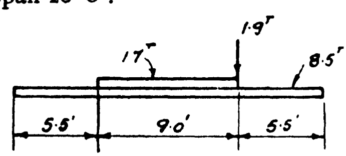
<table>
<tr><th>Loads and Calculations.</th><th>R/L. Tons.</th><th>R/R. Tons.</th><th>Section.</th><th>Beam Ref.</th></tr>
<tr><td>Span 10′ 0″.
As 93 at 4th floor</td><td>1·9</td><td>1·9</td><td>7″×4″×16 lb. I</td><td>93.</td></tr>
<tr><td>Span 14′ 6″.


FIG. 220.
O.W. & C. $= \cdot 8^T$
Stairs $\frac{9{\cdot}0\times 2{\cdot}12}{11{\cdot}2} = 1{\cdot}7^T$
Wall (△ lar) $= \frac{9{\cdot}0\times 5{\cdot}5}{2\times 25{\cdot}0} = 1{\cdot}0^T$
Wall $\frac{5{\cdot}5\times 5{\cdot}5}{25{\cdot}0} = 1{\cdot}2$
Landing $\frac{5{\cdot}5\times 2{\cdot}5}{11{\cdot}2} = 1{\cdot}2$
$2{\cdot}4^T$
2·4
1·0
1·7
·8
5·9
$R_L = \cdot 4 + \frac{1{\cdot}7\times 4{\cdot}5 + 1{\cdot}0\times 6{\cdot}0 + 2{\cdot}4\times 11{\cdot}75}{14{\cdot}5}$
$= \cdot 4 + \frac{7{\cdot}7 + 6{\cdot}0 + 28{\cdot}2}{14{\cdot}5}$
$= \cdot 4 + \frac{41{\cdot}9}{14{\cdot}5} = 3{\cdot}3^T.$
$\therefore R_R = 5{\cdot}9 - 3{\cdot}3 = 2{\cdot}6^T$
Zero shear occurs at 7·5′ from R.H. End.
$\therefore$ Z reqd.
$= 1{\cdot}5[2{\cdot}6\times 7{\cdot}5 - 1{\cdot}4\times 3{\cdot}75 - \cdot 7\times 2{\cdot}5 - \cdot 5\times 3{\cdot}75]$
$= 1{\cdot}5[19{\cdot}5 - 5{\cdot}2 - 1{\cdot}8 - 1{\cdot}9]$
$= 1{\cdot}5[19{\cdot}5 - 8{\cdot}9]$
$= 1{\cdot}5\times 10{\cdot}6 = 15{\cdot}9\text{ in.}^3$</td><td>3·3</td><td>2·6</td><td>9″×4″×21 lb. I
(18·0)</td><td>Cranked raker 98.</td></tr>
</table>

First Floor Steel—*continued*

Loads and Calculations.	R/L. Tons.	R/R. Tons.	Section.	Beam Ref.
Span 20′ 0″.  FIG. 221. Wall and O.W. & C. as 98 at 3rd floor $= 8 \cdot 5^T$ Stairs as 98 at 3rd floor $= 1 \cdot 7^T$ 1·7 10·0 4·5 8·5 24·7 $R_L = 5 \cdot 0 + \cdot 85 + 4 \cdot 25 + \frac{4 \cdot 5 \times 5 \cdot 5}{20 \cdot 0}$ $= 10 \cdot 1 + 1 \cdot 2 = 11 \cdot 3^T$ $\therefore R_R = 24 \cdot 7 - 11 \cdot 3^T = 13 \cdot 4^T$ Zreqd. $= 1 \cdot 5[11 \cdot 3 \times 10 \cdot 0 - \cdot 85 \times 2 \cdot 25 - 4 \cdot 25 \times 5 \cdot 0]$ $= 1 \cdot 5[113 \cdot 0 - 1 \cdot 9 - 21 \cdot 3]$ $= 1 \cdot 5[113 \cdot 0 - 23 \cdot 2]$ $= 1 \cdot 5 \times 89 \cdot 8 = 134 \cdot 7$ in.^{3}	11·3	13·4	22″ × 7″ × 75 lb. I (152·4)	96.
Span 20′ 0″.  FIG. 222. Wall and O.W. as 92 at 3rd floor $= 8 \cdot 5^T$ Stairs as 92 at 3rd floor $= 1 \cdot 7^T$ 1·7 1·9 8·5 12·1 $R_L = \cdot 85 + 4 \cdot 25 + \frac{1 \cdot 9 \times 5 \cdot 5}{20 \cdot 0}$ $= 5 \cdot 1 + \cdot 5 = 5 \cdot 6^T$ $\therefore R_R = 12 \cdot 1 - 5 \cdot 6 = 6 \cdot 5^T$	5·6	6·5	14″ × 6″ × 46 lb. I (63·2)	92.

First Floor Steel—*continued*

<table>
<tr><th>Loads and Calculations.</th><th>R/L. Tons.</th><th>R/R. Tons.</th><th>Section.</th><th>Beam Ref.</th></tr>
<tr><td>Zero shear from L.H. End
$= 5{\cdot}5' + \dfrac{5{\cdot}6 - \dfrac{5{\cdot}5 \times 8{\cdot}5}{20{\cdot}0}}{\dfrac{1{\cdot}7}{9{\cdot}0} + \dfrac{8{\cdot}5}{20{\cdot}0}}$
$= 5{\cdot}5' + \dfrac{5{\cdot}6 - 2{\cdot}34}{{\cdot}189 + {\cdot}425} = 5{\cdot}5' + \dfrac{3{\cdot}26}{{\cdot}614}$
$= 5{\cdot}5' + 5{\cdot}3' = 10{\cdot}8'$ from L.H. End.
∴ Z reqd.
$= 1{\cdot}5[5{\cdot}6 \times 10{\cdot}8 - 2{\cdot}34 \times 8{\cdot}05 - 3{\cdot}26 \times 2{\cdot}65]$
$= 1{\cdot}5[60{\cdot}5 - 18{\cdot}8 - 8{\cdot}7]$
$= 1{\cdot}5[60{\cdot}5 - 27{\cdot}5]$
$= 1{\cdot}5 \times 33 = 49{\cdot}5$ in.^{3}</td><td></td><td></td><td>14″ × 6″ × 46 lb. I
(63·2)</td><td>92
cont.</td></tr>
<tr><td>Span 20′ 0″.

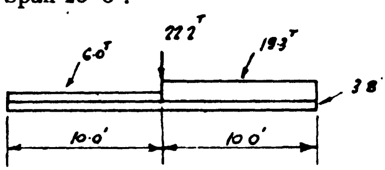



FIG. 223.

O.W. & C. as 91 at 3rd floor = 3·8 T
Floor as 91 at 3rd floor = 19·3 T
Landing and wall as 91 at 3rd floor = 6·0 T
Point load at centre
from A54 = 8·8
from A96 = 13·4
22·2 T

6·0
22·2
19·3
3·8
51·3

$R_L = 1{\cdot}9 + 11{\cdot}1 + \dfrac{6{\cdot}0 \times 3}{4} + \dfrac{19{\cdot}3}{4}$
$= 13{\cdot}0 + 4{\cdot}5 + 4{\cdot}9 = 22{\cdot}4^T$
∴ $R_R = 51{\cdot}3 - 22{\cdot}4 = 28{\cdot}9^T$
Z reqd.
$= 1{\cdot}5[22{\cdot}4 \times 10{\cdot}0 - 6{\cdot}0 \times 5{\cdot}0 - 1{\cdot}9 \times 5{\cdot}0]$
$= 1{\cdot}5[224{\cdot}0 - 30{\cdot}0 - 9{\cdot}5]$
$= 1{\cdot}5[224{\cdot}0 - 39{\cdot}5]$
$= 1{\cdot}5 \times 184{\cdot}5 = 276{\cdot}8$ in.^{3}</td><td>22·4</td><td>28·9</td><td>24″ × 7½″ × 95 lb. I
+
2/10″ × ½″ Pls
(293)</td><td>91.</td></tr>
</table>

First Floor Steel—*continued*

Loads and Calculations.	R/L. Tons.	R/R. Tons.	Section.	Beam Ref.
Span 20′ 0″.				

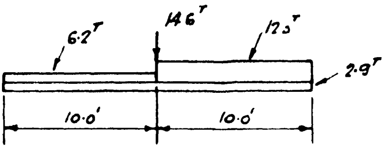



FIG. 224.				
O.W. & C. as 131 at 3rd floor $= 2{\cdot}9^{\mathrm{T}}$ 6·2				
Wall and floor as 131 at 3rd floor $= 12{\cdot}3^{\mathrm{T}}$ 14·6				
12·3				
Wall and landing as 131 at 3rd floor $= 6{\cdot}2^{\mathrm{T}}$ 2·9				
Point load at centre 36·0	16·5	19·5	24″ × 7½″ × 95 lb. I (211·1)	131.
from A96 $= 11{\cdot}3$				
from A98 $= 3{\cdot}3$				
$14{\cdot}6^{\mathrm{T}}$				
$R_L = 1{\cdot}45 + 7{\cdot}3 + \frac{12{\cdot}3}{4} + \frac{6{\cdot}2 \times 3}{4}$				
$= 8{\cdot}75 + 3{\cdot}08 + 4{\cdot}65 = 16{\cdot}5^{\mathrm{T}}$				
$\therefore R_R = 36{\cdot}0 - 16{\cdot}5 = 19{\cdot}5^{\mathrm{T}}$				
Z reqd.				
$= 1{\cdot}5[16{\cdot}5 \times 10{\cdot}0 - 6{\cdot}2 \times 5{\cdot}0 - 1{\cdot}45 \times 5{\cdot}0]$				
$= 1{\cdot}5[165{\cdot}0 - 31{\cdot}0 - 7{\cdot}2]$				
$= 1{\cdot}5[165{\cdot}0 - 38{\cdot}2]$				
$= 1{\cdot}5 \times 126{\cdot}8 = 190{\cdot}2$ in.^{3}				

Plate Curtailment in Compound Girders

In cases where single plates (top and bottom) only are used, these plates are run right through to the ends of the compound beam, except for short lengths at the ends which are cut off to clear the cleats connecting the compound beam to the members supporting it.

In cases where the total thickness of plate required is not greater than ⅝″, single plates only are used. For thicknesses over ⅝″, two or more plates are used; the component plates

usually are $\frac{3}{8}''$, $\frac{1}{2}''$ and $\frac{5}{8}''$ in thickness, the thickest plate being next to the joist section.

Fig. 225 shows a typical plate curtailment diagram, the compound girder in this case consisting of $1/24'' \times 7\frac{1}{2}'' \times 95$ lb. I section $+ \; 3/10'' \times \frac{1}{2}''$ plates on each flange. The method is similar to that given for the plate curtailment in Chapter XV.

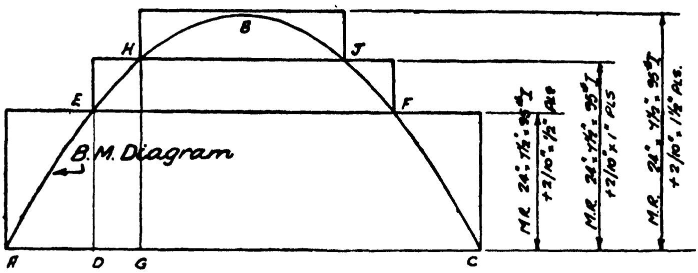


FIG. 225.—CURTAILMENT OF FLANGE PLATES.

The theoretical lengths EF and HJ for the outer $\frac{1}{2}''$ plates are extended by about 9 to 12 inches at each end, as in the case of plate girder plates.

Riveting in Compound Girders.—The tendency for one plate to slide horizontally over another, or the horizontal shear, is catered for by the rivets connecting the plates to the root section. The necessary pitch of the rivets is determined by a treatment usually regarded as being more accurate than the S/D method, for the case of shallow sections.

Horizontal shear per foot run $= s_H = \dfrac{S_V \times a \times \bar{y}}{I} \times 12$ tons (see Chapter IX).

where S_V tons = vertical shear load at the section (being usually greatest at the ends of the girder).

a in.^{2} = the area of the plate or portion to be attached by the rivets.

y ins. = the distance of the centre of gravity of the portion to be attached from the neutral axis of the beam.

I in.^{4} = the moment of inertia of the whole section, i.e. the portion to be attached plus the root section to which it is attached.

The pitch of rivets in inches $= \dfrac{12 \times N \times V}{s_H}$,

where N = the number of rows of rivets used.

V = value in tons of one rivet in single shear.

(Compare formula for plate girder riveting: $p = \dfrac{VD}{S}$.)

The maximum usual pitch of rivets in structural work is 6″. Other common pitches used are $4\frac{1}{2}$″, 4″ and 3″.

In the steel frame under consideration, the following compound beams have one plate only on each flange and therefore these plates will run the full length of the beams (see Fig. 229).

A, B, C and D. 62 and 102.

A, B and C. 91.

The following beams have more than one plate on each flange (see Fig. 230).

A, B, C and D. 22, 32, 72 and 112.

D51.

D91.

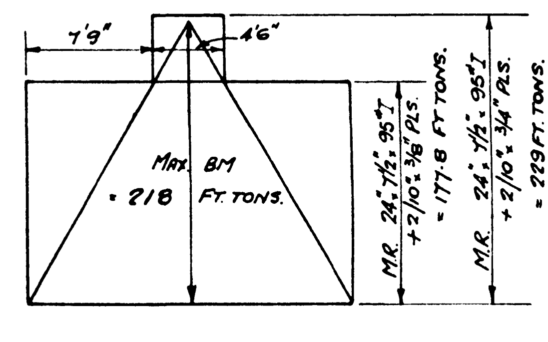


FIG. 226.

Theoretical length of top 10″ × $\frac{3}{8}$″ plates = 4′ 6″

Make actual length = 6′ 0″

. Plating is as follows:

1/10″ × $\frac{3}{8}$″ PL. Top and bottom, full length.

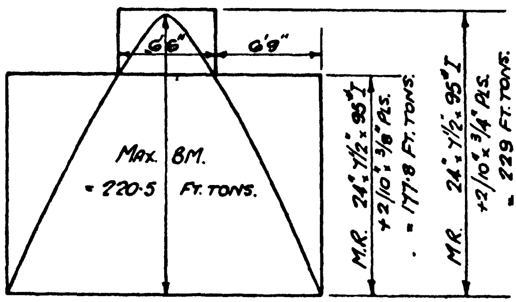
1/10″ × $\frac{3}{8}$″ PL. Top and bottom, 6′ long, central.

Riveting.—Horizontal shear $= \dfrac{22{\cdot}8 \times 10 \times \frac{3}{8} \times 12{\cdot}1875 \times 12}{3299}$

$= 3{\cdot}8$ tons per foot run.

Using $\frac{7}{8}''$ rivets, pitch $= \dfrac{12 \times 2 \times 3{\cdot}6}{3{\cdot}8} = 22{\cdot}7''$.

Use 2 rows of $\frac{7}{8}''$ rivets at 6″ pitch throughout.



D51.

FIG. 227.

Theoretical length of top plate $= 6'\ 6''$
Make actual length $= 8'\ 0''$

∴ Plating is as follows:

1/10″ × $\frac{3}{8}''$ PL. Top and bottom, full length.
1/10″ × $\frac{3}{8}''$ PL. Top and bottom, 8′ long, central.

Riveting. $s_H = \dfrac{32{\cdot}7 \times 10 \times \frac{3}{8} \times 12{\cdot}1875 \times 12}{3299}$

$= 5{\cdot}43$ tons per foot run.

Using $\frac{7}{8}''$ rivets, pitch $= \dfrac{12 \times 2 \times 3{\cdot}6}{5{\cdot}43} = 15{\cdot}9''$.

Use 2 rows of $\frac{7}{8}''$ rivets at 6″ pitch throughout.

Riveting for Compound Beams A, B, C and D, 62 *and* 102

Maximum shear = 17·1 tons, which is less than occurs in D51, and as the section at the ends is less than that for D51, obviously 2 rows of $\frac{7}{8}''$ rivets at 6″ pitch throughout will be sufficient.

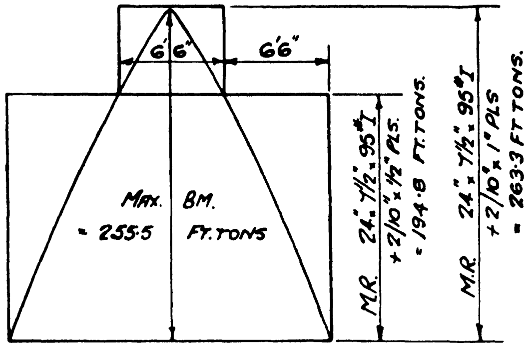


FIG. 228.

Theoretical length of top plate = 6′ 6″
Make actual length = 8′ 6″

∴ Plating is as follows:

1/10″ × ½″ PL. Top and bottom, full length.
1/10″ × ½″ PL. Top and bottom, 8′ 6″ long, central.

Riveting. $s_H = \dfrac{36{\cdot}1 \times 10 \times \frac{1}{2} \times 12{\cdot}25 \times 12}{3650}$

$= 7{\cdot}27$ tons per foot run.

Using ⅞″ rivets, pitch $= \dfrac{12 \times 2 \times 3{\cdot}6}{7{\cdot}27} = 11{\cdot}88''$.

Use 2 rows of ⅞″ rivets at 6″ pitch throughout.

Riveting for Compound Beams A, B and C, 91

Maximum shear = 28·9 tons, which is less than for D91, and as the section at the ends is the same as for D91, obviously 2 rows of rivets at 6″ pitch throughout will be sufficient.

Stanchion Calculations

In the subsequent calculations the following symbols are employed:

f_0 = Direct stress $= \dfrac{\text{Direct load}}{\text{Area of section}}$.

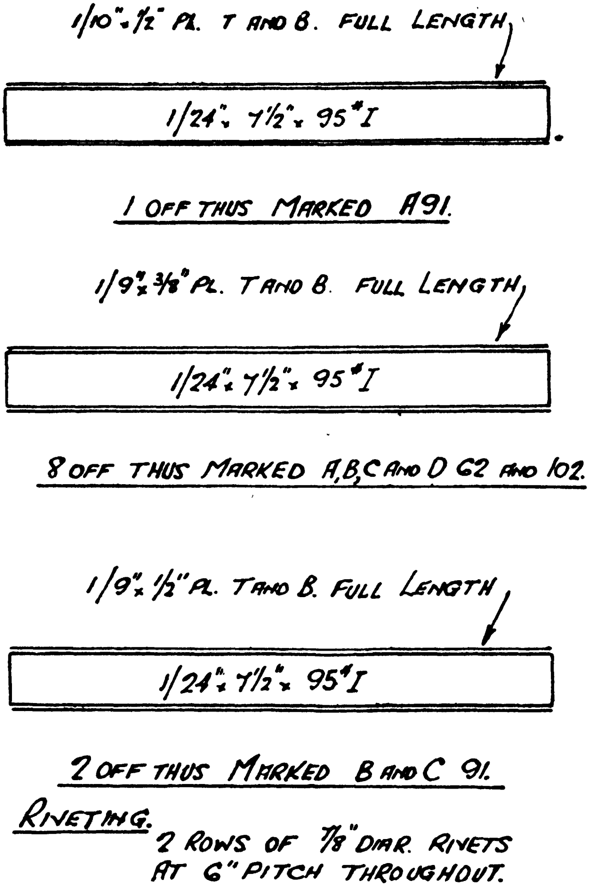


FIG. 229.—COMPOUND GIRDERS WITH PLATES FULL LENGTH TOP AND BOTTOM.

f_{Bxx} = Bending stress (XX axis)

$$= \frac{\text{Eccentric moment about XX axis}}{2 \times \text{modulus for XX axis}}.$$

f_{Byy} = Bending stress (YY axis)

$$= \frac{\text{Eccentric moment about YY axis}}{2 \times \text{modulus for YY axis}}.$$

F_1 = allowable stress on stanchion.

F_2 = allowable stress, claiming allowance for eccentric load.

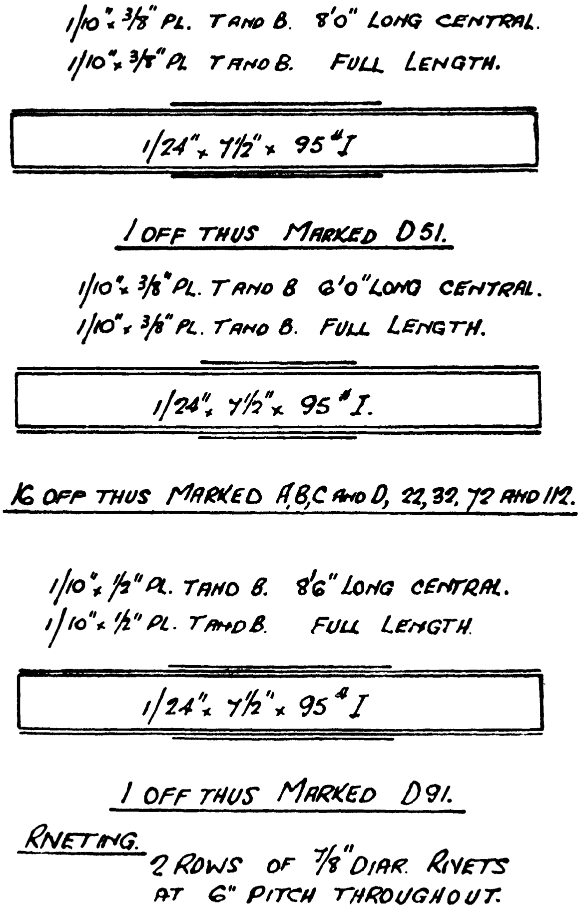


FIG. 230.—COMPOUND GIRDERS WITH PLATES CURTAILED.

The figure 2 appears in the denominator of the expressions for f_{Bxx} and f_{Byy}, as it is common practice to divide the eccentric moments on continuous lengths of stanchions equally between the lengths above and below the point at which the loads are applied (see Chapter XI).

The eccentric moment about any axis is the algebraic sum of the moments of the loads about that axis. If the eccentricities of two loads about any axis (one load on each side of stanchion)

are the same, the moment is the difference between the loads multiplied by the eccentricity.

Where cap connections are used the loads are considered to cause no eccentric moment on the stanchion.

The eccentricities used in the case of beams coming centrally up to stanchions are taken in these calculations as half the depth of the section for the XX axis, and 2″ for the YY axis (see Fig. 231). Various regulations are laid down with respect to the

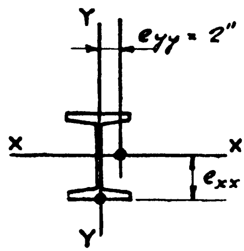


FIG. 231.

eccentricity which should be taken in the case of flange connections, and these recommend that the load should be taken as acting at distances up to about 2″ from the flange face, according to the nature of the connection. It has been considered desirable in the following computations to adopt half the overall section depth as the eccentricity, in order that the detailed calculations involved shall be made more readily comprehensible to the reader. No eccentric distance is taken less than 2″ from the principal axes under any circumstances.

Effective column lengths are taken as explained in Chapter XI.

Explanation of Calculations.—Consider the stanchion with the reference 1 (4 and 16 being similar). The top length carries beams R12 (reaction = 1·3 tons, see Beam reference 12, page 301) and R11 (reaction = 3·0 tons, see Beam reference 11, page 300). Adding in ·5 tons (estimated own weight and casing) the total load = 4·8 tons. This is carried to the previous column headed 'Totals.' The design of this stanchion length is deferred until the total load for the bottom length of stanchion is computed, and that length designed.

Length D–C contributes a total of 26·4 tons, the beams,

D12 and D11 giving reaction loads which may be checked by the reader if he refers to the tables of beam calculations under the references D12 and D11 respectively. The figure 26·4 is carried to the previous column, as before, giving a new total of 31·2 tons. Note that R11 is looked up in Roof Steel sheet and that MR refers to beams in motor roof. Similarly the letters D, C, B and A are prefixed to beams in 4th, 3rd, 2nd and 1st floors respectively. Thus D12 refers to beam 12 in 4th floor steel. G indicates ground floor.

Lengths C–B and B–A are similarly entered up and we find that length A–G has to support a total load of 116·1 tons. This length is now designed to support an axial load of 116·1 tons and the eccentric bending moments produced by the reaction of beam A12 (14·1 tons) and A11 (13·3 tons).

Length A–G.

$$f_C = \text{Direct stress} = \frac{\text{Total load}}{\text{Area}} = \frac{116 \cdot 1}{26 \cdot 3} = 4 \cdot 42 \text{ tons/in.}^2.$$

$$f_{Bxx} = \frac{14 \cdot 1 \times 5}{2 \times 87 \cdot 3} = \cdot 41 \text{ tons/in.}^2.$$

$$f_{Byy} = \frac{13 \cdot 3 \times 2}{2 \times 25 \cdot 8} = \cdot 52 \text{ tons/in.}^2.$$

This gives a total $= 5 \cdot 35$ tons/in.².

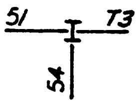
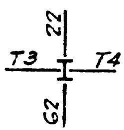
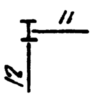
F_1 for the section chosen for this stanchion length $= 5 \cdot 46$ tons/in.², obtained as follows :

Effective length $= \cdot 875 \times 156''$ (·875 is taken as this is a two-way connection on a bottom stanchion length, see page 204).

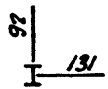
$$\frac{l}{r} = \frac{\cdot 875 \times 156}{1 \cdot 98} = 69.$$

F_1 (by by-law values) $= 5 \cdot 46$ tons/in.².

The basis section, 8″ × 6″ × 35 lb. B.S.B., is carried through the whole stanchion length and there is a splice where marked ×. The symbol × (against the 'stanchion reference' column) indicates in the case of each stanchion that there is to be a splice at that point. These splices are required for erection and fabrication purposes, as it is inconvenient, and costly, to handle lengths of stanchions greater than about 35′.

Sketch	Calculations.	Allowable Stresses. $^T/in.^2$	$\frac{L}{r}$	Totals. Tons.	Reactions. Amount Tons.	Reactions. Beam Ref.	Portion and Length.	Section.	Stanchion Ref.
 FIG. 232.					TIE 2·8 1·4 ·4	MR51 MR54 MR54× O.W. & C.	MR 8′ 0″	As below	(5×) (9×)
				4·6	4·6				
	Length R–D. $f_c = \frac{17·7}{7·35} = 2·41$ $f_{Bxx} = \frac{7·2 \times 3}{2 \times 15·05} = ·72$ $f_{Byy} = \frac{3·4 \times 2}{2 \times 3·95} = ·86$ Total $= 3·99\ ^T/in.^2$	$F_1 = 5·41$	$\frac{·75 \times 108}{1·16}$ $= 70$	13·1	4·4 1·0 7·2 ·5	R51 T3 R54 O.W. & C.	Roof 9′ 0″	6″×5″×25 lb. I	
				17·7	13·1				
 FIG. 233.	As Stan. (5×)				1·0 1·4 2·0 1·8 ·5	T3 T4 R22 R62 O.W. & C.	Roof 9′ 0″	6″×5″×25 lb. I	(6×) (10×)
				6·7	6·7				
 FIG. 234.					1·3 3·0 ·5	R12 R11 O.W. & C.	Roof 9′ 0″	8″×6″×35 lb. I	(1) (4) (16)
				4·8	4·8				

			26·4 — 31·2	13·3 12·4 ·7 — 26·4	D12 D11 O.W. & C.	4th 11′ 0″	As below	(1) (4) (16) *cont.*
Length C–B. $f_c = \frac{59·5}{16·3} = 3·65$ $f_{Bxx} = \frac{14·1 \times 4·38}{2 \times 49·4} = ·63$ $f_{Byy} = \frac{13·3 \times 2}{2 \times 12·5} = 1·07$ Total $= 5·35$ T/in.²	$F_1 = 5·61$	$\frac{·875 \times 132}{1·75}$ $= 66$	28·3 — 59·5	14·1 13·3 ·9 — 28·3	C12 C11 O.W. & C.	3rd 11′ 0″	8″ × 6″ × 35 lb. I + 2/8″ × ⅜″ PLs	
Length B–A. $f_c = \frac{87·8}{20·3} = 4·32$ $f_{Bxx} = \frac{14·1 \times 4·62}{2 \times 64·2} = ·51$ $f_{Byy} = \frac{13·3 \times 2}{2 \times 17·8} = ·75$ Total $= 5·58$ T/in.²	$F_1 = 5·80$	$\frac{·875 \times 132}{1·87}$ $= 62$	28·3 — 87·8	14·1 13·3 ·9 — 28·3	B12 B11 O.W. & C.	2nd 11′ 0″	8″ × 6″ × 35 lb. I + 2/8″ × ⅝″ PLs	×
Length A–G. $f_c = \frac{116·1}{26·3} = 4·42$ $f_{Bxx} = \frac{14·1 \times 5}{2 \times 87·3} = ·41$ $f_{Byy} = \frac{13·3 \times 2}{2 \times 25·8} = ·52$ Total $= 5·35$ T/in.²	$F_1 = 5·46$	$\frac{·875 \times 156}{1·98}$ $= 69$	28·3 — 116·1	14·1 13·3 ·9 — 28·3	A12 A11 O.W. & C.	1st 13′ 0″	8″ × 6″ × 35 lb. I + 2/8″ × 1″ PLs	

Sketch.	Calculations.	Allowable Stresses. T/in.²	$\frac{L}{r}$	Totals. Tons.	Reactions: Amount Tons.	Reactions: Beam Ref.	Portion and Length.	Section.	Stanchion Ref.
 FIG. 235.					1·3 3·0 ·5	R92 R131 O.W. & C.	Roof 9′ 0″	As below	(13)
				4·8	4·8				
	Length D–C. $f_o = \frac{24·8}{10·3} = 2·41$ $f_{Bxx} = \frac{5·6 \times 4}{2 \times 28·8} = ·39$ $f_{Byy} = \frac{13·7 \times 2}{2 \times 6·5} = 2·11$ Total $= 4·91^{T}/\text{in.}^2$	$F_1 = 4·66$ $F_2 = 5·29$	$\frac{·875 \times 132}{1·38} = 84$	20·0	5·6 13·7 ·7	D92 D131 O.W. & C.	4th 11′ 0″	8″ × 6″ × 35 lb. I	
				24·8	20·0				
	Length C–B. As Stan. (1)			23·8	7·5 15·4 ·9	C92 C131 O.W. & C.	3rd 11′ 0″	8″ × 6″ × 35 lb. I + 2/8″ × ⅜″ PLS	×
				48·6	23·8				
				23·8	7·5 15·4 ·9	B92 B131 O.W. & C.	2nd 11′ 0″	As below	
				72·4	23·8				

Length A–G. $f_o = \frac{94 \cdot 4}{22 \cdot 3} = 4 \cdot 23$ $f_{Bxx} = \frac{5 \cdot 6 \times 4 \cdot 75}{2 \times 71 \cdot 8} = \cdot 19$ $f_{Byy} = \frac{16 \cdot 5 \times 2}{2 \times 20 \cdot 5} = \cdot 81$ Total $= \underline{5 \cdot 23^{T}/\text{in.}^2}$	$F_1 = 5 \cdot 36$	$\frac{\cdot 875 \times 156}{1 \cdot 92}$ $= 71$	22·0 94·4	5·6 16·5 ·9 22·0	A92 A131 O.W. & C.	1st 13′ 0″	8″ × 6″ × 35 lb. I + 2/8″ × ¾″ PLS	(13) *cont.*
			6·6	3·0 3·1 ·5 6·6	R11 R21 O.W. & C.	Roof 9′ 0″	8″ × 6″ × 35 lb. I	(2) (3) (15)
			48·4 55·0	12·4 12·4 22·8 ·8 48·4	D11 D21 D22 O.W. & C.	4th 11′ 0″	As below	
Length C–B. $f_o = \frac{105 \cdot 4}{19 \cdot 3} = 5 \cdot 46$ $f_{Bxx} = \frac{22 \cdot 8 \times 4 \cdot 5}{2 \times 57 \cdot 5} = \cdot 89$ Total $= \underline{6 \cdot 35^{T}/\text{in.}^2}$	$F_1 = 6 \cdot 33$ $F_2 = 6 \cdot 37$	$\frac{\cdot 75 \times 132}{2 \cdot 02}$ $= 49$	50·4 105·4	13·3 13·3 22·8 1·0 50·4	C11 C21 C22 O.W. & C.	3rd 11′ 0″	8″ × 6″ × 35 lb. I + 2/9″ × ½″ PLS	×

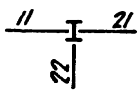
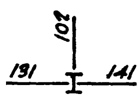
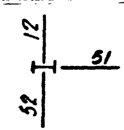


FIG. 236.

Sketch.	Calculations.	Allowable Stresses. ^{T}/in.^{2}.	$\frac{L}{r}$	Totals. Tons.	Reactions. Amount Tons.	Reactions. Beam Ref.	Portion and Length.	Section.	Stanchion Ref.
	Length B–A. $f_o = \frac{155 \cdot 8}{28 \cdot 7} = 5 \cdot 42$ $f_{Bxx} = \frac{22 \cdot 8 \times 5 \cdot 62}{2 \times 114} = \cdot 56$ Total $= 5 \cdot 98^{T}/\text{in.}^2$	$F_1 = 6 \cdot 57$	$\frac{\cdot 75 \times 132}{2 \cdot 35} = 42$	50·4 155·8	13·3 13·3 22·8 1·0 50·4	B11 B21 B22 O.W. & C.	2nd 11′ 0″	10″ × 8″ × 55 lb. I + 2/10″ × ⅝″ PLS	(2) (3) (15) *cont.*
	Length A–G. $f_o = \frac{206 \cdot 4}{36 \cdot 2} = 5 \cdot 71$ $f_{Bxx} = \frac{22 \cdot 8 \times 6}{2 \times 149} = \cdot 46$ Total $= 6 \cdot 17^{T}/\text{in.}^2$	$F_1 = 6 \cdot 37$	$\frac{\cdot 75 \times 156}{2 \cdot 47} = 48$	50·6 206·4	13·3 13·3 22·8 1·2 50·6	A11 B21 B22 O.W. & C.	1st 13′ 0″	10″ × 8″ × 55 lb. I + 2/10″ × 1″ PLS	
 FIG. 237.				6·6	3·0 3·1 ·5 6·6	R131 R141 O.W. & C.	Roof 9′ 0″	8″ × 6″ × 35 lb. I	(14)
				47·1 53·7	16·8 12·4 17·1 ·8 47·1	D131 D141 D102 O.W. & C.	4th 11′ 0″	As below	

Length C–B. $f_c = \frac{103 \cdot 6}{19 \cdot 3} = 5 \cdot 36$ $f_{Bxx} = \frac{17 \cdot 1 \times 4 \cdot 5}{2 \times 57 \cdot 5} = \cdot 67$ $f_{Byy} = \frac{5 \cdot 2 \times 2}{2 \times 17 \cdot 5} = \cdot 30$ Total $= 6 \cdot 33^{T}/\text{in.}^2$	$F_1 = 6 \cdot 33$	$\frac{\cdot 75 \times 132}{2 \cdot 02}$ $= 49$	 49·9 103·6	18·5 13·3 17·1 1·0 49·9	C131 C141 C102 O.W. & C.	3rd 11′ 0″	8″ × 6″ × 35 lb. I + 2/9″ × ½″ PLS	(14) *cont.*
Length B–A. $f_c = \frac{153 \cdot 5}{28 \cdot 7} = 5 \cdot 35$ $f_{Bxx} = \frac{17 \cdot 1 \times 5 \cdot 62}{2 \times 114} = \cdot 42$ $f_{Byy} = \frac{5 \cdot 2 \times 2}{2 \times 31 \cdot 8} = \cdot 17$ Total $= 5 \cdot 94^{T}/\text{in.}^2$	$F_1 = 6 \cdot 57$	$\frac{\cdot 75 \times 132}{2 \cdot 35}$ $= 42$	 49·9 153·5	18·5 13·3 17·1 1·0 49·9	B131 B141 B102 O.W. & C.	2nd 11′ 0″	10″ × 8″ × 55 lb. I + 2/10″ × ⅝″ PLS	
Length A–G. $f_c = \frac{204 \cdot 4}{36 \cdot 2} = 5 \cdot 65$ $f_{Bxx} = \frac{17 \cdot 1 \times 6}{2 \times 149} = \cdot 35$ $f_{Byy} = \frac{6 \cdot 2 \times 2}{2 \times 44 \cdot 3} = \cdot 14$ Total $= 6 \cdot 14^{T}/\text{in.}^2$	$F_1 = 6 \cdot 37$	$\frac{\cdot 75 \times 156}{2 \cdot 47}$ $= 48$	 50·9 204·4	19·5 13·3 17·1 1·0 50·9	A131 A141 B102 O.W. & C.	1st 13′ 0″	10″ × 8″ × 55 lb. I + 2/10″ × 1″ PLS	
 FIG. 238.			 3·2	2·8 TIE ·4 3·2	MR52 MR51 O.W. & C.	MR 8′ 0″	As below	(5)

Sketch.	Calculations.	Allowable Stresses. T/in.²	$\frac{L}{r}$	Totals. Tons.	Reactions. Amount Tons.	Reactions. Beam Ref.	Portion and Length.	Section.	Stanchion Ref.
				14·3	8·1 1·3 4·4 ·5	R52 R12 R51 O.W. & C.	Roof 9′ 0″	8″ × 6″ × 35 lb. I	(5) *cont.*
				17·5	14·3				×
				48·4	4·8 13·3 29·5 ·8	D52 D12 D51 O.W. & C.	4th 11′ 0″	As below	
				65·9	48·4				
	Length C–B. $f_c = \frac{108\cdot2}{20\cdot3} = 5\cdot33$ $f_{Bxx} = \frac{21\cdot3 \times 4\cdot5}{2 \times 64\cdot9} = \cdot74$ $f_{Byy} = \frac{8\cdot2 \times 2}{2 \times 20\cdot2} = \cdot41$ Total $= 6\cdot48^{T}/\text{in.}^2$	$F_1 = 6\cdot48$	$\frac{\cdot75 \times 132}{2\cdot23}$ $= 45$	42·3	5·9 14·1 21·3 1·0	C52 C12 C51 O.W. & C.	3rd 11′ 0″	8″ × 6″ × 35 lb. I + 2/10″ × ½″ Pls	
				108·2	42·3				×
	Length B–A. $f_c = \frac{150\cdot5}{28\cdot7} = 5\cdot24$ $f_{Bxx} = \frac{21\cdot3 \times 5\cdot62}{2 \times 114} = \cdot50$ $f_{Byy} = \frac{8\cdot2 \times 2}{2 \times 31\cdot8} = \cdot26$ Total $= 6\cdot00^{T}/\text{in.}^2$	$F_1 = 6\cdot57$	$\frac{\cdot75 \times 132}{2\cdot35}$ $= 42$	42·3	5·9 14·1 21·3 1·0	B52 B12 B51 O.W. & C.	2nd 11′ 0″	10″ × 8″ × 55 lb. I + 2/10″ × ⅝″ Pls	
				150·5	42·3				

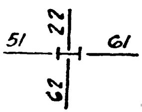
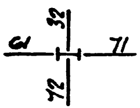
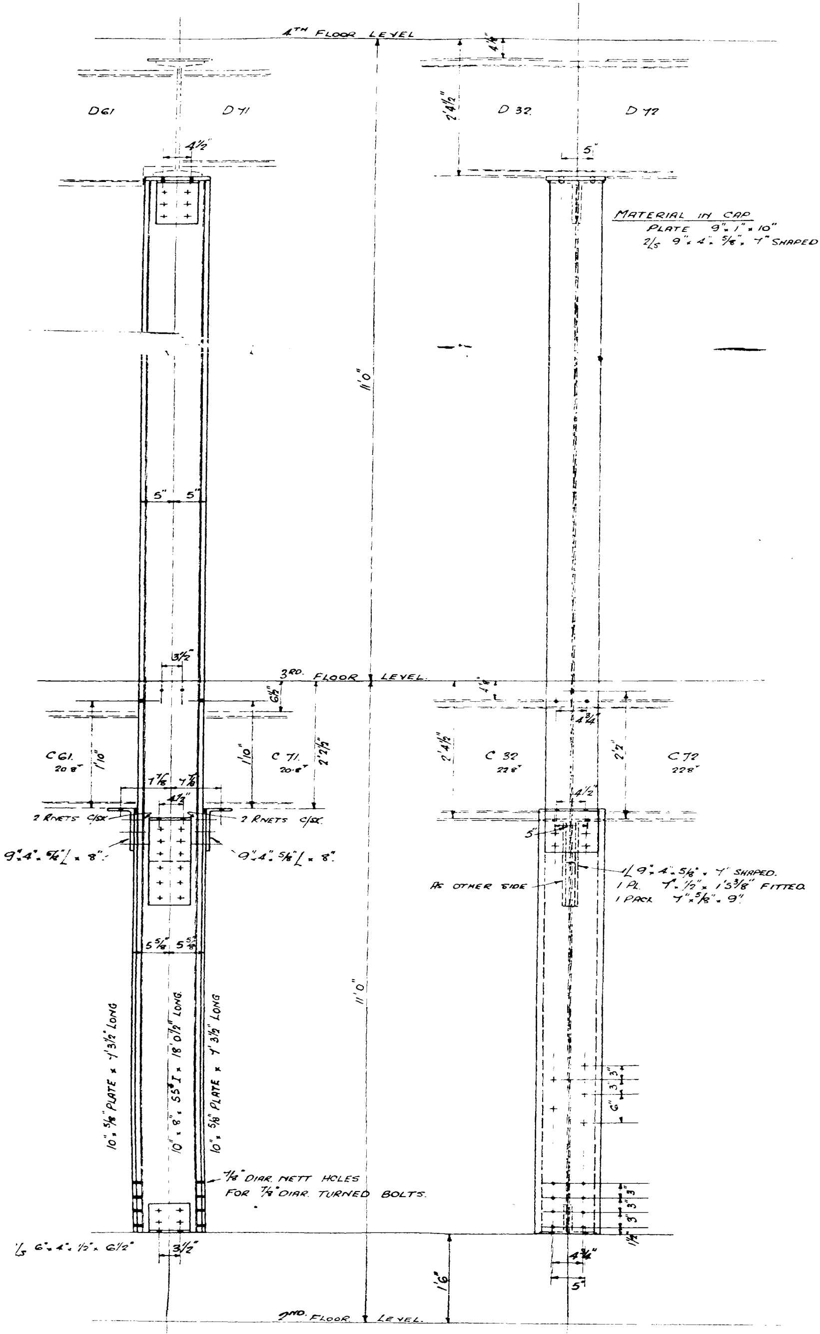


FIG. 239.

Length A–G. $f_c = \frac{193\cdot0}{36\cdot2} = 5\cdot33$ $f_{Bxx} = \frac{21\cdot3 \times 6}{2 \times 149} = \cdot43$ $f_{Byy} = \frac{8\cdot2 \times 2}{2 \times 44\cdot3} = \cdot19$ Total $= 5\cdot95^{T}/\text{in.}^2$	$F_1 = 6\cdot37$	$\frac{\cdot75 \times 156}{2\cdot47}$ $= 48$	42·5 193·0	5·9 14·1 21·3 1·2 42·5	A52 A12 A51 O.W. & C.	1st 13′ 0″	10″ × 8″ × 55 lb. I + 2/10″ × 1″ PLS	(5) *cont.*
Length D–C. Cap Connections. $f_c = \frac{98\cdot3}{23\cdot7} = 4\cdot15\ ^{T}/\text{in.}^2$	$F_1 = 6\cdot22$	$\frac{\cdot875 \times 132}{2\cdot25}$ $= 52$	98·3	32·7 24·7 22·8 17·1 1·0 98·3	D51 D61 D22 D62 O.W. & C.	4th 11′ 0″	10″ × 8″ × 55 lb. I + 2/10″ × ⅜″ PLS	(6)
Length C–B. $f_c = \frac{184\cdot3}{31\cdot2} = 5\cdot9$ $f_{Bxx} = \frac{3\cdot3 \times 5\cdot75}{2 \times 126} = \cdot08$ $f_{Byy} = \frac{5\cdot9 \times 2}{2 \times 35\cdot9} = \cdot17$ Total $= 6\cdot15\ ^{T}/\text{in.}^2$	$F_1 = 6\cdot57$	$\frac{\cdot75 \times 132}{2\cdot4}$ $= 42$	86·0 184·3	24·1 20·8 22·8 17·1 1·2 86·0	C51 C61 C22 C62 O.W. & C.	3rd 11′ 0″	10″ × 8″ × 55 lb. I + 2/10″ × ¾″ PLS	
Length B–A. $f_c = \frac{270\cdot5}{43\cdot2} = 6\cdot26$ $f_{Bxx} = \frac{3\cdot3 \times 6\cdot12}{2 \times 184} = \cdot06$ $f_{Byy} = \frac{5\cdot9 \times 2}{2 \times 62\cdot3} = \cdot10$ Total $= 6\cdot42\ ^{T}/\text{in.}^2$	$F_1 = 6\cdot81$	$\frac{\cdot75 \times 132}{2\cdot96}$ $= 34$	86·2 270·5	24·1 20·8 22·8 17·1 1·4 86·2	B51 B61 B22 B62 O.W. & C.	2nd 11′ 0″	10″ × 8″ × 55 lb. I + 2/12″ × 1⅛″ PLS	

Sketch.	Calculations.	Allowable Stresses. ^{T}/in.^{2}	$\frac{L}{r}$	Totals. Tons.	Reactions. Amount Tons.	Reactions. Beam Ref.	Portion and Length.	Section.	Stanchion Ref.
	Length A–G. $f_o = \frac{357 \cdot 1}{55 \cdot 2} = 6 \cdot 47$ $f_{Bxx} = \frac{3 \cdot 3 \times 6 \cdot 62}{2 \times 244 \cdot 5} = \cdot 05$ $f_{Byy} = \frac{5 \cdot 7 \times 2}{2 \times 86 \cdot 8} = \cdot 07$ Total $= \underline{\underline{6 \cdot 59\ ^T/\text{in.}^2}}$	$F_1 = 6 \cdot 69$	$\frac{\cdot 75 \times 156}{3 \cdot 07}$ $= 38$	86·6 357·1	24·1 20·8 22·8 17·1 1·8 86·6	A51 A61 A22 A62 O.W. & C.	1st 13′ 0″	10″ × 8″ × 55 lb. I + 2/12″ × 1⅝″ PLS	(6) *cont.*
 FIG. 240.	*Length D–C.* Cap Connections. $f_o = \frac{92 \cdot 1}{16 \cdot 18} = \underline{\underline{5 \cdot 7\ ^T/\text{in.}^2}}$	$F_1 = 5 \cdot 75$	$\frac{\cdot 875 \times 132}{1 \cdot 84}$ $= 63$	92·1	24·7 20·8 22·8 22·8 1·0 92·1	D61 D71 D32 D72 O.W. & C.	4th 11′ 0″	10″ × 8″ × 55 lb. I	(7) (11)
	Length C–B. $f_o = \frac{180 \cdot 5}{28 \cdot 7} = \underline{\underline{6 \cdot 29\ ^T/\text{in.}^2}}$	$F_1 = 6 \cdot 57$	$\frac{\cdot 75 \times 132}{2 \cdot 35}$ $= 42$	88·4 180·5	20·8 20·8 22·8 22·8 1·2 88·4	C61 C71 C32 C72 O.W. & C.	3rd 11′ 0″	10″ × 8″ × 55 lb. I + 2/10″ × ⅝″ PLS	×
	Length B–A. $f_o = \frac{269 \cdot 1}{40 \cdot 2} = \underline{\underline{6 \cdot 7\ ^T/\text{in.}^2}}$	$F_1 = 6 \cdot 81$	$\frac{\cdot 75 \times 132}{2 \cdot 92}$ $= 34$	88·6 269·1	20·8 20·8 22·8 22·8 1·4 88·6	B61 B71 B32 B72 O.W. & C.	2nd 11′ 0″	10″ × 8″ × 55 lb. I + 2/12″ × 1″ PLS	



ALL RIVETS AND BOLTS 7/8" DIAR. UNLESS OTHERWISE NOTED.
ALL HOLES 15/16" DIAR. " " "
ENDS OF STANS MACHINED DEAD SQUARE.

2 STANS THUS MARK 7(B-D) & 11 (B-D) 18'1 1/2" EXACT OVERALL.

Length A–G.

As Stan. (6)

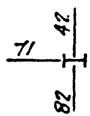
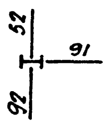
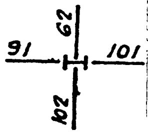


FIG. 241.

		20·8	A61	1st	10″ × 8″ × 55 lb. I	(7) (11) *cont.*
		20·8	A71		+	
		22·8	A32		2/12″ × 1⅝″ PLS	
		22·8	A72	13′ 0″		
	89·0	1·8	O.W. & C.			
	358·1	89·0				
		1·4	T4	Roof	8″ × 6″ × 35 lb. I	(8) (12)
		1·3	R42			
		1·3	R82			
		·4	O.W. & C.	9′ 0″		
	4·4	4·4				
		20·8	D71	4th	As below	
		13·3	D42			
		13·3	D82			
	48·2	·8	O.W. & C.	11′ 0″		
	52·6	48·2				
		20·8	C71	3rd	8″ × 6″ × 35 lb. I	
		14·1	C42		+	
		14·1	C82		2/9″ × ½″ PLS	
	50·0	1·0	O.W. & C.	11′ 0″		
	102·6	50·0				×
		20·8	B71	2nd	10″ × 8″ × 55 lb. I	
		14·1	B42		+	
		14·1	B82		2/10″ × ⅝″ PLS	
	50·0	1·0	O.W. & C.	11′ 0″		
	152·6	50·0				

Sketch.	Calculations.	Allowable Stresses. T/in.².	$\frac{L}{r}$	Totals. Tons	Reactions. Amount Tons.	Reactions. Beam Ref.	Portion and Length.	Section.	Stanchion Ref.
	As Stan. (2)			 50·2	20·8 14·1 14·1 1·2	A71 A42 A82 O.W. & C.	1st 13′ 0″	10″ × 8″ × 55 lb. I + 2/10″ × 1″ PLs	(8) (12) *cont.*
				202·8	50·2				
 FIG. 242.					2·8 TIE ·4	MR52 MR91 O.W. & C.	MR 8′ 0″	As below	(9)
				3·2	3·2				
				   14·3	8·1 1·3 4·4 ·5	R52 R92 R91 O.W. & C.	Roof 9′ 0″	8″ × 6″ × 35 lb. I	
				17·5	14·3				×
				   40·3	4·8 5·3 29·4 ·8	D52 D92 D91 O.W. & C.	4th 11′ 0″	As below	
				57·8	40·3				

								(9) *cont*
Length C–B. $f_C = \frac{94 \cdot 1}{18 \cdot 3} = 5 \cdot 14$ $f_{Bxx} = \frac{21 \cdot 9 \times 4 \cdot 5}{2 \times 57 \cdot 2} = \cdot 86$ $f_{Byy} = \frac{1 \cdot 6 \times 2}{2 \times 15 \cdot 2} = \cdot 10$ Total $= 6 \cdot 10^{T}/\text{in.}^2$	$F_1 = 6 \cdot 10$	$\frac{\cdot 75 \times 132}{1 \cdot 82}$ $= 55$	36·3 94·1	5·9 7·5 21·9 1·0 36·3	C52 C92 C91 O.W. & C.	3rd 11′ 0″	8″ × 6″ × 35 lb. I + 2/8″ × ½″ PLS	
Length B–A. $f_C = \frac{130 \cdot 4}{26 \cdot 2} = 4 \cdot 98$ $f_{Bxx} = \frac{21 \cdot 9 \times 5 \cdot 5}{2 \times 103} = \cdot 59$ $f_{Byy} = \frac{1 \cdot 6 \times 2}{2 \times 27 \cdot 6} = \cdot 06$ Total $= 5 \cdot 63^{T}/\text{in.}^2$	$F_1 = 6 \cdot 54$	$\frac{\cdot 75 \times 132}{2 \cdot 30}$ $= 43$	36·3 130·4	5·9 7·5 21·9 1·0 36·3	B52 B92 B91 O.W. & C.	2nd 11′ 0″	10″ × 8″ × 55 lb. I + 2/10″ × ½″ PLS	
Length A–G. $f_C = \frac{166 \cdot 4}{33 \cdot 7} = 4 \cdot 94$ $f_{Bxx} = \frac{22 \cdot 4 \times 5 \cdot 88}{2 \times 137} = \cdot 48$ $f_{Byy} = \frac{\cdot 6 \times 2}{2 \times 40 \cdot 1} = \cdot 02$ Total $= 5 \cdot 44^{T}/\text{in.}^2$	$F_1 = 6 \cdot 37$	$\frac{\cdot 75 \times 156}{2 \cdot 44}$ $= 48$	36·0 166·4	5·9 6·5 22·4 1·2 36·0	A52 A92 A91 O.W. & C.	1st 13′ 0″	10″ × 8″ × 55 lb. I + 2/10″ × ⅞″ PLS	

Sketch.	Calculations.	Allowable Stresses. T/in.².	L/r	Totals. Tons.	Reactions. Amount Tons.	Reactions. Beam Ref.	Portion and Length.	Section.	Stanchion Ref.
					36·1	D91	4th	10″×8″×55 lb. I	(10)
Fig. 243.					24·7	D101	11′ 0″	+	
					17·1	D62		2/10″×⅜″ Pls	
					17·1	D102			
					1·0	O.W. & C.			
				96·0	96·0				
					28·5	C91	3rd	10″×8″×55 lb. I	
					20·8	C101	11′ 0″	+	
					17·1	C62		2/10″×¾″ Pls	
					17·1	C102			
				84·7	1·2	O.W. & C.			
				180·7	84·7				
					28·5	B91	2nd	10″×8″×55 lb. I	
					20·8	B101	11′ 0″	+	
					17·1	B62		2/12″×1⅛″ Pls	
					17·1	B102			
				84·9	1·4	O.W. & C.			
				265·6	84·9				
					28·9	A91	1st	10″×8″×55 lb. I	
	As Stan. (6)				20·8	A101	13′ 0″	+	
					17·1	A62		2/12″×1⅝″ Pls	
					17·1	A102			
				85·7	1·8	O.W. & C.			
				351·3	85·7				

Foundation Calculations

Allowable pressures:

1. On soil $= 3$ tons per sq. ft.
2. On concrete $= 30$ tons per sq. ft.

Slab bases will be used throughout in the example, the L.C.C. By-laws (1938) formulæ for determining the thickness of slabs being as follow:

$$t = \sqrt{\frac{3W}{4f} \cdot \frac{(B - b)}{D}} \quad \text{or} \quad \sqrt{\frac{3W}{4f} \cdot \frac{(D - d)}{B}}$$

whichever is the greater.

(For explanation of terms see Chapter XII.)

Where the stanchion base loads are approximately the same, they will be grouped together.

Stanchions 1, 4 *and* 16.

Load $=$ 117 tons.

Weight of concrete base $=$ (say) 12 tons (10% of stanchion load).

Total $=$ 129 tons.

Concrete area required $= \frac{129}{3} = 43$ sq. ft.

Make base $6' \, 9'' \times 6' \, 9'' = 45{\cdot}6$ sq. ft.

Steel area required $= \frac{117}{30} = 3{\cdot}9$ sq. ft.

Make slab base $2' \times 2' = 4{\cdot}0$ sq. ft.

$$t = \sqrt{\frac{3 \times 117}{4 \times 9} \times \frac{(24 - 8)}{24}} = \sqrt{\frac{3 \times 117 \times 16}{4 \times 9 \times 24}}$$

$$= \sqrt{6{\cdot}5} = 2{\cdot}55'', \text{ say } 2\tfrac{3}{4}''.$$

Maximum overhang $= \frac{6' \, 9'' - 2'}{2} = 2' \, 4\tfrac{1}{2}''.$

Depth of concrete base (if unreinforced) should be about 6″ more than the maximum overhang of the concrete beyond the slab base.

∴ Make concrete base 3′ deep.

Concrete base $= 6' \, 9'' \times 6' \, 9'' \times 3'$ deep.

Slab base $= 2' \times 2' \times 2\tfrac{3}{4}''$.

Stanchion 13.

$$\begin{aligned} \text{Load} &= 95 \text{ tons.} \\ \text{Weight of concrete base} &= 10 \text{ tons.} \\ \text{Total} &= 105 \text{ tons.} \end{aligned}$$

$$\text{Concrete area required} = \frac{105}{3} = 35 \text{ sq. ft.}$$

$$\text{Make base } 6' \times 6' = 36 \text{ sq. ft.}$$

$$\text{Steel area required} = \frac{95}{30} = 3{\cdot}17 \text{ sq. ft.}$$

$$\text{Make slab base } 1'\ 10\tfrac{1}{2}'' \times 1'\ 10\tfrac{1}{2}'' = 3{\cdot}52 \text{ sq. ft.}$$

$$t = \sqrt{\frac{3 \times 95}{4 \times 9} \times \frac{(22{\cdot}5 - 8)}{22{\cdot}5}} = \sqrt{\frac{3 \times 95 \times 14{\cdot}5}{4 \times 9 \times 22{\cdot}5}}$$

$$= \sqrt{5{\cdot}1} = 2{\cdot}26'', \text{ say } 2\tfrac{1}{2}''.$$

$$\text{Maximum overhang} = \frac{6' - 1'\ 10\tfrac{1}{2}''}{2} = 2'\ 0\tfrac{3}{4}''.$$

$$\begin{aligned} \text{Make depth} &= 2'\ 6''. \\ \text{Concrete base} &= 6' \times 6' \times 2'\ 6'' \text{ deep.} \\ \text{Slab base} &= 1'\ 10\tfrac{1}{2}'' \times 1'\ 10\tfrac{1}{2}'' \times 2\tfrac{1}{2}''. \end{aligned}$$

Stanchions 2, 3, 8, 12, 14 *and* 15.

$$\begin{aligned} \text{Maximum load} &= 207 \text{ tons.} \\ \text{Weight of concrete base} &= 21 \text{ tons.} \\ \text{Total} &= 228 \text{ tons.} \end{aligned}$$

$$\text{Concrete area required} = \frac{228}{3} = 76 \text{ sq. ft.}$$

$$\text{Make base } 8'\ 9'' \times 8'\ 9'' = 76{\cdot}8 \text{ sq. ft.}$$

$$\text{Steel area required} = \frac{207}{30} = 6{\cdot}9 \text{ sq. ft.}$$

$$\text{Make slab base } 2'\ 7\tfrac{1}{2}'' \times 2'\ 7\tfrac{1}{2}'' = 6{\cdot}9 \text{ sq. ft.}$$

$$t = \sqrt{\frac{3 \times 207}{4 \times 9} \times \frac{(31{\cdot}5 - 10)}{31{\cdot}5}} = \sqrt{\frac{3 \times 207 \times 21{\cdot}5}{4 \times 9 \times 31{\cdot}5}}$$

$$= \sqrt{11{\cdot}8} = 3{\cdot}43'', \text{ say } 3\tfrac{1}{2}''.$$

$$\text{Maximum overhang} = \frac{8'\ 9'' - 2'\ 7\tfrac{1}{2}''}{2} = 3'\ 0\tfrac{3}{4}''.$$

$$\begin{aligned} \text{Make depth} &= 3'\ 6''. \\ \text{Concrete base} &= 8'\ 9'' \times 8'\ 9'' \times 3'\ 6'' \text{ deep.} \\ \text{Slab base} &= 2'\ 7\tfrac{1}{2}'' \times 2'\ 7\tfrac{1}{2}'' \times 3\tfrac{1}{2}''. \end{aligned}$$

Stanchion 5.

Load = 193 tons.
Weight of concrete base = 20 tons.
Total = 213 tons.

$$\text{Concrete area required} = \frac{213}{3} = 71 \text{ sq. ft.}$$

Make base 8′ 6″ × 8′ 6″ = 72·3 sq. ft.

$$\text{Steel area required} = \frac{193}{30} = 6{\cdot}43 \text{ sq. ft.}$$

Make slab base 2′ $7\frac{1}{2}$″ × 2′ $7\frac{1}{2}$″ = 6·9 sq. ft.

$$= \sqrt{\frac{3 \times 193}{4 \times 9} \times \frac{(31{\cdot}5 - 10)}{31{\cdot}5}} = \sqrt{\frac{3 \times 193 \times 21{\cdot}5}{4 \times 9 \times 31{\cdot}5}}$$

$$= \sqrt{10{\cdot}1} = 3{\cdot}18'', \text{ say } 3\tfrac{1}{4}''.$$

$$\text{Maximum overhang} = \frac{8'\,6'' - 2'\,7\frac{1}{2}''}{2} = 2'\,11\tfrac{1}{4}''.$$

Make depth = 3′ 6″.
Concrete base = 8′ 6″ × 8′ 6″ × 3′ 6″ deep.
Steel base = 2′ $7\frac{1}{2}$″ × 2′ $7\frac{1}{2}$″ × $3\frac{1}{4}$″.

Stanchions 6, 7, 10 *and* 11.

Maximum load = 359 tons.
Weight of concrete base = 36 tons.
Total = 395 tons.

$$\text{Concrete area required} = \frac{395}{3} = 131{\cdot}7 \text{ sq. ft.}$$

Make base 11′ 6″ × 11′ 6″ = 132·5 sq. ft.

$$\text{Steel area required} = \frac{359}{30} = 11{\cdot}97 \text{ sq. ft.}$$

Make slab base 3′ 6″ × 3′ 6″ = 12·25 sq. ft.

$$t = \sqrt{\frac{3 \times 359}{4 \times 9} \times \frac{(42 - 12)}{42}} = \sqrt{\frac{3 \times 359 \times 30}{4 \times 9 \times 42}}$$

$$= \sqrt{21{\cdot}4} = 4{\cdot}62'', \text{ say } 4\tfrac{3}{4}''.$$

$$\text{Maximum overhang} = \frac{11'\,6'' - 3'\,6''}{2} = 4'.$$

Make depth = 4′ 6″.
Concrete base = 11′ 6″ × 11′ 6″ × 4′ 6″ deep.
Slab base = 3′ 6″ × 3′ 6″ × $4\frac{3}{4}$″.

Stanchion 9.

Load = 167 tons.
Weight of concrete base = 17 tons.
Total = 184 tons.

$$\text{Concrete area required} = \frac{184}{3} = 61 \cdot 3 \text{ sq. ft.}$$

Make base 8′ × 8′ = 64 sq. ft.

$$\text{Steel area required} = \frac{167}{30} = 5 \cdot 57 \text{ sq. ft.}$$

Make slab base = 2′ 4½″ × 2′ 4½″ = 5·64 sq. ft.

$$t = \sqrt{\frac{3 \times 167}{4 \times 9} \times \frac{(28 \cdot 5 - 10)}{28 \cdot 5}} = \sqrt{\frac{3 \times 167 \times 18 \cdot 5}{4 \times 9 \times 28 \cdot 5}}$$

$$= \sqrt{9 \cdot 04} = 3 \cdot 01'', \text{ say } 3''.$$

$$\text{Maximum overhang} = \frac{8' - 2'\ 4\tfrac{1}{2}''}{2} = 2'\ 9\tfrac{3}{4}''.$$

Make depth = 3′ 3″.
Concrete base = 8′ × 8′ × 3′ 3″ deep.
Slab base = 2′ 4½″ × 2′ 4½″ × 3″.

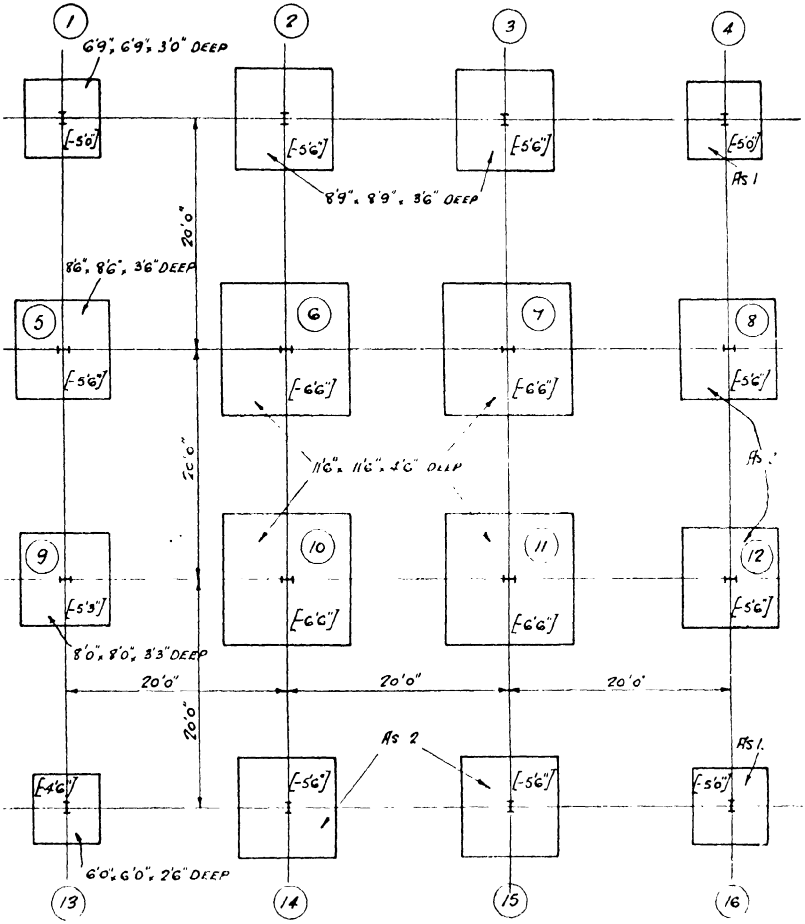


PLATE XI.—FOUNDATION PLAN.

Notes.

1. Figures thus [— 5′ 6″] give levels of undersides of bases below ground floor.
2. Concrete mixture: 1 cwt. Portland cement, $2\frac{1}{2}$ cu. ft. fine aggregate passing $\frac{3}{16}$″ sieve, 5 cu. ft. coarse aggregate passing 2″ sieve.

APPENDIX I

SELECTED LIST OF BRITISH STANDARD SPECIFICATIONS

COMPLETE and Sectional (e.g. 'Building') lists of B.S.S. may be obtained from the British Standards Institution, Publications Dept., 28 Victoria Street, London, S.W.1.

The price of each specification is 2*s.* net (2*s.* 2*d.* post free). Special discounts are arranged for Educational Authorities.

The B.S.S. marked by an asterisk are referred to in the text of the book.

B.S.S. No.

*449–1937.—Use of Structural Steel in Building. [Add. 1940.]

* 4–1932.—Channels and Beams for Structural Purposes. Dimensions and Properties of. [Add. April 1934.] [Partly superseding No. 6—1924.]

* 4A–1934.—Equal Angles, Unequal Angles and Tee Bars for Structural Purposes. Dimensions and Properties of. [Partly superseding No. 6—1924.]

* 15–1936.—Steel for Bridges, etc., and General Building Construction, Structural. [Add. 1938 & 1941.]

12–1940.—Ordinary Portland and Rapid-hardening Portland Cement.

308–1927.—Engineering Drawing Office Practice.

373–1938.—Testing Small Clear Specimens of Timber, Methods of.

405–1930.—Expanded Metal (Steel).

476–1932.—Fire-resistance, Incombustibility and Non-inflammability of Building Materials and Structures, Definitions for. [Including Methods of Test.]

492–1933.—Precast Concrete Partition Slabs (Solid). [Add. 1937.]

499–1933.—Welding and Cutting, Nomenclature, Definitions and Symbols for.

*538–1940.—Metal Arc Welding in Mild Steel as Applied to General Building Construction. [Add. 1940.]

*548–1934.—High Tensile Structural Steel for Bridges, etc., and General Building Construction. [Add. 1936 & 1938.]

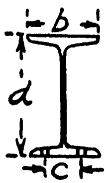
*275–1927.—Dimension of Rivets. [Add. 1941.]

*153–Parts 1 and 2–1933.—Girder Bridges. Materials [Add. 1941] and Workmanship.

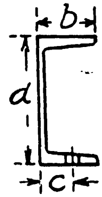
648–1935.—Unit Weights of Building Materials, Schedule of.

'Add.' signifies that an Addendum or Corrigendum is issued with this specification.

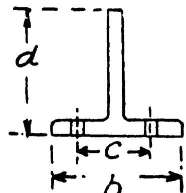
RIVET HOLE POSITIONS IN STANDARD SECTIONS



B.S B



B.S.C



B S.T

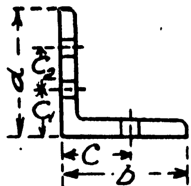
Size d × b ins.	c ins.	Size d × b ins.	c ins.
24 × 7½	4·5	10 × 8	4·75
22 × 7	4·0	10 × 6	3·5
20 × 7½	4·5	10 × 5	2·75
20 × 6½	3·75	10 × 4½	2·5
18 × 8	4·75	9 × 7	4·0
18 × 7	4·0	9 × 4	2·25
18 × 6	3·5	8 × 6	3·5
16 × 8	4·75	8 × 5	2·75
16 × 6_H	3·5	8 × 4	2·25
16 × 6_L	3·5	7 × 4	2·25
15 × 6	3·5	6 × 5	2·75
15 × 5	2·75	6 × 4½	2·5
14 × 8	4·75	6 × 3	1·5
14 × 6_H	3·5	5 × 4½	2·5
14 × 6_L	3·5	5 × 3	1·5
13 × 5	2·75	4¾ × 1¾	·875
12 × 8	4·75	4 × 3	1·5
12 × 6_H	3·5	4 × 1¾	·875
12 × 6_L	3·5	3 × 3	1·5
12 × 5	2·75	3 × 1½	·75

Size d × b ins.	c ins.
17 × 4	2·25
15 × 4	2·25
13 × 4	2·25
12 × 4	2·25
12 × 3½	2·0
11 × 3½	2·0
10 × 3½	2·0
10 × 3	1·75
9 × 3½	2·0
9 × 3	1·75
8 × 3½	2·0
8 × 3	1·75
7 × 3½	2·0
7 × 3	1·75
6 × 3½	2·0
6 × 3	1·75
5 × 2½	1·375
4 × 2	1·125
3 × 1½	·875

There are several standard sections for each value of $d \times b$, obtained by increasing web thickness.

Size b × d × t ins.	c ins.
6 × 6 × ⅝	3·5
6 × 6 × ½	3·5
6 × 4 × ⅝	3·5
6 × 4 × ½	3·5
6 × 3 × ½	3·5
6 × 3 × ⅜	3·5
5 × 4 × ½	2·75
5 × 4 × ⅜	2·75
5 × 3 × ½	2·75
5 × 3 × ⅜	2·75
4 × 4 × ½	2·25
4 × 4 × ⅜	2·25
4 × 3 × ½	2·25
4 × 3 × ⅜	2·25
3 × 3 × ⅜	1·5
2½ × 2½ × ⅜	1·375
2½ × 2½ × ¼	1·375
2 × 2 × ¼	1·125
1½ × 1½ × ¼	·75

For rivet positions in depth of tee use values given for same depth of angle.

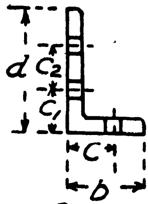


B.S.E.A.

Size d × b × t ins.	C ins.	C_1 ins.	C_2 ins.
8 × 8 × 7/8	4·5	3	3
8 × 8 × 3/4	4·5	3	3
8 × 8 × 5/8	4·5	3	3
7 × 7 × 3/4	4·0	2·5	3
7 × 7 × 5/8	4·0	2·5	3
7 × 7 × 1/2	4·0	2·5	3
6 × 6 × 3/4	3·5	2·25	2·25
6 × 6 × 5/8	3·5	2·25	2·25
6 × 6 × 1/2	3·5	2·25	2·25
6 × 6 × 3/8	3·5	2·25	2·25
5 × 5 × 5/8	3·0	2·0	1·75
5 × 5 × 1/2	3·0	2·0	1·75
5 × 5 × 3/8	3·0	2·0	1·75
4½ × 4½ × 5/8	2·5	—	—
4½ × 4½ × 1/2	2·5	—	—
4½ × 4½ × 3/8	2·5	—	—
4 × 4 × 5/8	2·25	—	—
4 × 4 × 1/2	2·25	—	—
4 × 4 × 3/8	2·25	—	—
3½ × 3½ × 1/2	2·0	—	—
3½ × 3½ × 3/8	2·0	—	—

Size d × b × t ins.	C ins.
3½ × 3½ × 1/4	2·0
3 × 3 × 1/2	1·75
3 × 3 × 3/8	1·75
3 × 3 × 1/4	1·75
2½ × 2½ × 3/8	1·375
2½ × 2½ × 5/16	1·375
2½ × 2½ × 1/4	1·375
2¼ × 2¼ × 5/16	1·25
2¼ × 2¼ × 1/4	1·25
2¼ × 2¼ × 3/16	1·25
2 × 2 × 5/16	1·125
2 × 2 × 1/4	1·125
2 × 2 × 3/16	1·125
1¾ × 1¾ × 1/4	·875
1¾ × 1¾ × 3/16	·875
1½ × 1½ × 1/4	·875
1½ × 1½ × 3/16	·875

The B.S.S. gives 4 smaller standard sections, and a number of other sections not possessing the standard *profile*.



B.S.U.A.

Size $d \times b \times t$ ins.	C ins.	C_1 ins.	C_2 ins.	Size $d \times b \times t$ ins.	C ins.	C_1 ins.
$9 \times 4 \times \frac{3}{4}$	2·25	3·0	4·0	$4 \times 3\frac{1}{2} \times \frac{1}{2}$	2·0	2·25
$9 \times 4 \times \frac{1}{2}$	2·25	3·0	4·0	$4 \times 3\frac{1}{2} \times \frac{3}{8}$	2·0	2·25
$8 \times 6 \times \frac{3}{4}$	3·5	3·0	3·0	$4 \times 3\frac{1}{2} \times \frac{5}{16}$	2·0	2·25
$8 \times 6 \times \frac{1}{2}$	3·5	3·0	3·0	$4 \times 3 \times \frac{1}{2}$	1·75	2·25
$8 \times 4 \times \frac{5}{8}$	2·25	3·0	3·0	$4 \times 3 \times \frac{3}{8}$	1·75	2·25
$8 \times 4 \times \frac{1}{2}$	2·25	3·0	3·0	$4 \times 3 \times \frac{5}{16}$	1·75	2·25
$8 \times 3\frac{1}{2} \times \frac{1}{2}$	2·0	3·0	3·0	$4 \times 2\frac{1}{2} \times \frac{3}{8}$	1·375	2·25
$8 \times 3\frac{1}{2} \times \frac{7}{16}$	2·0	3·0	3·0	$4 \times 2\frac{1}{2} \times \frac{5}{16}$	1·375	2·25
$7 \times 4 \times \frac{5}{8}$	2·25	2·5	3·0	$4 \times 2\frac{1}{2} \times \frac{1}{4}$	1·375	2·25
$7 \times 4 \times \frac{1}{2}$	2·25	2·5	3·0	$3\frac{1}{2} \times 3 \times \frac{1}{2}$	1·75	2·0
$7 \times 3\frac{1}{2} \times \frac{1}{2}$	2·0	2·5	3·0	$3\frac{1}{2} \times 3 \times \frac{3}{8}$	1·75	2·0
$7 \times 3\frac{1}{2} \times \frac{7}{16}$	2·0	2·5	3·0	$3\frac{1}{2} \times 3 \times \frac{5}{16}$	1·75	2·0
$6 \times 4 \times \frac{5}{8}$	2·25	2·25	2·25	$3\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{8}$	1·375	2·0
$6 \times 4 \times \frac{1}{2}$	2·25	2·25	2·25	$3\frac{1}{2} \times 2\frac{1}{2} \times \frac{5}{16}$	1·375	2·0
$6 \times 4 \times \frac{3}{8}$	2·25	2·25	2·25	$3\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$	1·375	2·0
$6 \times 3\frac{1}{2} \times \frac{1}{2}$	2·0	2·25	2·25	$3 \times 2\frac{1}{2} \times \frac{3}{8}$	1·375	1·75
$6 \times 3\frac{1}{2} \times \frac{3}{8}$	2·0	2·25	2·25	$3 \times 2\frac{1}{2} \times \frac{5}{16}$	1·375	1·75
$6 \times 3 \times \frac{1}{2}$	1·75	2·25	2·25	$3 \times 2\frac{1}{2} \times \frac{1}{4}$	1·375	1·75
$6 \times 3 \times \frac{3}{8}$	1·75	2·25	2·25	$3 \times 2 \times \frac{3}{8}$	1·125	1·75
$5 \times 4 \times \frac{1}{2}$	2·25	2·0	1·75	$3 \times 2 \times \frac{5}{16}$	1·125	1·75
$5 \times 4 \times \frac{3}{8}$	2·25	2·0	1·75	$3 \times 2 \times \frac{1}{4}$	1·125	1·75
$5 \times 3\frac{1}{2} \times \frac{1}{2}$	2·0	2·0	1·75	$2\frac{1}{2} \times 2 \times \frac{5}{16}$	1·125	1·375
$5 \times 3\frac{1}{2} \times \frac{3}{8}$	2·0	2·0	1·75	$2\frac{1}{2} \times 2 \times \frac{1}{4}$	1·125	1·375
$5 \times 3 \times \frac{1}{2}$	1·75	2·0	1·75	$2\frac{1}{2} \times 2 \times \frac{3}{16}$	1·125	1·375
$5 \times 3 \times \frac{3}{8}$	1·75	2·0	1·75	$2\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{4}$	·875	1·375
$5 \times 3 \times \frac{5}{16}$	1·75	2·0	1·75	$2\frac{1}{2} \times 1\frac{1}{2} \times \frac{3}{16}$	·875	1·375
$4\frac{1}{2} \times 3 \times \frac{1}{2}$	1·75	2·5	—	$2 \times 1\frac{1}{2} \times \frac{1}{4}$	·875	1·125
$4\frac{1}{2} \times 3 \times \frac{3}{8}$	1·75	2·5	—	$2 \times 1\frac{1}{2} \times \frac{3}{16}$	·875	1·125
$4\frac{1}{2} \times 3 \times \frac{5}{16}$	1·75	2·5	—			

The B.S.S. gives other angle thicknesses, but the table contains the standard sections.

GEOMETRICAL PROPERTIES OF A PARABOLA

THE properties given below will be found useful in bending moment and deflection problems.

Geometrical Construction of a Parabola

Fig. 244 shows a simple construction, given the base and the maximum central ordinate (as in a B.M. diagram for a simply supported beam with uniformly distributed load).

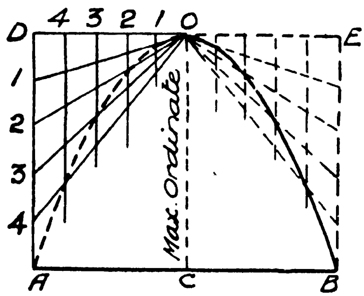


FIG. 244.

The rectangle ADEB is drawn of height CO. OD is divided into a chosen number of equal parts (say 5), and DA is also divided into the same number of equal parts. The point of intersection of the vertical through 1 and the radial line O1 will be a true point on the parabola. Similarly the intersection points of the other corresponding verticals and radials will give further points through which to draw in the required curve. The right half of the diagram may be drawn in, if desired, by the principle of symmetry.

Area of a Parabola

Area of parabola AOB (Fig. 244)

$$= \tfrac{2}{3} \text{ of the base} \times \text{height}$$

$$= \tfrac{2}{3}\, AB \times CO.$$

Area of semi-parabola AOC

$= \frac{2}{3}$ of the base $\times$ height

$= \frac{2}{3}$ AC $\times$ CO.

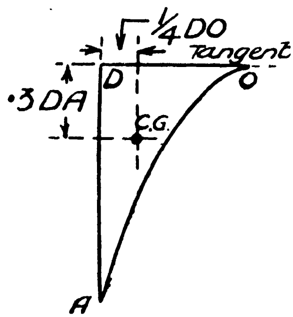


FIG. 245.

Area of parabolic diagram ADO (Fig. 245)

$= \frac{1}{3}$ of the base $\times$ height

$= \frac{1}{3}$ OD $\times$ DA.

Position of Centre of Gravity

In the parabolic diagram AOC (Fig. 246), the C.G. is at a point $\frac{2}{5}$ of CO, measured from CA, and $\frac{3}{8}$ of CA, measured from CO.

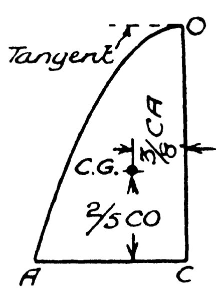


FIG. 246

In the case of diagram AOD (Fig. 245), the C.G. is $\frac{1}{4}$ DO from DA, and ·3DA from DO

ANSWERS TO NUMERICAL QUESTIONS

Exercises 1. Page 14

(1) 8 tons/in.^{2}.
(2) 3.
(3) 4 ins.
(4) 4 ft. square.
(5) (*a*) 5·75 tons. (*b*) 15 tons.
(6) 5·94 tons/in.^{2}. Necessary thickness = $\frac{7}{16}$ ins.
(7) Strain = ·0002. Contraction in length = ·0624 ins.
(8) 13,000 tons/in.^{2}.
(10) ·0282 ins.
(11) 10·4 tons.
(12) 3·9 tons/in.^{2}.

Exercises 2. Page 27

(1) Ult. stress = 32 tons/in.^{2}. Working stress = 6·4 tons/in.^{2}.
(2) Y.P. stress = 16·86 tons/in.^{2}. Ult. (commercial) stress = 29·16 tons/in.^{2}. Percentage elongation = 28·75. Percentage contraction in area = 54·3.
(3) Load at Y.P. = 10·08 tons. Maximum load = 16·8 tons. Elongation = 2·24 ins. Diameter at fracture = ·547 ins.
(4) Working stress = 8 tons/in.^{2}. Thickness = ·75 ins.
(6) 6·63 tons.
(7) 4.
(8) Stress exceeds elastic limit.

Exercises 4. Page 63

(1) 2·65 tons; 7·22 tons.
(2) 4·42 tons.
(3) Section 1: 16·76 tons. Section 2: 16·17 tons.
Rivet strength = 18·55 tons.
Safe load (L) = 16·17 tons.
Percentage efficiency = 80·8.
(4) 10·6 tons.
(5) No. of rivets = 5.
Suitable plate width = $5\frac{1}{4}$ ins.
Cover thickness = $\frac{5}{16}$ ins.
Percentage efficiency = 86·6.
(6) 3·61 tons.

EXERCISES 5. PAGE 84

(2) Centre: 5 cwts. ft. (negative) and 2 cwts.
Maximum: 18 cwts. ft. (negative) and 6 cwts.
(3) Maximum values: 2 tons ft. (negative) and 1 ton.
Given section: 1·125 tons ft. (negative) and ·75 tons.
(4) Given section: 24 cwts. ft. and 4 cwts.
B.M.s: 16 cwts. ft.; 28 cwts. ft.; 12 cwts. ft.
(5) (*a*) 1200 lb. ft.; (*b*) 22·1 tons ft.
(6) 6′ – 8″.
(7) From left end of beam (in cwts. ft.):
0; 40 (neg.); 25; 35; 40 (neg.); 0.
(8) Left end support: 144 lb. ft. (neg.).
Right end support: 225 lb. ft. (neg.).
Maximum B.M. (in central portion) = 576 lb. ft. (treated as simple beam between the supports for purpose of diagram construction).

EXERCISES 6. PAGE 112

(1) 26·8 tons.
(2) 12″ × 8″ × 65 lb. B.S.B.
(3) 8 tons/in.^{2}.
(4) 20·3 ins.^{3}.
(5) 9·34 tons.
(6) Timber beams 2 ins. × 7 ins.
Steel beams 7″ × 4″ × 16 lb. B.S.B.

EXERCISES 7. PAGE 125

(1) I_{XX} = 405·66 ins.^{4}; I_{YY} = 36·1 ins.^{4}.
(2) 3·41 ins.
(4) 35·54 ins.^{4}.
(5) $I_{maximum}$ = 809 ins.^{4}; $Z_{maximum}$ = 134·8 ins.^{3}.
(6) 40·2 tons.
(7) $I_{maximum}$ = 363 ins.^{4}; $Z_{maximum}$ = 66 ins.^{3}.
(8) 86·4 tons.

EXERCISES 8. PAGE 146

(1) ·393 ins.
(2) (*a*) 8 tons/in.^{2}; (*b*) ·105 ins.
(3) ·0033 radians.
(4) 1,000,000 lb./in.^{2}.
(5) 28 ft.
(6) ·44 ins.
(8) ·53 ins.

Exercises 9. Page 163

(1) S.F. diagram crosses base line at 5 ft. from left end.
(2) At 9 ft. from left end. B.M. max. = 209 c.f.
(3) Loads from left: 2 tons; 6 tons; 5 tons.
(4) 45·56 tons ft. at 5·25 ft. from left end.
(5) $2\frac{3}{4}$ tons/in.^{2}. Maximum = 3 tons/in.^{2}.
(6) ·155 tons/in.^{2}; 2·48 tons/in.^{2}; 2·99 tons/in.^{2}.
(7) Two rows 4″ pitch in each leg of angles.

Exercises 10. Page 186

(1) (*a*) 24·4 ins.^{3}; (*b*) 16·25 ins.^{3}.
B.M.s maximum (*a*) 195 tons ins.; (*b*) 130 tons ins.
Fixing moment (*b*) 130 tons ins.
(2) 9·2 tons ft.; 7·94 tons/in.^{2}.
(3) Left end: 28 tons ft. Right end: 14 tons ft.
(5) 63 tons ft.
Reactions (from left): 6·75 tons; 38·75 tons; 14·5 tons.
(7) B.M.s in cwts. ft. (from left):
o; 15·48 (neg.); 14·34 (neg.); o.
Reactions in cwts. (from left):
3·84; 32·35; 28·59; 1·22.
(8) Fixing moments: (left end) 15·75 tons ft.
(right end) 11·25 tons ft.
Reactions: (left end) 7·375 tons.
(right end) 4·625 tons.

Exercises 11. Page 222

(2) (*a*) ·866 in.; (*b*) 1·5 ins.; (*c*) 1·38 ins.; (*d*) 2·47 ins.
(3) 67·9 tons.
(5) 94·5 tons.
(6) Direct: 1·481 tons/in.^{2}.
Bending: 1·54 tons/in.^{2}.
Equivalent concentric load = 48·9 tons. (48·48 by coeff.)
Maximum stresses: 3·02 tons/in.^{2} and ·06 tons/in.^{2} (tension).

Exercises 12. Page 247

(1) 3 ins.
(2) Upper tier: 3 No. 12″ × 5″ × 32 lb. B.S.B.s.
Lower tier: 14 No. 6″ × 3″ × 12 lb. B.S.B.s.
(3) 23·3 lb./in.^{2}.
(4) 5 tons/sq. ft.
(5) $1\frac{3}{4}$ ins.
(6) 3/20″ × $6\frac{1}{2}$″ × 65 lb. B.S.B.s will be suitable.

EXERCISES 13. PAGE 258

(3) Maximum stress = 7·4 tons/in.^{2}. Load = 11·8 tons.
(4) Working stress = 8 tons/in.^{2}.
Safe load = 9·1 tons.
(5) 9·8 tons.
(6) 364 lb. per sq. ft.

EXERCISES 14. PAGE 274

(2) (i) 6 tons/in.^{2}; (ii) 5 tons/in.^{2}.
(3) (i) ·219″.
(ii) 2·45 in.^{2}.
(iii) 5″.
(4) (i) ·875 tons.
(ii) 1·575 tons.
(iii) 5 tons.
(5) 23·6 tons.
(6) 24 tons.
(7) Safe end reaction = 8·31 tons.
Safe uniformly distributed load = 16·62 tons.
(8) Max. horizontal shear = 2·18 tons per inch.

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